No. 24-1822

United States Court of Appeals for the Federal Circuit

CONSTELLATION DESIGNS, LLC

Plaintiff-Appellee,

v.

LG ELECTRONICS INC., LG ELECTRONICS U.S.A., INC., LG ELECTRONICS ALABAMA, INC.

Defendants-Appellants

APPEAL FROM THE UNITED STATES DISTRICT COURT FOR THE EASTERN DISTRICT OF TEXAS IN CASE NO. 2:21-CV-00448-JRG

NON-CONFIDENTIAL OPENING BRIEF OF DEFENDANTS-APPELLANTS

Michael J. McKeon
Christian A. Chu
R. Andrew Schwentker
Michael J. Ballanco
Benjamin J. Christoff
FISH & RICHARDSON P.C.
1000 Maine Avenue, S.W.
Suite 1000
Washington, DC 20024
Tel: (202) 783-5070

Ashley A. Bolt
FISH & RICHARDSON P.C.
1180 Peachtree Street NE
21st Floor
Atlanta, GA 30309
Tel: (404) 492-5008

August 19, 2024

Attorneys for Defendants-Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc.

EXEMPLAR CLAIMS AT ISSUE ON APPEAL

Claim 17 of U.S. Patent No. 8,842,761 (Appx141)

17.A digital communication system, comprising:

a receiver configured to receive signals transmitted via a communication channel using a QAM symbol constellation;

wherein the receiver, comprises:

a demodulator configured to demodulate the signal received via the communication channel;

a demapper configured to estimate likelihoods of symbols in a QAM symbol constellation from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper using an LDPC code; and

wherein the QAM symbol constellation is a geometrically spaced symbol constellation optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a QAM signal constellation that maximizes d_{min}.

Claim 1 of U.S. Patent No. 10,693,700 (Appx197)

1. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a multidimensional symbol constellation selected from a plurality of multidimensional symbol constellations; and a decoder, coupled to the demapper, capable of using the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code;

wherein the plurality of multidimensional symbol constellations comprises a plurality of different non-uniform multidimensional symbol constellations having the same number of constellation points, where the constellation points are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

wherein the receiver is capable of selecting an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs, where each of the plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs.

Claims 21, 23 of U.S. Patent No. 11,019,509 (Appx260)

21. A communication system, comprising a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver uses a symbol constellation to transform the received signals into received bits, and the symbol constellation includes constellation points at a plurality of unique point locations, where:

the plurality of unique point locations are unequally spaced;

the constellation points each have a location and a different label; and

the locations of at least two of the constellation points are the same.

23. The communication system of claim 21, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations;

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where unequally spaced symbol constellations of the first type include at least two constellation points having identical locations and different labels;

the receiver selects an LDPC code rate and the unequally spaced symbol constellation as a pair from a plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs; and

each of the plurality of unequally spaced symbol constellations is only included in one of the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs.

Claim 24 of U.S. Patent No. 11,018,922 (Appx447)

24. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a non-uniform quadrature amplitude modulation 1024-point symbol constellation (NU-QAM 1024); and

a decoder, coupled to the demapper, capable of using likelihoods determined by the demapper to provide a sequence of received bits based upon a Low Density Parity Check (LDPC) code;

wherein the NU-QAM 1024 constellation comprises an in-phase component and a quadrature component, where each component comprises 32 levels of amplitude such that the amplitudes scaled by a scaling factor are within 0.55 from the following set of amplitudes:

−38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

CERTIFICATE OF INTEREST

Counsel for Defendants-Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc. (collectively, "LG") certifies the following:

1. **Provide the full names of all entities represented by undersigned counsel in this case.**

   LG Electronics Inc., LG Electronics U.S.A., Inc., LG Electronics Alabama, Inc.

2. **Provide the full names of all real parties in interest for the entities. Do not list the real parties if they are the same as the entities.**

   None/Not Applicable

3. **Provide the full names of all parent corporations for the entities and all publicly held companies that own 10% or more stock in the entities.**

   LG Corporation, LG Electronics Inc., LG Electronics U.S.A., Inc.

4. **List all law firms, partners, and associates that (a) appeared for the entities in the originating court or agency or (b) are expected to appear in this court for the entities. Do not include those who have already entered an appearance in this court. Fed. Cir. R. 47.4(a)(4).**

   Fish & Richardson P.C.: Bailey K. Benedict, Claire Chang, Elizabeth Ranks, Ilya "Eli" Svetlov, Leeron G. Kalay, Meghana Thadani, Ruffin B. Cordell, Ryan M. Teel (no longer with firm), Joshua P. Carrigan (no longer with firm), and Thomas H. Reger, II

   Gillam & Smith, LLP: Melissa R. Smith

5. **Other than the originating case(s) for this case, are there related or prior cases that meet the criteria under Fed. Cir. R. 47.5(a)?**

   No.

6. **Provide any information required under Fed. R. App. P. 26.1(b) (organizational victims in criminal cases) and 26.1(c) (bankruptcy case debtors and trustees). Fed. Cir. R. 47.4(a)(6).**

None/Not Applicable

Dated: August 19, 2024

/s/ Michael J. McKeon
Michael J. McKeon

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The public version of this document redacts information that third-party Realtek has designated confidential. The redaction appears on page 48.

TABLE OF AUTHORITIES

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STATEMENT OF RELATED CASES

No party to this case has brought any previous appeal in or from the same civil action before this or any other appellate court. Counsel for Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc. (together "LG") is unaware of any case that will directly affect or be directly affected by this Court's decision in this appeal.

STATEMENT OF JURISDICTION

The District Court had jurisdiction over this patent case. The District Court issued orders denying LG's post-trial motions on April 23-24, 2024, Appx43-52; Appx53-77; Appx78-82, and entered an Amended Final Judgment on April 26, 2024. Appx83-86. LG timely filed its notice of appeal on May 15, 2024. This Court has jurisdiction under 28 U.S.C. §1295(a)(1).

STATEMENT OF THE ISSUES

1. Whether the District Court erred in granting summary judgment of patent eligibility, even though the inventor conceded that his invention is the black-box mathematical operation of optimizing a constellation for parallel decoding capacity, and no other limitations provide any inventive concept?

2. Whether the District Court erroneously denied judgment as a matter of law ("JMOL") of noninfringement, where Appellee Constellation Designs, LLC ("Constellation") relied solely on an industry standard (or purportedly related evidence) for at least one limitation in each asserted claim without ever proving that any asserted claim is standard essential?

3. Whether the District Court erroneously denied JMOL of noninfringement for accused products having a Realtek chip, where Constellation

never obtained discovery from Realtek to show that this chip has the claimed demapper, decoder, likelihoods, and constellations?

4. Whether the District Court erroneously denied JMOL of no damages, where the accused products are multi-component products with numerous unaccused technologies, and Constellation's damages expert supposedly relied on a built-in apportionment theory by using third-party Zenith's licenses, involving distinct technology and distinct patents, that were executed 15 years before the hypothetical negotiation.

INTRODUCTION

After jury trial in this case, two things are clear: Constellation's four patents asserted against LG are ineligible under §101, and Constellation failed to show infringement of those patents. Each of these errors separately requires reversal.

On eligibility, the inventor conceded that his invention was restricted to the mathematical concept of optimizing "constellations" used in broadcast transmissions—a concept reflected only in a single limitation of the '761 patent's asserted claims. The District Court seized on that single limitation to grant summary judgment of eligibility for all four patents, despite the claims of the '700, '509, and '922 patents having no limitation reciting such optimization. In making this ruling, the District Court ignored precedent holding that amorphous optimization, unbound by any specific requirement, is an ineligible abstract idea. And across the asserted patents, the remaining limitations capture conventional features of digital communication systems, and do not confer eligibility.

As to infringement, the jury's verdict lacked any legally sufficient factual basis. Constellation violated *Fujitsu* and its progeny by presenting a mix-and-match infringement case, using evidence about an industry standard (or evidence supposedly related to it) for at least one limitation of each asserted claim. Yet, Constellation never showed that the asserted claims are essential to the standard, or that LG's accused televisions comply with the relied-upon standard or with the related non-mandatory "recommended guidelines." And for the televisions having a Realtek chip, Constellation never obtained the necessary discovery from Realtek to show that this chip has the claimed demapper, decoder, likelihoods, and constellations.

Lastly, the flawed damages award cannot stand because it depends entirely on what Constellation labeled (after trial, as its expert never discussed apportionment at trial) as a built-in apportionment theory. This supposedly built-in apportionment, however, relies on the supposedly "sufficient comparability" of licenses that cannot begin to "bake in" apportionment for Constellation's asserted claims—namely, third-party Zenith's 15-year-old licenses covering unrelated, prior-generation modulation technology, different patents, and distinct products.

STATEMENT OF THE CASE

In December 2021, Constellation sued LG in the Eastern District of Texas on seven patents. Appx1007-1058. Constellation initially asserted 239 claims against LG's televisions that are compatible with a broadcast television standard from the Advanced Television Systems Committee ("ATSC") called "ATSC 3.0." Appx1015; Appx1060-1061.

On May 1, 2023, Constellation moved for summary judgment of patent eligibility. Appx1067-1104. On June 15 and 27, 2023, the District Court granted Constellation's §101 motion. Appx2.

A jury trial took place from July 5-11, 2023, on nine claims in four patents: claims 17, 21, 24, and 28 of the '761 patent; claim 5 of the '700 patent; claims 24 and 44 of the '922 patent; and claims 21 and 23 of the '509 patent. Appx20423 (29:12-16). On July 11, 2023, the jury found all asserted claims valid and infringed, and awarded $1,684,469 in damages. Appx32-39.

On September 19, 2023, LG filed post-trial motions for, *inter alia*, JMOL of noninfringement, JMOL of no damages, and/or new trial. Appx1326-1347; Appx1348-1370; Appx1371-1408. On April 23-24, 2024, the District Court upheld the jury's findings and damages award. Appx43-52; Appx53-77; Appx78-82. The District Court entered an Amended Final Judgment on April 26, 2024, that granted, *inter alia*, an ongoing forward-looking royalty at the rate of $6.75 per unit. Appx83-86. LG timely filed its notice of appeal on May 15, 2024. Appx1001-1006.

Because each issue raised in this appeal involves discrete procedural and factual backgrounds, LG provides background relevant to each issue in the corresponding Argument sections.

SUMMARY OF THE ARGUMENT

This Court should reverse the District Court's amended judgment for multiple reasons.

First, the asserted claims are patent ineligible. The named inventor conceded that he invented a black box mathematical operation of optimizing a constellation to maximize parallel decoding capacity. The asserted claims together cover only this mathematical operation, coupled to conventional aspects of digital communication systems that provide no inventive concept. Further, the District Court's grant of summary judgment of eligibility at *Alice* step one is fatally flawed at least for the '700, '509, and '922 patents, because it rested on a purported optimization-based technical improvement reflected only in the asserted claims of the '761 patent, without identifying any improvement in the claims of the other three patents.

Second, the jury's infringement verdict lacks a legally sufficient basis, because Constellation presented a mix-and-match infringement theory, contrary to *Fujitsu* and its progeny, by using product-specific evidence for some limitations and using evidence about an industry standard (or evidence supposedly related to it) for at least one limitation of each asserted claim. Yet, Constellation failed to prove that any asserted claims are essential to the standard. Without a showing of standard essentiality, Constellation cannot rely on the industry standard and its infringement case necessarily fails.

The infringement verdict also fails for televisions having a Realtek chip because Constellation's evidence does not satisfy each limitation of the asserted claims. Having taken no discovery from Realtek, Constellation relied solely on the Realtek chip's ability to process ATSC 3.0 transmitted signals and on a single PowerPoint slide that does not even disclose the claim limitations of demapper, decoder, likelihoods, or constellations.

Third, the damages award cannot stand because Constellation's damages expert never apportioned, even though LG's accused televisions include numerous unaccused technologies and components. As its expert never mentioned apportionment at trial, Constellation argued *ex post facto* that he used a built-in apportionment theory. That theory, however, improperly rests on wholly different licenses granted by third-party Zenith 15 years earlier for distinct patents, technology, and products. Zenith's licenses are not "sufficiently comparable" to support built-in apportionment, leaving the damages award unsupported.

ARGUMENT

I. STANDARD OF REVIEW

In reviewing a denial of a JMOL motion, this Court applies regional circuit law, here the Fifth Circuit. *Transocean Offshore Deepwater Drilling, Inc. v. Maersk Drilling USA, Inc.*, 699 F.3d 1340, 1346-47 (Fed. Cir. 2012). The Fifth Circuit reviews a JMOL decision *de novo* by reapplying the JMOL standard. *Ford v. Cimarron Ins. Co., Inc.*, 230 F.3d 828, 830 (5th Cir. 2000). JMOL is appropriate when "a party has been fully heard on an issue during a jury trial and the court finds that a reasonable jury would not have a legally sufficient evidentiary basis to find for the party on that issue." Fed. R. Civ. P. 50(a)(1).

This Court also reviews the admission of expert testimony under Fifth Circuit law for an abuse of discretion. *Ericsson, Inc. v. D-Link Sys., Inc.*, 773 F.3d 1201, 1225 (Fed. Cir. 2014).

The Court further reviews a district court's grant of summary judgment *de novo* according to Fifth Circuit law, *Luv n' Care, Ltd. v. Laurain*, 98 F.4th 1081, 1100 (Fed. Cir. 2024), "without deference to the trial court whether there are disputed material facts, and [it] review[s] independently whether the prevailing party is entitled to judgment as a matter of law." *SunTiger, Inc. v. Sci. Rsch. Funding Grp.*, 189 F.3d 1327, 1333 (Fed. Cir. 1999).

II. THE DISTRICT COURT ERRONEOUSLY GRANTED SUMMARY JUDGMENT OF PATENT ELIGIBILITY

A. Relevant Background

1. Nearly All of the Claimed Concepts Were Already Known

This case involves constellations used in digital communication systems. But the named inventors did not invent either digital communication systems or constellations; both existed long before the asserted patents. Appx20186 (286:17-288:25).

In a typical digital communication system, a transmitter uses a coder (a.k.a., encoder), mapper, and modulator to transform a bitstream into a signal for transmission over a communication channel. Appx92 (Fig. 2); Appx136 (3:30-31).

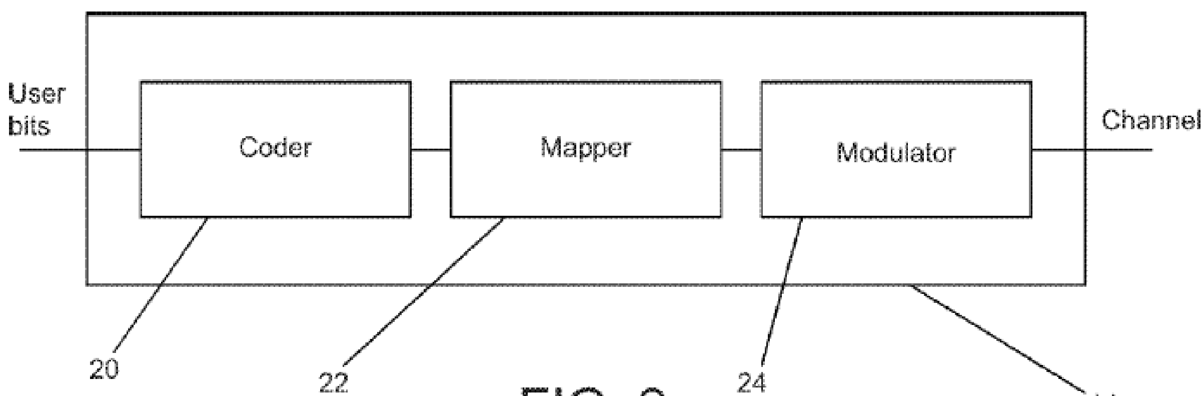
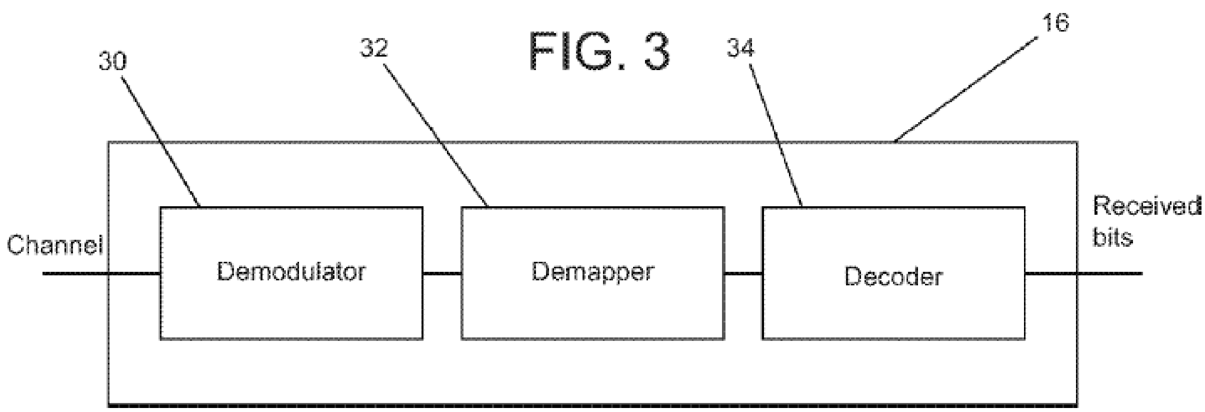


FIG. 2

At the communication channel's other end, a receiver receives the signal and transforms it back into a bitstream using, for example, a demodulator, demapper, and decoder. Appx92 (Fig. 3); Appx136 (3:32-33).

FIG. 3



Constellations are traditionally stored in the mapper and demapper. Appx137 (5:29-34). A constellation is a group of "symbols," representable on a Cartesian plane or in a chart, that serves as a key to convert between groups of digital bits, on the one hand, and waveforms, on the other. Appx137 (5:29-34); *see also* Appx135 (1:29-32); Appx105 (Fig. 11a); Appx127 (Fig. 20); Appx20167 (211:24-25); Appx20168-20169 (213:22-220:10).

Typically, constellations have particular characteristics, including dimensionality, sizes, shapes, and capacity. A constellation can be represented as one-dimensional or two-dimensional on a Cartesian plane. Appx140 (12:8-34); Appx105 (Fig. 11a); Appx128 (Fig. 21). A quadrature amplitude modulation ("QAM") constellation is an example of a two-dimensional constellation. *Id.* A constellation also has a size, which is normally identified as a power of two, such as 16QAM or 32QAM. *E.g.*, Appx139 (9:26-10:33). The greater the size, the more bits each constellation symbol represents. Appx20172 (231:4-23). Moreover, a constellation has a shape which is affected by whether the constellation's points are evenly-spaced (uniform) or unevenly-spaced (non-uniform). Appx135 (1:25-28); Appx138 (7:40-42); *see also* Appx127-128 (Figs. 20-21). And a constellation has a "capacity," which is a quantification of the information existing at corresponding points in the transmitter and receiver. Appx137 (6:23-51). For example, a constellation's "parallel decode capacity"[1] ("PD capacity") compares the information leaving the transmitter's encoder against the information entering the receiver's decoder. *Id.*

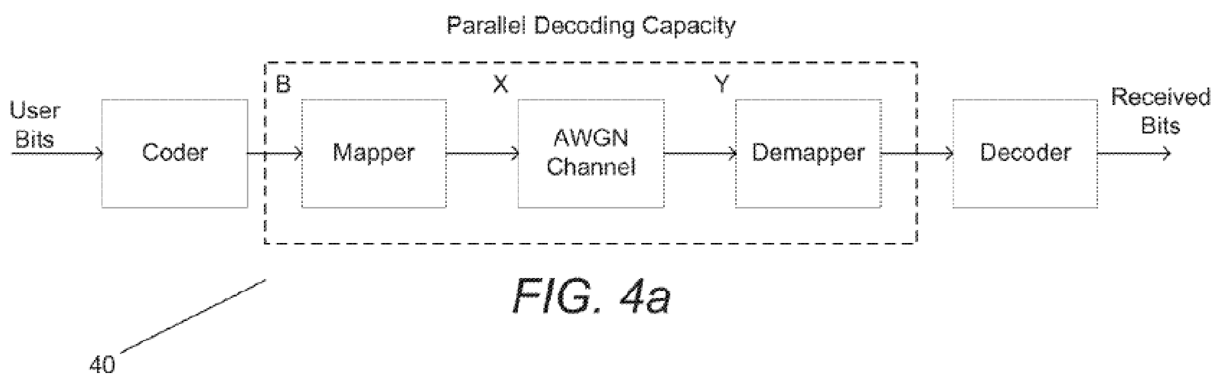


FIG. 4a

The prior art created many different types of constellations by modifying these characteristics. For instance, as the inventor and asserted patents admitted, the prior art already taught optimized non-uniform constellations. Appx20186 (286:17-288:25); Appx135 (1:59-62). As another example, the prior art already improved a constellation's PD capacity by changing the constellation's shape, thus resulting in so-called "shaping gain." The Sommer reference—published years before the asserted patents' provisional applications and cited during the patents' prosecution—designed constellations that achieved a "shaping gain" of nearly 1 dB in PD capacity by arranging the constellations' points non-uniformly. Appx1202-1204.

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[1] The patents uses "parallel decod***e*** capacity" and "parallel decod***ing*** capacity" interchangeably. *Compare* Appx88_(Abstract), *with* Appx141 (13:22-27).

To improve constellations, the prior art also paired constellations with different "code rates" (which indicate the rate of data encoding) and then used "mod cod pairs" to "gear shift" between constellations. Appx1218-1224 (149:11-155:21). Technical standards predating the asserted claims, such as DVB-S2, used such mod cod pairs. Appx1221 (152:9-12); Appx1223-1224 (154:21-155:15); Appx1297; Appx1303, Appx1306-1307, Appx1310; Appx1124, Appx1128 (¶¶1845, 1848); Appx1318-1319, Appx1316 (¶17, Fig. 7).

Thus, well before the filing of the asserted patents' provisional application, extensive work on constellations had already occurred and the constellation-related art itself was already mature. Appx20180 (261:14-17); Appx20193 (315:2-15).

2. The Asserted Claims

As named inventor Chris Jones conceded, he did not invent receivers, demodulators, demappers, decoders, codes, or constellations. Appx20186 (286:17-287:12). Nor did he invent non-uniform constellations or optimized non-uniform constellations. Appx20193 (315:2-15). Instead, he invented a "very constrained aspect of that," namely, "a very specific type of … optimized non-uniform constellation for PD capacity." Appx20193 (315:2-15).

Despite this admission, the asserted patents broadly describe the concept of "optimizing" a constellation for PD capacity, without limiting this optimization to any particular process or any constraints. Appx138-139 (8:57-9:10). Constellation's technical expert even argued that "any method or [] optimization process" would meet the claim language, because it was up to the constellation designer to select whether and what optimization constraints to use. Appx1198-1199 (152:25-153:9); Appx1194-1195 (105:9-106:14); Appx1196-1197 (148:20-149:15). Under this logic, the designer may arbitrarily set the threshold improvement level at which an iterative optimization process ceases and the designer may declare the result to be optimized. Appx138 (7:58-8:7); Appx1196-1197 (148:20-149:15).

In 2016—after the development of the ATSC 3.0 standard had largely concluded, Appx20346 (12:14-24), and years before the filing of the '700, '509, and '922 patents—the named inventors found an Institute of Electrical and Electronics Engineers ("IEEE") article discussing constellations used in the ATSC 3.0 standard. Appx20178 (253:17-254:2); Appx20177 (252:5-12). Although none of the named inventors had participated in the development of ATSC 3.0, Appx20183 (275:14-276:8), named inventor Chris Jones asserted that, according to the IEEE article, he and his co-inventor "were the first ones to suggest" the idea of using PD capacity to "mov[e] the constellation points" to increase performance. Appx20177-20178 (252:5-253:5). But he conceded that ATSC did not literally copy his work. Appx20184 (279:3-280:18).[2]

[2] This testimony is consistent with the uncontroverted testimony of Dr. Jeong, who testified that he developed many ATSC 3.0 constellations on his own. Appx20351 (30:18-23).

After learning about the ATSC 3.0 standard, Chris Jones analyzed ATSC 3.0's constellations, Appx20185 (281:7-15), and began adjusting his patents to read onto the standard. For example, he amended the specification of a then-pending application[3] to add ten columns of constellations derived by comparing ATSC 3.0's constellations to constellations already in the patent, and then sought claims on these new constellations. Appx1229-1279; Appx1226 (331:12-18); Appx447 (24:42-51). Constellation's patent attorney also testified that he was aware of the ATSC 3.0 standard when prosecuting all asserted patents, except for the '761 patent. Appx1288-1292 (149:14-153:7).

This awareness and targeting of ATSC 3.0 are apparent in the asserted claims. The '761 patent's claim 17[4]—which is representative of this patent's claims for this appeal—is narrower than the claims of other patents because its "wherein" clause requires optimizing a constellation for PD capacity. Appx1211-1212 (17:12-18:18). The claims of other asserted patents, which were prosecuted after Chris Jones learned about ATSC 3.0, no longer require optimization for PD capacity. For example, the '700 patent's claim 1—which is illustrative of this patent in this appeal—does not mention optimization and instead covers a receiver using mod cod pairs. As another example, the '509 patent's two asserted claims similarly do not

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[3] This application led to a patent which Constellation dropped before trial.

[4] All representative claims are reproduced in this brief's inner cover.

recite optimization and instead cover overlapping constellation points and mod cod pairing. And the ’922 patent’s illustrative claim 24 again does not recite optimization, and instead covers any constellation whose point magnitudes fall within 0.55 of the recited list of magnitudes when scaled by any amount (zero to infinity).

3. Dr. Akl’s Expert Analysis on Patent Eligibility

In his expert report, LG’s technical expert, Dr. Robert Akl (“Akl”) analyzed all relevant evidence under the *Alice* framework and opined that the asserted claims are ineligible under §101. Appx1105-1151.

For the ’761 patent, Akl explained that the asserted claims are “directed to the idea of a ‘geometrically spaced’ constellation that is optimized for capacity.” Appx1107-1109 (¶¶1811, 1815). He further opined that the claim limitations, alone or together, “lack any inventive concept because they recite only well-known, routine and conventional components and functions of a communication system to implement the abstract and/or mathematical of optimizing a constellation to result in a shaping gain.” Appx1111 (¶1820). On this point, Akl discussed numerous prior art references showing that the claimed limitations—including “receiver,” “demodulator,” “decoder,” constellations, and code rates—cover conventional subject matter. Appx1111-1118 (¶ 1820-1826). As he further explained, the ’761 patent’s asserted claims preempt the mathematical concept of optimizing a constellation for PD capacity to accomplish a shaping gain—a concept already disclosed in the Sommer reference—"because the claims provide no meaningful boundaries or limitations that would avoid" preempting the entire mathematical operation. Appx1118-1119 (¶¶1827-1829).

Turning to the '700 patent, Akl opined that this patent's asserted claims "are directed to the abstract notion of using a particular constellation-code rate pair that has been selected from a plurality of such pairs"—i.e., mod cod pairs. Appx1124-1130 (¶¶1845-1850). He also opined that these asserted claims lack an inventive concept, and instead recite only "well-known, routine, and conventional features of communication systems to implement" mod cod pairs. Appx1130-1131 (¶¶1851-1855). These claims, as he explained, unduly preempt the concept of mod cod pairs by, for example, covering broad ranges of code rates already widely used in the field. Appx1131 (¶1855).

Regarding the '509 patent, Akl incorporated by reference his analyses of the '761 patent regarding conventional components and the '700 patent (such as mod cod pairs) given the substantial overlap with '509 patent's claims. Appx1131-1132 (¶¶1857-1858). As he opined, the '509 patent's claims "are directed to the abstract concept of using overlapping constellation [points]," Appx1132 (¶1859), and "only recite patent-ineligible and abstract concepts devoid of particular technical details that could be addressed to solving a particular, technical problem." Appx1136

(¶1861). Akl then explained that the claimed constellation sizes, code rate ranges, and power level ranges failed to provide any inventive concept, as they were written in a preemptive manner. Appx1136-1138 (¶¶1862-1865).

For the ’922 patent, Akl explicated that this patent’s asserted claims “are directed to the patent ineligible concept of a signal represented in the form of a constellation.” Appx1138 (¶1868). “The remaining limitations” in this patent’s asserted claims, Akl explained, “recite well-known, routine and conventional concepts that fail to meaningfully narrow the claims to a concrete or particular solution to a specific problem.” *Id*. As he discussed, these asserted claims’ limitations reciting equipment, constellation size, mod cod pairs, and shaping gain failed to provide any inventive concept, because they were already well-known, routine, and conventional. Appx1140-1142 (¶¶1871-1876).

4. The District Court’s Grant of Summary Judgment of Patent Eligibility at *Alice*’s Step One

Despite Akl’s expert opinions on §101, the District Court granted Constellation’s summary judgment motion of patent eligibility at the pretrial conference.

During oral arguments on Constellation’s motion, the District Court asked Constellation to provide “tangible evidence from the claim limitations read in the light of the specification and the intrinsic record that will show that it is an improvement in the technology and it’s directed to solving a technical problem[.]”

Appx20019 (73:16-21). The District Court essentially asked Constellation to identify particular claim limitations and "explain … how they are directed to a concept that's not abstract." Appx20019 (72:3-11). Receiving no helpful answer, Appx20019 (72:12-73:3), the District Court then pointed to the "wherein" clause in claim 17 of the '761 patent—the only limitation in the asserted claims that mentions optimization of constellations—and asked Constellation to explain "how ***that*** limitation is an improvement over the prior technology, how it is something that's directed at a solution to a technical problem," Appx20019 (73:4-14).[5] Unsatisfied with Constellation's answer to this additional question, Appx20020 (74:5-18), the District Court again took the matter upon itself: "And that benefit is improved capacity?" Appx20020 (74:19). Constellation answered affirmatively. Appx20020 (75:20).

On this thinnest of reeds, the District Court held that all asserted claims are patent eligible at *Alice*'s step one because they purportedly contain a technical improvement, namely, they "achieve improved, i.e., optimized, channel capacity and more efficient over-the-air data transmission." Appx20025 (94:13-18); *see also* Appx2. The District Court did not explain how its stated benefit for the "wherein" clause of the '761 patent's claim 17 could apply to any other asserted claim, as no other asserted independent claim recites optimization. *See id.* Nonetheless, it

[5] Unless noted, all emphases added.

concluded that the asserted claims are "not directed primarily to an abstract concept" based on "the evidence before the Court presented by both the parties on this matter." Appx20025 (94:19-95:1). It then struck Akl's analysis on patent ineligibility as inconsistent with its summary judgment ruling. Appx20025 (95:2-11); Appx2.

B. The Asserted Claims Are Ineligible Under § 101

1. The Asserted Claims Fail *Alice*'s Step One

a. Black-Box "Optimization" Does Not Make the Claims Eligible

Step one of *Alice* asks whether patent claims are "directed to a patent-ineligible concept like an abstract idea." *Hawk Tech. Sys., LLC v. Castle Retail, LLC*, 60 F.4th 1349, 1356 (Fed. Cir. 2023); *see also Alice Corp. Pty. v. CLS Bank Int'l*, 573 U.S. 208, 218 (2014). The asserted claims answer this test with a resounding "yes."

"Under step one's directed-to inquiry, we ask ***what the patent asserts to be the focus of the claimed advance over the prior art*** to determine whether the claim's character as a whole is directed to ineligible subject matter." *Simio, LLC v. FlexSim Software Prod., Inc.*, 983 F.3d 1353, 1359 (Fed. Cir. 2020) (internal citations omitted). Here, the alleged advance(s) cannot overcome this hurdle. There is no disagreement between named inventor Chris Jones and Akl that communication systems with nearly all (if not all) claimed components and concepts existed long before the alleged invention. Appx20186 (286:17-288:25); Appx1110, Appx1112, Appx1122, Appx1128, Appx1132, Appx1139-1140, Appx1144, Appx1149 (¶¶1817, 1821, 1838, 1847, 1858, 1870, 1882, 1894). As the named inventor admitted, his patents are directed to a "very constrained aspect" of constellations, namely "optimized non-uniform constellation for PD capacity." Appx20193 (315:5-15).

But this purported optimization cannot imbue the asserted claims with patent eligibility because optimization is an unpatentable mathematical operation, and moreover, the optimization parameters are never claimed. *See, e.g.*, *In re Bd. of Trs. of Leland Stanford Junior Univ.*, 991 F.3d 1245, 1252 (Fed. Cir. 2021) (holding claims covering complex mathematical algorithm ineligible). The '761 patent—the only asserted patent whose claims recite optimization—describes an "optimization" process without providing any details, such as constraint values, necessary to achieve this optimization. Appx1194-1195 (105:9-106:14). Instead, the patent's optimization process is a black box that, according to Constellation's own expert, covers "any method or [] optimization process." Appx1198-1199 (152:25-153:9). As a result, a designer could optimize by arbitrarily setting a predetermined quantity of signal-to-noise improvement, however small the improvement may be. Appx1196-1197 (148:20-149:15). As these admissions make clear, the '761 patent's asserted claims are drawn fundamentally to the general mathematical concept of optimizing constellations to yield ***any*** improvement in PD capacity. The claimed optimization process is therefore user-defined, result-oriented, and reserved for the public domain.

Claims written in this manner epitomize an abstract concept, as this Court has consistently held. *E.g.*, *Two-Way Media Ltd. v. Comcast Cable Commc'ns, LLC*, 874 F.3d 1329, 1337 (Fed. Cir. 2017) (deeming abstract at step one claims written "using result-based functional language" that "does not sufficiently describe how to achieve these results in a non-abstract way" (cleaned up)). For example, much like the claims at issue here, the challenged claims in *Hawk Technology* covered the transformation of image data through encoding and converting format for transmission. 60 F.4th at 1353, 1357. Faced with those claims, this Court explained "that encoding and decoding image data and converting formats, including when data is received from one medium and sent along through another, are by themselves abstract ideas." *Id.* at 1357 (cleaned up). Rejecting the patentee's argument that its claims resulted in images with optimized quality while conserving bandwidth, this Court stated that "the claims themselves do not disclose performing any 'special data conversion' or otherwise describe how the alleged goal of 'conserving bandwidth while preserving data' is achieved." *Id.* And just like the black-box claims here, the *Hawk Technology* claims did not "explain what the [claimed] parameters are or how they should be manipulated" and thus lacked "sufficient recitation of *how* the purported invention improves the functionality" of the system and were instead "recited at such a level of result-oriented generality that those claims amount to a mere implementation of an abstract idea." *Id.* at 1357-58 (italics in original; citation omitted).

That is precisely the deficiency that renders the '761 patent's asserted claims patent ineligible. All that is claimed here is the outcome of optimization, and this "essentially result-focused, functional character of claim language has been a frequent feature of claims held ineligible under §101." *Elec. Power Grp., LLC v. Alstom S.A.*, 830 F.3d 1350, 1355-56 (Fed. Cir. 2016). For such claims only reciting "generic functional language to achieve these purported solutions," the "[i]nquiry therefore must turn to any requirements for how the desired result is achieved." *Two-Way Media*, 874 F.3d at 1339. But the desired "optimization" here is amorphous because the claims provide no limit or instruction on how to achieve optimization. Constellation's expert even admitted that "any method or [] optimization process" would satisfy the claim language. Appx1198-1199 (152:25-153:9); Appx1194-1195 (105:9-106:14); Appx1196-1197 (148:20-149:15). With no limit on how optimization is achieved, the claims are abstract. *See Two-Way Media*, 874 F.3d at 1337 (holding claims directed to "functional results," but without "describ[ing] how to achieve these results in a non-abstract way," ineligible); *SAP America, Inc. v. InvestPic, LLC*, 898 F.3d 1161, 1167 (Fed. Cir. 2018) (distinguishing claims held eligible as "they had the specificity required to transform a claim from one claiming only a result to one claiming a way of achieving it").

b. No Other Claimed Feature Provides Eligibility

The District Court relied on "optimization" to further declare the '700, '509, and '922 patents eligible. Appx20025 (94:13-18); Appx2. But there is ***no*** optimization limitation in these three patents' asserted claims. Appx197 (Cl. 1); Appx260 (Cls. 21, 23); Appx447 (Cl. 24).

This fact alone highlights a fundamental flaw in the District Court's ruling. As this Court's "case law [makes] clear[,] the §101 inquiry must be based 'on the language of the Asserted Claims themselves.'" *Packet Intel. LLC v. NetScout Sys., Inc.*, 965 F.3d 1299, 1318 (Fed. Cir. 2020). Yet, the District Court seized on a limitation absent from the asserted claims of the '700, '509, and '922 patents to grant summary judgment. That is a fundamental error.[6] And while these patents' respective specifications discuss optimizing constellations for PD capacity, relying on this unclaimed aspect is also error. *See Packet Intel.*, 965 F.3d at 1318 ("[A]

[6] The District Court's summary judgment for the'700, '509, and '992 patents cannot stand for this reason alone. Should this Court decline to enter judgment as a matter of law for LG as discussed below, LG respectfully requests reversal of the summary judgment and remand. *See Realtime Data LLC v. Reduxio Sys., Inc.*, 831 F. App'x 492, 497-499 (Fed. Cir. 2020) (vacating §101 ruling and remanding where district court "failed to tie those descriptions to any specific claim").

court cannot rely on unclaimed details in the specification as the 'focus' of the claim for §101 purposes.").

Had the District Court not erred, it would have realized the abstract nature of the '700, '509, and '922 patents. Starting with the '700 and '509 patents, they are both directed to the abstract notion of a mod cod pair selected from a plurality of such pairs. Appx1124, Appx1128, Appx1131-1132 (¶¶1845, 1848, 1857). As the named inventor admitted, mod cod pairs and their use have been long prevalent in digital communication systems. Appx1218-1224 (149:11-155:21). That is, "the idea of having multiple types of constellations and code rates to select from is not novel, but has been implemented in prior standards long before the invention," such as the prior art DVB-S1 and DVB-S2 standards. Appx1128-1129 (¶1849); Appx1297; Appx1303, Appx1306-1307, Appx1310; Appx1318-1319 (¶17); Appx1316 (Fig. 7).

Other claims reciting similar selection have fallen as ineligible. *E.g., In re Rudy*, 956 F.3d 1379 (Fed. Cir. 2020). The claim in *Rudy*, for example, was "directed to the abstract idea of ***selecting*** a fishing hook ***based on*** observed water conditions." *Id.* at 1384-85. Although *Rudy*'s claim recited a basis for selection by linking water conditions and the hooks to be selected, this greater level of specificity could not save that claim from being abstract. *Id.* at 1381. If *Rudy*'s claim could not survive scrutiny despite the greater level of detail, the asserted claims of the '700 and ’509 patents that are far less specific must fall as well. After all, making a selection of mod cod pairs from a known set, with no selection parameters or restrictions, improperly preempts all such selections. *See Cisco Sys., Inc. v. Uniloc 2017 LLC*, 813 F. App’x 495, 497 (Fed. Cir. 2020) (“The general recitation of the familiar concept[] of … ***selecting*** leaves the claimed method ‘untethered to any specific or concrete way of implementing it’” and is ineligible).

As for the ’922 patent, it is merely directed to representing a signal as a constellation. Appx1138 (¶1868). As the named inventor admitted, constellations were well-known in digital communication systems before the asserted patents. Appx20186 (286:17-288:25). As a mathematical representation of a signal in a different format, Appx137 (5:29-34), a constellation and its use cannot avoid abstractness. *See Hawk Tech.*, 60 F.4th at 1357 (“[W]e have held that encoding and decoding image data and converting formats, including when data is received from one medium and sent along through another, are by themselves abstract ideas.” (cleaned up)).

2. No Inventive Concept Exists to Save the Claims at *Alice*’s Step Two

Because the District Court granted Constellation’s summary judgment motion at *Alice* step one, it did not address *Alice*’s step two. But this Court can and should. *See, e.g., Chamberlain Grp., Inc. v. Techtronic Indus. Co.*, 935 F.3d 1341, 1346, 1348 (Fed. Cir. 2019) (analyzing step two, when district court did not, after reversing district court's step one conclusion that claims were not directed to abstract idea).

Examining the asserted claims at *Alice*'s step two leads to the inescapable conclusion that they lack any inventive concept which transforms their abstract ideas into patent-eligible subject matter. *Alice*, 573 U.S. at 218. For the '761 patent, Akl explained—with citations to dozens of prior art references—that the hardware components of receivers (¶1822), demodulators (¶1823), and decoders (¶1824) were well-known, routine, and conventional. Appx1112-1117. He further stated that the patent did not describe these components with any specificity that goes beyond their conventional nature. *Id*. For the '700 patent, he first incorporated this analysis of the '761 patent, Appx1130 (¶1852), and then explained that the '700 patent's use of "the concept of using selectable code rate-constellation pairs was also well known, routine and conventional because it was already used in prior digital transmission standards, including DVB as one example, before the purported invention." Appx1130 (¶1853). For the '509 patent, Akl explicated that this patent's claims also implicated the non-inventive concept of constellation-code rate pairs (like the '700 patent), and were further "directed to the abstract and mathematical concept of using overlapping constellation points." Appx1136-1137 (¶¶1862, 1864). For the '922 patent, Akl reiterated and incorporated his analysis of the claimed aspects common with the other patents, and then discussed how the additionally claimed shaping gains and the particular constellations were also well-known in the prior art. Appx1141 (¶¶1874-1875). Essentially, the limitations of the asserted claims, taken alone or together, recite well-known, routine and conventional features of communication systems that cannot make the asserted claims patent eligible. *Voter Verified, Inc. v. Election Sys. & Software LLC*, 887 F.3d 1376, 1386 (Fed. Cir. 2018) ("The case law has consistently held that ... standard components [of general purpose computers] are not sufficient to transform abstract claims into patent-eligible subject matter.").

The named inventor's admissions at trial further reinforce the claims' lack of inventive concept and their recitation of well-known, routine, and conventional aspects. For example, Chris Jones admitted that the claimed hardware components—including receivers, demodulators, demappers, and decoders—were known long before the asserted claims. Appx20186 (286:17-287:3). So too were low-density parity check ("LDPC") codes. *Id.* (287:4-6). And he conceded that constellations, including optimized non-uniform constellations, existed well before the asserted claims. *Id.* (287:7-288:25).

Constellation, which bore the burden of showing a lack of genuine dispute of material fact for summary judgment, argued for step two that the claimed hardware is "specially programmed" to implement the invention. Appx1097. But this argument amounts to "simply stating the abstract idea while adding the words 'apply it.'" *Rudy*, 956 F.3d at 1385. This tactic is inadequate because there must be something more than the elements of the abstract idea to save the claims at step two. *See id.*; *BSG Tech LLC v. Buyseasons, Inc.*, 899 F.3d 1281, 1290 (Fed. Cir. 2018) ("It has been clear since *Alice* that a claimed invention's use of the ineligible concept to which it is directed cannot supply the inventive concept that renders the invention 'significantly more' than that ineligible concept.").

Constellation also presented alleged praise for the named inventors' work and for the accused ATSC 3.0 and A/322 Standards. Appx1097-1098. This alleged praise, however, is not co-extensive with the claims-in-suit, which must remain the focus of any §101 analysis. But even if this purported praise were relevant, praise does not confer patent eligibility. Although many of the canonical patent-ineligibility cases involved widely-praised developments, the Supreme Court warned that "[g]roundbreaking, innovative, or even brilliant discovery does not by itself satisfy the §101 inquiry" and found ineligibility nonetheless. *Ass'n for Molecular Pathology v. Myriad Genetics, Inc.*, 569 U.S. 576, 591 (2013); *see also Synopsys, Inc. v. Mentor Graphics Corp.*, 839 F.3d 1138, 1151 (Fed. Cir. 2016) ("[A] claim for a *new* abstract idea is still an abstract idea." (italicization in original)).

Simply put, there is nothing more to the asserted claims than their abstract ideas, and *Alice*'s step two does not rescue these claims from ineligibility.

3. This Court May Enter Judgment of Ineligibility in the Current Posture

As discussed above, the asserted patents fail the *Alice* test and are patent ineligible. The Court should, at the least, reverse the District Court's improperly-granted summary judgment.

This Court should go further by entering judgment of ineligibility in favor of LG. There is no procedural hurdle preventing the Court from granting such relief. While LG opposed summary judgment on eligibility because fact issues remained pre-trial and because factual admissions at trial would be relevant to the inquiry, that has proven to be true. *See* Appx1168-1169. For example, the named inventor made many significant admissions at trial, as discussed above, that confirm the ineligibility analysis. *See, e.g.*, Appx20186 (286:17-288:25); Appx20193 (315:2-15).

Under such circumstances, this Court has recognized that it may direct judgment on appeal in certain situations. *See, e.g.*, *Litton Indus. Prod., Inc. v. Solid State Sys. Corp.*, 755 F.2d 158, 164 (Fed. Cir. 1985) ("We recognize that, in some cases, it may be proper for an appellate court which disagrees with a district court's decision granting summary judgment in favor of the moving party, to reverse and remand with instructions to award summary judgment in favor of the nonmoving party."). In light of the trial record, this is such a situation. It would be wasteful to remand for a trial, when the trial record has confirmed the relevant facts. There can be no reasonable factual dispute that judgment of ineligibility should be entered in

LG's favor. *Cf. Berkheimer v. HP Inc.*, 881 F.3d 1360, 1368 (Fed. Cir. 2018) ("When there is no genuine issue of material fact regarding whether the claim element or claimed combination is well-understood, routine, conventional to a skilled artisan in the relevant field, this issue can be decided on summary judgment as a matter of law.").

III. NO REASONABLE JURY COULD HAVE FOUND INFRINGEMENT

A. Noninfringement Background

1. Constellation's Infringement Argument at Trial

This case implicates digital televisions compatible with the over-the-air broadcast standard called "ATSC 3.0." Appx20216 (86:2-8). ATSC 3.0 is not, however, a single standard; it is a suite of standards covering multiple technologies and features. Appx20241 (185:20-186:16); Appx20359-20361 (64:13-68:20, 69:1-71:5). One standard within the ATSC 3.0 suite is "A/322," and it governs the physical layer of signals transmitted by broadcasters. Appx20218 (93:15-94:1).

Constellation and its technical expert, Dr. Mark Jones ("Jones"),[7] did not rely exclusively on compliance with ATSC 3.0 or A/322 to argue literal infringement at trial. Rather, for the same asserted claim, Jones relied on standard-related evidence

[7] Two Constellation trial witnesses were named "Jones": Mark Jones, a technical expert, and Chris Jones, a corporate witness and named inventor.

for some limitations, and on evidence about LG's accused televisions for other limitations.

Jones adopted this "mix-and-match" approach because ATSC 3.0 and A/322 govern transmitters. Appx20216 (87:15-88:4); Appx20248-49 (216:20-217:8). As Jones admitted, A/322 "discusses the transmission[,]" and specifically "the formation of the signal or the wave form that goes out" from broadcasters like CBS and ABC. Appx20248-49 (216:20-217:8); Appx20218 (93:15-94:1); Appx20219 (99:21-100:12); Appx20220 (101:13-102:2).

But LG's accused televisions are not transmitters—they are receivers. That is, LG's accused televisions receive transmissions sent over the air by broadcasters. Because ATSC 3.0 (and the standards within this suite) only addresses transmitters, ATSC 3.0 is not mandatory for televisions and other receivers. Appx20240 (182:6-8). In fact, the ATSC standard body does not impose any mandatory standard on receivers, and instead offers ***recommended guidelines*** for receivers in a document called "A/327." Appx20249 (218:10-22); Appx20219 (100:13-20).

Although A/322 does not govern receivers, Jones nevertheless relied on A/322 to prove infringement by LG's accused televisions by assuming—without ever proving—that these televisions "reverse" A/322's operations. Appx20216 (87:23-88:4); Appx20220 (101:13-19, 103:24-104:2). Under this assumption, Jones relied on A/322's transmitter constellation point values to meet certain limitations in the asserted independent receiver claims of three out of four patents (the ’922, ’700, and ’509 patents). Appx20229-30 (139:13-142:12); Appx20231 (145:4-146:13); Appx20232-33 (152:16-155:17); Appx20233-35 (156:6-163:6).

The independent receiver claim of the fourth patent (the ’761 patent), however, recites a “wherein” limitation requiring the “optimization” of constellations. Appx141 (13:22-27). With A/322 saying nothing about optimization, Jones tried to meet this “wherein” clause using three documents predating A/322’s November 2017 final adoption: (1) a 2013 proposal by Samsung and Sony to ATSC; (2) a 2014 proposal by LG to ATSC; and (3) a 2016 IEEE article. Appx20231-32 (148:5-150:7). He provided no separate evidence to show that those documents accurately describe the final, as-adopted A/322 standard. *Id.*

Jones used the A/322 standard for his infringement analysis despite having access to LG’s source code for two accused LG chips (B17+ and O22 chips).[8] For example, when addressing infringement of the ’922 patent, Jones only compared the constellation point values in the claims to those in A/322, ***not*** to those in LG chips’ source code. Appx20229 (140:2-9). He could not use the constellation point values in LG’s source code because, as he conceded, those values must be “rounded” first before they could allegedly “represent” the values in A/322. Appx20224 (117:6-

[8] Constellation did not obtain source code for third-party Realtek’s K8Hp chips during discovery.

118:5); Appx20251 (226:22-227:15); Appx20256 (245:7-14). As LG's expert, Akl, showed at trial, the values in LG's source code do not match the A/322 values, which Jones did not dispute. Appx20362-63 (75:6-77:9); Appx20251 (225:16-19).

While Jones relied on A/322 (and supposedly related documents) for some limitations of the asserted independent claims, he never compared A/322 to other limitations in the same claims that cover conventional structures—such as the preambles and the "receiver," "demodulator," "demapper," and "decoder" limitations. For these structural limitations in independent claim 24 of the '922 patent, for example, Jones used technical documents about LG's accused B17+ and O22 chips without mentioning A/322 or even ATSC 3.0. Appx20227-29 (132:20-139:12). When addressing the structural limitations in the remaining patents' independent claims, he simply referred back to his analysis of the '922 patent's structural limitations.[9] Appx20231 (146:14-148:4); Appx20232 (151:13-152:15); Appx20233 (156:6-18).

[9] While the '509 patent's asserted claims recite "wherein the receiver uses a symbol constellation to transform the received signals into received bits," Jones expressly tied this limitation to his analysis of the "demodulator, demapper, and decoder" limitations in the '922 patent. Appx20233-20234 (156:19-157:8) (stating that the '509 patent's "wherein" limitation is "met in the evidence I described for the demodulator, demapper, and decoder that I described both at the board in a standard as well as with respect to those particular limitations in the '922 patent"). Therefore, infringement of the '509 patent rises or falls with the '922 patent.

2. LG's Development of the A/322 Transmitter Standard and Its B17+ and O22 Television Chips

At trial, Dr. ByeongKook Jeong ("Jeong"), LG's chief research engineer, testified about LG's contributions to developing ATSC 3.0 and A/322, as well as LG's design of its accused televisions. *See* Appx20346-54 (9:4-43:13).

As Jeong explained, LG helped develop ATSC 3.0, partly because LG had significant technical knowledge to contribute. Appx20346 (11:1-10). A fifteen-employee LG team worked on ATSC 3.0's physical layer (A/322), and additional teams worked on other ATSC 3.0 standards. *Id.* (11:11-12:2). Jeong's A/322 team worked full-time after the standard body called for proposals in March 2013, and submitted a system proposal six months later. Appx20346-47 (12:17-13:21). Jeong personally developed non-uniform transmitter constellations to include in the proposal, with the goal of maximizing BICM capacity.[10] Appx20347 (15:16-16:23). Jeong came up with 19 of the 60 transmitter constellations adopted into A/322. Appx20348 (20:13-14).

Once the standards body approved A/322 in 2015, Jeong turned to developing LG's B17+ chip, which was completed in 2018. Appx20349 (23:8-13, 24:2-6). He then developed LG's O22 chip, which was completed in 2021. *Id.* (24:16-19). In designing these LG chips, Jeong intentionally developed and incorporated

[10] A constellation's BICM capacity is synonymous with its parallel decode capacity. Appx20215 (84:13-20).

constellation values ***different*** from those in A/322. Appx20350 (26:3-27:24). While the transmitter constellations in A/322 were maximized for BICM capacity, the constellation values in LG's chips "sacrifice[d] some BICM capacity" to "achieve a small size and better power consumption." Appx20350 (25:21-27:24). No witness contradicted Jeong's testimony.

3. The District Court's Denial of LG's Motion for JMOL and a New Trial

Despite Jones' mix-and-match approach, the jury found that LG's accused televisions infringe all asserted claims. Appx32-39. LG moved for JMOL of noninfringement, in part because Constellation failed to comply with this Court's precedent. Appx1371-1408.

The District Court denied LG's motion. Appx54-62. The District Court "h[eld] that the reasoning of *Fujitsu* … applies on a limitation-by-limitation basis," Appx57, even though it acknowledged that this holding "extend[s] the reasoning the Federal Circuit laid out in *Fujitsu* from a claim-by-claim basis to a limitation-by-limitation basis." Appx59. It also allowed assessment of standard essentiality on a limitation-by-limitation basis, Appx57, because:

> [A] patent owner may rely solely on a standard to show that a product practices a ***limitation*** of a claim if (1) the relevant portion of the standard is sufficiently specific to show that practicing it would always result in practicing that ***limitation***, and (2) the relevant portion of the standard is mandatory, or, if it is optional, there is evidence showing that the accused device implements that portion of the standard.

Appx59. The District Court thus cabined this Court's *Fujitsu* and *INVT* precedent to the scenario "where a standard was relied upon to show infringement of an entire claim, not a particular limitation." Appx56-59. The District Court reasoned that its approach would conserve judicial resources by reducing discovery costs and streamlining infringement proof for certain product classes. Appx57. Applying its novel legal standard, the District Court held that "Constellation may mix and match evidence of standard compliance with a direct comparison" of the claim language to the accused products. Appx60.

B. Constellation's Mix-and-Match Infringement Case Violates This Court's Standard Essentiality Precedent

The District Court disregarded this Court's strict requirement that infringement of a standard-essential patent may be proven by compliance with an industry standard ***only*** when an asserted ***claim*** covers every possible implementation of the standard. *See* Appx59-60. This error led the District Court to allow the mix-and-match approach that misled the jury into finding infringement. Under the proper legal framework, this Court should reverse the District Court's denial of JMOL of noninfringement. But even if this Court adopts the District Court's novel legal standard, no reasonable jury could have found infringement.

1. Precedent and Policy Do Not Support the District Court's Limitation-by-Limitation Analysis

The default rule is that direct infringement requires comparing the claims to the accused products or systems. *Fujitsu Ltd. v. Netgear Inc.*, 620 F.3d 1321, 1327 (Fed. Cir. 2010). *Fujitsu* permitted a narrow exception when claims are standard essential: If "the reach of the ***claims*** includes any device that practices a standard," direct infringement may be proven by comparing the claims to the standard. *Id.* But *Fujitsu* imposed a threshold requirement when using industry standards to prove infringement: "***Only*** in the situation ***where a patent covers every possible implementation*** of a standard will it be enough to prove infringement by showing standard compliance." *Id.* at 1328.

This threshold requirement was necessary because, "in many instances, an industry standard does not provide the level of specificity required to establish that practicing that standard would always result in infringement." *Id.* at 1327. Likewise, if the asserted claims encompass an optional (rather than mandatory) section of the standard, then "standards compliance alone would not establish that the accused infringer chooses to implement the optional section." *Id.* at 1327-28; *see also Godo Kaisha IP Bridge 1 v. TCL Commc'n Tech. Holdings Ltd.*, 967 F.3d 1380, 1384 (Fed. Cir. 2020) ("*Fujitsu* teaches that where, but only where, a patent covers mandatory aspects of a standard, is it enough to prove infringement by showing standard compliance."). "In these instances, it is not sufficient for the patent owner to establish infringement by arguing that the product admittedly practices the standard, therefore it infringes." *Fujitsu*, 620 F.3d at 1328. Instead, "the patent owner must compare the ***claims*** to the accused products or, if appropriate, prove that the accused products implement any relevant optional sections of the standard." *Id.*

A recent precedent—*INVT SPE LLC v. International Trade Commission*, 46 F.4th 1361 (Fed. Cir. 2022)—confirms *Fujitsu*'s narrow reach. The patentee in *INVT* tried to prove infringement by reading its claims onto the 4G LTE cellular standard. *Id.* at 1368. The International Trade Commission found that the patentee's claims—which this Court construed to recite capability—were not standard-essential and thus failed to meet the threshold requirement. *Id.* INVT could not prove infringement because it failed to show, as it must for non-standard-essential patents, that the accused products practiced its claims. *Id.* This Court affirmed, emphasizing that "[i]nfringement can be proven based on an accused product's use of an industry standard if the asserted ***claim*** is standard essential." *Id.* at 1377 (citing *Fujitsu*, 620 F.3d at 1326-29). "Claims are standard essential if 'the reach of the ***claims*** includes any device that practices the standard.'" *Id.* (quoting *Fujitsu*, 620 F.3d at 1327). "In other words, 'all implementations of a standard infringe the ***claim***' and the 'patent covers every possible implementation of a standard.'" *Id.* (quoting *Fujitsu*, 620 F.3d at 1327-28).

Critically, INVT did not meet *Fujitsu*'s requirements because, for two claimed limitations, it "failed to show ... infringement based on the ***claim*** being essential to the standard[.]" *Id.* at 1377. The patentee presented "no evidence that a standard-compliant user device ever receives data modulated and encoded with the claimed parameters." *Id.* at 1378. Because INVT failed to prove standard essentiality, it had to prove infringement by comparing the claims to the products—something it failed to do—and thus this Court affirmed noninfringement. *Id.* at 1380.

In accordance with *Fujitsu* and *INVT*, the threshold question for the District Court should have been whether the asserted ***claims*** were standard essential. But the District Court stretched existing precedent by holding "that the reasoning of *Fujitsu* also applies on a limitation-by-limitation basis." Appx59; *see also* Appx57. Yet, the District Court did not cite any case in which any court, much less this Court, applied *Fujitsu* on a limitation-by-limitation basis. *Fujitsu* is instead clear that its narrow exception applies to claims, not limitations. For this reason alone, the District Court's legal analysis should be reversed.

Probably aware that its reasoning stood on thin ice, the District Court advanced a public policy argument that a limitation-by-limitation, mix-and-match approach conserves judicial resources. Appx57. Any possible conservation of judicial resources is dwarfed by the massive financial burden that companies will face by this opening of the floodgates to unpoliced reliance on standards as a proxy for proving infringement. Applying a limitation-by-limitation test, in essence, blesses a patentee taking impermissible liberties and shortcuts to arrive at an infringement determination, as Constellation did here. But this approach is inherently flawed because it does not require a patentee to demonstrate that a full claim is standard essential. Thus, the inferential support for using a standard to demonstrate infringement recognized by *Fujitsu* crumbles.

Abuses that could flow from the District Court's approach are evident. For example, if a patentee is not required to demonstrate a full claim's standard essentiality, it could permissibly cite a portion of a standard as circumstantial evidence for a limitation even if the standard has no nexus to the claims. In other words, patentees could draft patent claims on a standard, but avoid the expense and effort of proving either that (1) the patent claim is truly standard essential such that it covers every possible implementation of the standard; or (2) the accused product meets each and every limitation of a patent claim. Alleged infringers, on the other hand, will be left to defend against and disprove, at great expense, inherently flawed infringement theories. To the extent *Fujitsu* and *INVT* leave open the possibility that their reasoning could apply on a limitation-by-limitation basis, public policy requires rejection of this possibility. Holding otherwise would create an unjustified windfall to opportunistic patentees while placing an expensive and unfair burden on companies who routinely fall victim to targeting by patent infringement campaigns.

2. No Reasonable Jury Could Find Infringement Using Standards-Based Evidence

Despite the legal standard articulated in *Fujitsu* and reinforced in *INVT*, Constellation relied on the A/322 transmission standard for infringement without showing that its asserted claims are standard essential. There is no substantial evidence to support the infringement verdict. But even if a mix-and-match analysis were appropriate, Constellation failed to show that LG actually implements the relevant part of the standard.

a. Constellation Failed to Establish that the Asserted Claims Are Essential to A/322

Constellation never established that the asserted claims are standard essential, a threshold requirement to rely on a standard as proof of infringement. *Fujitsu*, 620 F.3d at 1327-28. Indeed, Jones never compared the standard in question (A/322) to ***all*** limitations of any asserted claim. *E.g.*, Appx20227-29 (132:20-139:12) (failing to rely on A/322 for the preamble, "receiver," "demodulator," "demapper," and "decoder" limitations of the '922 patent's claim 24); Appx20231 (146:14-148:4) (same for '761 pat.); Appx20232 (151:13-152:15) (same for '700 pat.); Appx20233 (156:6-18) (same for '509 pat.); *see also* Appx20231-32 (148:5-150:7) (relying on documents preceding adoption of A/322, rather than on A/322, for "wherein" limitation of '761 patent's claim 17). The District Court essentially conceded that Jones never established the asserted claims' standard essentiality by deciding that

Jones was not required to do so. Appx60. Jones and Constellation thus failed to show that any asserted claim is, in fact, standard essential. *See INVT*, 46 F.4th at 1377; *Fujitsu*, 620 F.3d at 1327.

Even had Constellation attempted to prove that the asserted claims were essential to A/322, it would have failed. The A/322 standard sets forth requirements for ***transmitters*** and therefore does not address the claimed structural limitations for receivers like televisions, including "likelihood," "demodulator," "demapper," "decoder." Appx20219 (99:21-100:12); Appx20220 (101:13-102:2). Unsurprisingly, then, Jones never established that A/322 is mandatory for receivers, like televisions. *See Godo Kaisha*, 967 F.3d at 1384 ("*Fujitsu* teaches that where, but only where, a patent covers ***mandatory*** aspects of a standard, is it enough to prove infringement by showing standard compliance."). At trial, he even conceded that ATSC 3.0, including the A/322 standard, is not mandatory for televisions. Appx20240 (182:6-7) ("Q. ATSC 3.0 is not mandatory. Correct? A. All TVs don't have to have it."). Because the A/322 transmitter standard is not mandatory for televisions, the asserted claims are not standard essential for televisions, and infringement by a television cannot rest on this standard. *See Fujitsu*, 620 F.3d at 1328; *Dynacore Holdings Corp. v. U.S. Philips Corp*., 363 F.3d 1263, 1276-78 (Fed. Cir. 2004) (affirming noninfringement where patentee did not show limitation was mandatory to standard).

To fill the wide evidentiary gaps, Jones turned to the A/327 "recommended practices and guidelines[.]" Appx20219-20220 (100:13-101:12); *see also* Appx20228 (136:5-19) (citing A/327 guidelines' "recommended practice" to discuss a constellation). But as its name indicates, the A/327 "guideline" is optional. As Jones admitted, A/327 is "not indicating what a company must do, but it is describing the recommendations." Appx20219 (100:13-20). Because it is not mandatory, A/327 cannot demonstrate infringement. *See Fujitsu*, 620 F.3d at 1327.

There is therefore no evidence from which a reasonable jury could find infringement because Constellation failed to prove that the asserted ***claims*** are essential to the A/322 standard or otherwise mandated by the recommended A/327 guidelines.

b. LG Products Do Not Practice the A/322 Standard

Even if Constellation could prove standard essentiality, *Fujitsu* instructs that "[a]n accused infringer is free to ... prove that it does not practice the standard." 620 F.3d 1321 at 1327. LG presented substantial evidence at trial that it can, and does, deviate from A/322's constellations.

Jeong, who designed the constellations in LG's accused televisions, testified that he "intentionally used constellations in LG's televisions that are different from the constellations in the A/322 standard" to reduce size and power consumption. Appx20350 (26:21-27:24). To confirm this fact, Akl performed binary-to-decimal computations, undisputed by Jones, and showed that the LG television's constellations do not match those in A/322. *Compare* Appx20362-20363 (75:6-77:9) (discussing computations of constellation values in LG's source code, and showing no match to any values in Annex C), *with* Appx20251 (225:16-19) (agreeing that Akl's computations are correct). This undisputed evidence shows that LG's accused televisions do not implement the standard's constellation values, thus "prov[ing] that it does not practice the standard." *Fujitsu*, 620 F.3d at 1327.

Faced with this evidence, the District Court advanced four points. First, it found that "the accused TVs comply with ... A/322" because "LG itself identifies the accused TVs as ***compatible*** with ATSC 3.0." Appx62 (citing Appx20216 (86:21-87:11) & Appx20359 (61:17-19)). But ***compatibility*** with ATSC 3.0, including the ability to decipher ATSC 3.0 signals, does not equal "compliance" with the A/322 transmitter standard. For example, LG's accused televisions do not use, and thus do not comply with, A/322's constellations. Appx20350 (26:21-27:24). But these televisions are compatible with the A/322-based transmitted signals because they have high-performing decoders that compensate for the differences in constellations. *Id.* (26:25-27:8).

Second, the District Court cited Jones's testimony that "the FCC mandated the use of the A/322 for ATSC 3.0 compatible televisions." Appx62 (citing Appx20218 (93:15-94:10)). But Jones testified only that, "by mandated, what [the

FCC] meant was that if a ***broadcaster*** was going to use a next generation or more advanced physical layer, that this was the physical layer that they were supposed to use." Appx20218 (93:15-94:10).

Third, the District Court relied on LG's statement to the FCC that "A/322 is the component of ATSC 3.0 that ensures that receivers in televisions and other consumer reception devices are able to demodulate an ATSC 3.0 signal," Appx62 (quoting Appx20317 (189:9-13)). This reliance assumed, erroneously, that A/322 only pertains to constellation values. Appx20361 (69:1-3). But A/322 covers a plethora of other technologies—such as single frequency network synchronization, "which makes it easier for a receiver like in your house to receive a signal from multiple transmitted antennas"—that facilitate demodulation. *Id.* (69:24-70:12).

Finally, the District Court reasoned that "a transmitter and a receiver need to use the same constellation," by citing the named inventor's testimony. Appx61 (citing Appx20171 (225:6-15)). But the cited testimony was not so categorical. Appx20171 (225:6-15). He simply stated that, theoretically, a receiver needs "to ***know***" the constellation used by the transmitter. *Id.* But he never said—nor could he say—that a receiver must use the exact same constellation applied by the transmitter. LG's televisions are clear evidence that receivers can, and do, use different constellation values from the ones in the transmitter. Appx20350 (26:21-24); Appx20362-20363 (75:6-77:9).

Accordingly, LG's accused televisions do not comply with the A/322 standard. Constellation had to compare every limitation of the asserted claims to LG's products, but it has not done so. The infringement verdict thus cannot stand.

C. Constellation Failed to Show Infringement for Televisions with a Realtek Chip

At trial, Constellation and its expert separated the accused LG ATSC 3.0-compatible televisions into three groups depending on whether the televisions incorporate a B17+, O22, or K8Hp chip. Appx20216 (86:21-87:5). While LG makes the B17+ and O22 chips, third-party Realtek makes the K8Hp chip. Appx20216 (87:6-8); Appx17171 (JTX-037). Despite having no discovery from Realtek, Constellation asserted that LG's accused televisions using a Realtek chip infringe the asserted claims. In addition to improperly relying on the standard, *see* Sec. III.B, *supra*, Constellation relied on speculation about the Realtek chip and testimony unrelated to this chip. This is not substantial evidence. For this independent reason, the infringement verdict must be reversed for televisions with a Realtek chip.

1. There Is Insufficient Evidence that the Realtek Chip Meets the Structural Limitations of Any Asserted Claim

Realtek never provided discovery, such as source code, about its K8Hp chip. Appx20249 (219:18-22), Appx20251 (228:9-24). As the District Court instructed the jury, "Realtek is a third party. … It's their decision as to whether or not to make their source code available to anybody else. They have not made it available to either of these two parties in this case." Appx20365 (88:17-21). Nor does Realtek share its chip's technical details with a chipmaking competitor like LG, leaving LG without knowledge of how Realtek implements ATSC 3.0. Appx20350-20351 (28:12-29:13).

Lacking the necessary information, Jones asserted that the structural limitations were met based on a single slide (reproduced below) in a five-page, high-level Realtek document produced by LG:

REALTEK CONFIDENTIAL INFORMATION

[redacted]

Appx18048. This slide, according to Jones, supposedly shows that the Realtek chip meets the "demodulator," "demapper," and "decoder" structural limitations in the '922 patent. Appx20222 (111:21-112:5); Appx20228 (135:15-21); Appx20229

(137:10-18, 139:4-12). For similar limitations in the other independent asserted claims, he referred back to his discussion of the ’922 patent. Appx20231 (146:14-148:4), Appx20232 (151:13-152:15), Appx20233-34 (156:13-18, 157:3-8).

Jones’s assertions that the single slide somehow discloses the claimed “demapper” and “decoder” are unsupported. Nowhere on this slide do the words “demapper” and “decoder” (or variations thereof) appear. Appx18048. The slide is instead titled “ATSC 3.0 ***Demodulator*** Block Diagram,” and, at best, it depicts an “ATSC 3.0 ***Demodulator***” with several inner blocks. Appx18048. Under Jones’s logic, a decoder and demapper would be inside the demodulator, even though the “demodulator” in the claims is a separate structure coupled to the demapper, which is, in turn, coupled to the decoder. Appx141 (14:10-17); Appx197 (13:47-57); Appx447 (24:31-41). So Jones pointed to the word “BICM” in the slide’s last bullet point as both his demapper and his decoder, even though the slide does not depict any “BICM” block—it instead depicts two “2BICM” blocks, which are described separately from “BICM.” *Compare* Appx18048, *with* Appx20229 (137:10-18, 139:4-12). Jones never reconciled these inconsistencies, leaving his testimony without adequate support. *See ParkerVision, Inc. v. Qualcomm Inc*., 621 F. App’x 1009, 1017 (Fed. Cir. 2015) (affirming noninfringement JMOL where patentee’s expert made a “conclusory statement” and failed to offer “any explanatory testimony or other evidence” on how the product could operate as claimed).

Although Jones also mentions his own and LG's testing to show that the K8Hp chip can process ATSC 3.0 signals, Appx20221 (105:3-108:20); Appx20222 (110:3-111:20), this is insufficient to salvage infringement. Throughout his testimony about the '922 patent's structural limitations, he never mentioned any testing results. Appx20228-29 (134:15-139:12). As his testimony for related limitations in the other three patents referred back to his '922 patent testimony, the related limitations could not be met through testing results. Appx20231 (146:14-148:4); Appx20232 (151:13-152:15); Appx20233-34 (156:13-157:8). Without any testimony on point, a reasonable jury could not infer that any testing results show a demapper or a decoder in the Realtek chip. *See AquaTex Indus., Inc. v. Techniche Sols*., 479 F.3d 1320, 1329 n.7 (Fed. Cir. 2007) (requiring expert testimony).

2. There Is No Evidence that the Realtek Chip Uses Likelihoods

The asserted claims require "likelihoods," which, according to Jones, are "probabilities or LLRs, likelihood ratios, that indicate the probability of whether [the receiver] received a 1 or a 0."[11] Appx20219 (98:8-17). These "likelihoods" must be determined by a demapper and provided to a decoder. Appx447 (24:33-41).

[11] While the '509 patent's asserted claims do not expressly recite "likelihoods," Constellation and the District Court viewed these claims' infringement as rising or falling with the '922 patent. *See* Appx68 (District Court recognizing that Jones "referred to the '922 Patent to show that the other Asserted Patents were infringed and the '922 Patent recites 'likelihoods.'").

Even if the jury could "infer that the LG TVs with Realtek chips had demappers and decoders," Appx68, there is still no evidence that the Realtek chip uses likelihoods. Jones's cited slide does not mention or depict any likelihoods. Appx18048. Nor did he mention, much less show, any likelihoods in discussing the supposed demapper and decoder in Realtek's chip, even though he said he found likelihoods in LG's own chips. *Compare* Appx20229 (137:10-18 & 139:4-12) (Realtek chip), *with id.* (137:19-138:2, 138:22-139:3) (LG chips). And merely having a demapper or decoder, if any, does not mean that there are likelihoods. According to the named inventor, there are two alternative types of decoding operations in this field: (1) "soft decoding," which is "where you compute likelihoods," and (2) "hard decoding," which picks the closest symbol without using likelihoods. Appx20171 (226:11-17). Consistent with the inventor's testimony, the A/327 Recommended Practice Guidelines expressly acknowledge that ATSC 3.0-compatible receivers can use either "hard" or "soft" decoding. Appx15961-15962; Appx15972-15973. But there was no evidence about which type, if any, the Realtek chip implements. The jury therefore had no basis to find that the Realtek chip has a demapper generating likelihoods and a decoder using likelihoods. *See Forest Labs., Inc. v. Abbott Labs.*, 239 F.3d 1305, 1312 (Fed. Cir. 2001) (affirming noninfringement JMOL where patentee failed to present evidence that the accused product had claimed water percentages).

The District Court nevertheless found "substantial evidence that the ATSC 3.0 TVs with Realtek chips utilize 'likelihoods'" based on general testimony that "digital communications systems" and "ATSC 3.0 televisions use likelihoods." Appx69-70. But the cited testimony was not specific to Realtek chips. Appx20170-20171 (223:13-227:10); Appx20358-20359 (59:11-61:1); Appx20219 (98:9-99:10). This is especially problematic given the undisputed evidence that receivers can operate without likelihoods by using hard decoding. Appx20171 (226:11-17); Appx15961-15962. Accordingly, the jury had to speculate about the possible use of likelihoods in the Realtek chips, while the District Court ignored this evidentiary gap. *See* Appx70. This is legally erroneous and requires reversal.

3. There Is No Evidence the Realtek Chip Uses the Accused Constellations

For at least one limitation in each asserted claim, Jones relied on the constellations in A/322 (or on documents purporting to describe those constellations). Appx20229-20230 (139:13-142:12); Appx20231-20232 (148:5-150:7); Appx20233 (153:3-14); Appx20233 (154:3-17); Appx20234-20235 (157:9-161:14). Accordingly, Jones had to show that the Realtek chip uses the A/322 standard's constellations.

But he only pointed to his and LG's testing, which showed that LG televisions using Realtek chips could process ATSC 3.0 signals. Appx20221 (105:14-108:16); Appx20222 (110:3-111:20). As LG's own televisions show, a receiver does not need to use A/322's constellations to process an ATSC 3.0 signal. Appx20350 (26:21-27:24). Without Realtek's source code, Jones could not show that Realtek's chip uses A/322's constellations. Appx20251 (228:11-12); *see also* Appx20351 (29:9-30:3) (Jeong explaining source code is necessary to determine the constellations in Realtek's chip). Since Jones's infringement theory required A/322's constellations, the absence of such evidence left the jury with no basis to find infringement. *See Forest Labs.*, 239 F.3d at 1312.

Nonetheless, the District Court decided that A/322's constellations were in Realtek's chips based on the named inventor's testimony that supposedly "establishes that the TVs use the same constellations" as the transmitter. Appx70-71 (citing Appx20171 (225:6-15)). This decision was erroneous because the cited testimony addressed the background of the purported invention—not televisions, much less LG's televisions with Realtek chips. Appx20167-20174 (211:14-239:2). Further, the named inventor only stated that a demapper "***need[s] to know*** the constellation [] used in order to successfully demap a receive signal." Appx20171 (225:6-15). Knowing and using are two different things. Even if a Realtek chip knew the constellation used by a transmitter, it could ***use*** a different constellation to process ATSC 3.0 signals, as LG's chips do. Appx20350 (26:21-27:8). The cited testimony cannot fill the evidentiary gap on the claimed constellations.

In sum, there are large evidentiary gaps regarding the claimed structural limitations, likelihoods, and constellations with respect to Realtek's chips. No reasonable jury could have found infringement by LG televisions with Realtek chips.

IV. NO REASONABLE JURY COULD HAVE FOUND THE DAMAGES AWARDED AT TRIAL

The Court should reverse the District Court's denial of JMOL of no damages, or alternatively the denial of LG's damages-related *Daubert* motion, because the District Court applied an erroneous legal standard at both junctures. *Riles v. Shell Expl. & Prod. Co.*, 298 F.3d 1302, 1308 (Fed. Cir. 2002) (explaining JMOL decisions are reviewed for legal error or lack of substantial evidence); *Highmark Inc. v. Allcare Health Mgmt. Sys., Inc.*, 572 U.S. 559, 564 n.2 (2014) (explaining abuse of discretion results from "an erroneous view of the law or on a clearly erroneous assessment of the evidence").

The District Court erred by allowing Constellation's damages expert to stretch the built-in apportionment doctrine past its breaking point by using a third party's licenses—covering different patents, different technologies, and different product types—that are not "sufficiently comparable" and thus do not meaningfully reflect the value of Constellation's asserted patents. It further erred by permitting Constellation's expert to improperly increase the royalty rate for general historical inflation. Reversal of these errors is necessary.

A. Damages-Related Background

In this case, Constellation accused LG's ATSC 3.0-compatible televisions. Appx20273 (14:17-24). Constellation's technical expert testified that one chip allegedly implements Constellation's patents in these televisions, Appx20216 (86:25-87:11), but conceded that these televisions include a host of unaccused technologies and features. Appx20216 (86:15-20); Appx20240-20241 (184:3-185:8); Appx20291 (85:22-86:19); Appx20322 (211:25-212:12). He also admitted that a key aspect of his infringement analysis focuses on a single annex (Annex C) of a single standard specification (A/322) within the ATSC 3.0 suite of different technology standards. Appx20219-20220 (100:2-10, 101:20-102:2); Appx15764-15777 (A/322 Annex C); Appx20241-20242 (185:20-187:1); *see also* Appx20359-20361 (64:13-68:20, 69:1-71:5).

Constellation's damages expert, Ryan Sullivan ("Sullivan"), opined that LG owed $1,684,469 in past damages for the accused televisions. Appx20280 (42:8-10). Although the past damages amount may be perceived as low, Sullivan's royalty rate was a substantially high $6.75 per-unit for each television and the District Court adopted this rate in imposing an ongoing forward-looking royalty. Appx20273 (15:2-3); Appx83-86. Because Constellation never entered into a royalty-bearing license for any asserted patent, Sullivan used a $5 per-unit rate from three of nineteen licenses granted by third-party Zenith.[12] Appx20274-20275 (19:16-21:11); Appx20276 (27:7-24); Appx17112-17127; Appx16578-16611; Appx17128-17142. Sullivan used these three Zenith licenses even though they involved a different licensor (Zenith), preceded the 2020 hypothetical negotiations by about 15 years (2004-2005), and primarily covered a different technology called vestigial sideband ("VSB") signal modulation. Appx20248 (213:15-17, 214:5-9); Appx20274-20275 (19:16-21:11); Appx20277 (32:18-23). Sullivan then increased Zenith's $5 per-unit rate by 35% (to his final $6.75 rate) for inflation based on the general Consumer Price Index ("CPI"). Appx20277 (31:1-7); Appx20278 (33:20-22); Appx20285 (64:16-21).

The $6.75 rate is disproportional to the market because two existing patent pools of ATSC 3.0 patents charge much lower rates. The Avanci patent pool charges between $2 and $3 per unit for a license to over 11,000 declared patents covering the gamut of different technologies in the ATSC 3.0 suite of standards. Appx20279 (37:6-18); Appx20393-94 (197:2-198:17, 201:7-203:7). And a nascent patent pool formed by MPEG LA charges $2.75 per unit for 53 U.S. patents relevant to ATSC 3.0. Appx20279 (37:6-18); Appx20397 (213:24-214:5). Sullivan's $6.75 rate for four patents exceeds the sum of the rates for both ATSC 3.0 patent pools.

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[12] Although Zenith is a subsidiary of LG, Appx20264 (277:5-9), Zenith is not a named party in this case.

Nonetheless, the jury adopted Sullivan's $6.75 per-unit rate, as shown by its award of his exact damages amount. Appx38. The District Court denied LG's pre- and post-trial motions on damages, and adopted this rate in its award of an ongoing forward-looking royalty. Appx20058 (227:11-228:15); Appx43-52; Appx78-82; Appx83-86.

B. The Zenith Licenses Are Not "Sufficiently Comparable" For Built-In Apportionment

1. Apportionment Was Necessary

The apportionment requirement is well-established: "[T]he patentee … must in every case give evidence tending to separate or apportion … the patentee's damages between the patented feature and the unpatented features[.]" *LaserDynamics, Inc. v. Quanta Computer, Inc.*, 694 F.3d 51, 67 (Fed. Cir. 2012). Accordingly, when a patentee accuses multi-component products having both patented and unpatented features, "[t]he essential requirement is that the ultimate reasonable royalty award must be based on the incremental value that the patented invention adds to the end product." *Ericsson, Inc. v. D-Link Sys., Inc.*, 773 F.3d 1201, 1226 (Fed. Cir. 2014); *see also VirnetX, Inc. v. Cisco Sys., Inc.*, 767 F.3d 1308, 1326 (Fed. Cir. 2014).

Apportionment is essential in this case for at least three reasons. First, only one chip in the accused LG televisions allegedly implements Constellation's patents. Appx20216 (86:25-87:11). But LG's accused televisions undisputedly include a vast array of unaccused technologies and features. Appx20216 (86:15-20); Appx20240-20241 (184:3-185:8); Appx20291 (85:22-86:19); Appx20322 (211:25-212:12). Where, as here, "multi-component products are involved, the governing rule is that the ultimate combination of royalty base and royalty rate must reflect the value attributable to the infringing features of the product, and no more." *Ericsson*, 773 F.3d at 1226.

Second, Constellation's infringement analysis targets only a few constellations appearing on 14 pages of the 262-page A/322 standard, which in turn belongs to the large ATSC 3.0 suite of standards. Appx20241 (185:20-187:1); Appx20359-61 (64:13-68:20, 69:1-71:5). Yet, Sullivan equated the patented technology to ATSC 3.0 and tied the value of Constellation's patents to all of ATSC 3.0. Appx20275 (22:4-23:15); Appx20299 (117:19-118:13). Not only did he conflate the patents with ATSC 3.0, but he also failed to separate the allegedly patented features from the A/322 standard or from the ATSC 3.0 suite of standards. This approach violates precedent. *Ericsson*, 773 F.3d at 1232 (vacating award because "patentee's royalty must be premised on the value of the patented feature, not any value added by the standard's adoption of the patented technology").

Third, Sullivan failed to apportion out the value of non-asserted patents. The Zenith licenses granted rights to patents for VSB technology that Zenith, not Constellation, owned. Appx20293 (94:10-15). Sullivan improperly failed to account for these non-asserted patents. *See Ericsson*, 773 F.3d at 1227 (Fed. Cir. 2014) (explaining "allegedly comparable licenses may cover more patents than are at issue in the action," and so "[t]estimony relying on licenses must account for such distinguishing facts when invoking them to value the patented invention"); *see also Trell v. Marlee Elecs. Corp.*, 912 F.2d 1443, 1447 (Fed. Cir. 1990). Worse yet, his $6.75 rate covers all of Constellation's patents, not just the four patents at trial, because he opined that the same rate applies regardless how many patents or claims infringed. Appx20280 (42:11-23); Appx20294 (97:21-98:2). This non-apportioned one-rate-fits-all violates precedent. *Omega Pats., LLC v. CalAmp Corp.*, 13 F.4th 1361, 1376-77, 1379 (Fed. Cir. 2021) (rejecting award which applied "the same rate no matter how many claims or how many of the patents it infringes").

Apportionment was therefore necessary. Yet, Sullivan never mentioned the word "apportionment" or its derivatives at any point during his testimony.

2. The District Court Incorrectly Relied on Built-in Apportionment

Despite Sullivan's lack of apportionment, the District Court permitted the jury to hear his damages theory and then refused to correct the legal error post-trial. Before trial, the District Court denied LG's *Daubert* motion by reasoning that the Zenith licenses allegedly "are not so far removed as to be inherently non-comparable." Appx20057 (225:9-15). After trial, the District Court sustained the damages theory by finding that Sullivan applied built-in apportionment using the

2004-2005 Zenith agreements. Appx1420-1421; Appx48. As the District Court reasoned, "all that is required is that the [Zenith] license(s) be 'sufficiently comparable,'" Appx48-50, because "the Federal Circuit has never held" that the "'baked-in' rule cannot apply" where the relied-upon license involves distinct patents and different technology, Appx49. The District Court then denied LG's post-trial JMOL motion because Sullivan's analysis had survived *Daubert* and the District Court viewed LG's JMOL challenge as revisiting the *Daubert* challenge. Appx48-50. In denying LG's *Daubert* and JMOL motions, the District Court committed the same legal error in relying on built-in apportionment.

a. This Court's Built-in Apportionment Doctrine Is Narrowly Circumscribed

The built-in apportionment doctrine requires that "the license must be 'sufficiently comparable' in that 'principles of apportionment were effectively baked into' the purportedly comparable license." *Omega*, 13 F.4th at 1377 (citation omitted).

Omega is instructive on this doctrine. Relying on a built-in apportionment theory, patentee Omega advocated for a one-size-fits-all $5 rate (regardless of which patents were licensed) based on 18 licenses covering devices with the same functionality as the asserted '278 patent (although only two licenses covered this patent). *Id.* at 1379. This Court, however, rejected Omega's rate as legally insufficient because Omega "failed to show that these agreements attributed a $5.00-

per-unit royalty to the value of the ’278 patent” and because Omega could not show that its “patent/claim-independent approach” sufficiently accounted for apportionment. *Id.* And even for the two licenses which included the ’278 patent (along with many other patents), Omega did not “adequately account for substantial ‘distinguishing facts’ between the proffered licenses and a hypothetical negotiation over a single-patent license to the ’278 patent.” *Id.* at 1380. Omega’s expert “merely *identified* such differences.” *Id.* at 1381 (original emphasis). The Court therefore vacated the award in *Omega*.

Given the strict requirements for finding built-in apportionment, this Court has applied this doctrine in only a handful of cases. Almost always, the relied-on licenses involved the asserted patents. *EcoFactor, Inc. v. Google LLC*, 104 F.4th 243, 255 (Fed. Cir. 2024); *Pavo Sols. LLC v. Kingston Tech. Co.*, Inc., 35 F.4th 1367, 1380 (Fed. Cir. 2022); *Vectura Ltd. v. Glaxosmithkline LLC*, 981 F.3d 1030, 1041 (Fed. Cir. 2020); *Commonwealth Sci. & Indus. Resch. Org. v. Cisco Sys., Inc.*, 809 F.3d 1295, 1303 (Fed. Cir. 2015). When the relied-on licenses included additional patents, this Court required the patentee to appropriately account for those additional patents. *See EcoFactor*, 104 F.4th at 255; *Vectura*, 981 F.3d 1040-41 (evidence that the “key component” of the relied-upon license was closely related to the asserted patent); *see also Omega*, 13 F.4th at 1380-81.

In two other cases, the relied-on licenses involved extenuating circumstances (not relevant here) that justified permitting the application of built-in apportionment. *Bio-Rad Lab'ys, Inc. v. 10X Genomics Inc.*, 967 F.3d 1353, 1373-74 (Fed. Cir. 2020) (two licenses dealing with same microfluid technology, and the defendant did not seek to exclude the third license dealing with other technology); *Elbit Sys. Land & C4I Ltd. v. Hughes Network Sys., LLC*, 927 F.3d 1292, 1300 (Fed. Cir. 2019) (defendant presented no damages expert, and its technical expert agreed the license's technology was the "closest" comparator).

This Court's precedent thus consistently requires a relied-on license to reflect the value of the asserted patent itself for built-in apportionment to apply. *E.g.*, *Vectura*, 981 F.3d at 1041 ("Built-in apportionment effectively assumes that the negotiators of a comparable license settled on a royalty rate and royalty base combination ***embodying the value of the asserted patent***.").

b. This Court Has Never Sanctioned Built-In Apportionment Under These Circumstances

The District Court's approval of built-in apportionment based on a third-party's decades-old licenses of different patents in the context of different technology and products eviscerates the guardrails around built-in apportionment.

At the threshold, Sullivan committed the same error as Omega's expert—he presented the same impermissible "patent/claim-independent approach" that *Omega* rejected, by pushing for a $6.75 rate regardless of which or how many patent claims are infringed. Appx20280 (42:11-23); Appx20294 (97:21-98:2). He also merely identified differences between the Zenith licenses and the hypothetical negotiation, without ever accounting for these differences. *Compare* Appx20277 (30:21-25), *with Omega*, 13 F.4th at 1381. He should have done more because the differences between the hypothetical negotiation and the Zenith licenses are significant.

First, the patents are quite different, since none of the Zenith patents in these licenses overlap with the Constellation patents asserted at trial. Appx20276 (25:7-23); Appx20293 (94:10-15); Appx15551 (listing Zenith patents).

Second, the technologies involved are also substantially different, as Sullivan admitted. Appx20293 (94:7-9). Zenith's licenses relate to VSB, a type of signal modulation used in the prior generation standard called ATSC 1.0, which is technologically different from the accused ATSC 3.0 suite of standards. Appx20292 (91:2-13). For example, ATSC "1.0 used VSB while 3.0 uses OFDM," with OFDM replacing VSB in ATSC 3.0. *Id.* (91:5-6). This replacement of VSB is why ATSC "3.0 is not backward compatible with 1.0." *Id.* (91:10-13). As another example, ATSC 1.0 and 3.0 use different compression technologies. *Id.* (91:7-9). And as a third example, ATSC 1.0 used uniform constellations, Appx20238 (174:20-25), while Constellation's patents purportedly cover non-uniform constellations, Appx20257 (251:5-9).

Third, the Zenith licenses' age and licensor differ from the hypothetical negotiation. Zenith, not Constellation, was the licensor and executed its licenses in 2004-2005, more than 15 years before the date of the hypothetical negotiation in 2020. Appx20274 (17:16-22); Appx20290 (81:9-18); Appx20294 (97:16-20). Unlike Constellation, Zenith was an industry player that had manufactured products and substantially contributed to the standardization of ATSC 1.0. Appx20276 (25:9-23); Appx20183 (275:14-22); Appx20181-20182 (267:12-22, 268:25-269:4).

Fourth, the licensed products are quite different. Unlike the accused televisions at issue in the hypothetical negotiation, Zenith's licenses covered a broad range of consumer electronics, including televisions, computers, cable boxes, video recorders, camcorders, converter boxes, set-top boxes, and satellite boxes. Appx20293 (94:21-95:1, 96:3-21). Even when focusing on just televisions, the accused LG televisions are thinner, bigger, lighter, cheaper, and display much higher resolution than the older televisions sold in the mid-2000s. Appx20293 (94:21-95:1); Appx20290-20291 (82:7-86:19); Appx20286-20287 (68:19-70:14). In fact, the accused Smart TVs here incorporate technology for streaming shows, movies, and news over the internet, while the older televisions covered by the Zenith licenses did not have that technology. Appx20290-20291 (83:13-85:9, 86:4-19).

Fifth, the geographical scope of the Zenith licenses differs from the hypothetical negotiation. While the hypothetical negotiation focuses on U.S. rights, Zenith's licenses covered sales in the U.S., Canada, and Mexico. Appx20294 (97:7-11).

In sum, the patents, technologies, parties, timing, accused products, and geographical scope of the Zenith agreements are so different from the hypothetical negotiation that those licenses cannot be comparable or reflect any "baked-in apportionment." Sullivan acknowledged these differences but did not adjust for them.

c. Constellation's Experts Impermissibly Relied on Loose or Vague Points of Comparison

To circumvent the requirements of built-in apportionment and sweep the significant differences under the proverbial rug, Sullivan relied on vague points of comparison provided by Constellation's technical expert. Appx20276 (26:20-22).

This "technical" analysis involved running keyword searches on Zenith's VSB patents and the asserted Constellation patents for the following generic terms: (i) "ATSC" (regardless of which standard); (ii) "physical layer," (iii) "receiver," (iv) "demodulator," (v) "demapper," and (vi) "decoder." Appx20237 (170:24-171:4); Appx20276 (26:20-22). As the named inventor admitted, these components existed well before his invention. Appx20186-20187 (286:17-289:6). And although some Zenith patents lack certain terms, Appx20237 (171:20-22), Constellation's technical expert still deemed them technically "comparable" to Constellation's patents because the Zenith patents purportedly "are within the same use area [and] they contain many of the same elements." Appx20237 (171:23-172:2).

The mere fact that both sets of patents refer to generic, decades-old equipment cannot mean they are technologically comparable. *See Omega*, 13 F.4th at 1379. Rather, these points of alleged comparability are so vague and loose that they are no better than those rejected in *LaserDynamics*. 694 F.3d at 79 ("[A] loose or vague comparability between different technologies or licenses does not suffice."); *see also ADASA Inc. v. Avery Dennison Corp.*, 55 F.4th 900, 915 (Fed. Cir. 2022) (rejecting "RFID technology" as "too broad and vague a category").

To buttress this flimsy technical keyword search, Sullivan also pointed to alleged points of economic comparability between the Zenith licenses and the hypothetical negotiation, including the use of a running per-unit rate, the coverage of televisions, the duration for the patents' life, the non-exclusivity of the license, the licensors' non-manufacturing business, and LG as a licensee. Appx20276-20277 (25:9-10, 27:15-29:4, 31:8-13). These comparison points are much too loose and vague, with each one possibly implicating hundreds of licenses or more. But even if these loose comparison points found in many licenses could be economically sufficient, they cannot rescue the unduly vague technical comparability analysis. *See Wordtech Sys., Inc. v. Integrated Networks Sols., Inc.*, 609 F.3d 1308, 1319-20 (Fed. Cir. 2010) (comparability mandate is both technical and economic).

If these loose and vague technical and economic comparison points sufficed, all patentees would simply argue that non-comparable licenses, regardless of how different or dissimilar they may be to the hypothetical negotiation, have built-in apportionment based on broad, loose, or vague keywords or catch-phrases. The built-in apportionment doctrine would become the exception that swallows the rule.

C. Dr. Sullivan's Sole Adjustment, Increasing Damages by 35% for Inflation, Lacks Legal and Factual Support

Using the general CPI to inflate a comparable license's rate is both unprecedented and akin to using the 25% rule. Here, the general CPI is not only irrelevant, but inapplicable to the facts of this case.

First, Sullivan's 35% inflation uplift lacks case law support. Constellation's only cited case, *Minnesota Mining and Manufacturing Co. v. Johnson & Johnson Orthopaedics, Inc.*, 976 F.2d 1559, 1579 (Fed. Cir. 1992), is inapposite. There, in view of 3M's lost profits theory based on price erosion, this Court agreed that "3M would have been able to increase its prices 2% per annum during the period of infringement if [defendant] JJO had not been competing in the market," given that (1) 3M presented evidence it "would have raised prices approximately 4% per year ***to match the rate of inflation***," and (2) defendant and competitor JJO argued to the contrary that "there would have been zero inflation" in the relevant market. *Id.* at 1578-79. Here, in contrast, Constellation did not advance a theory of lost profits based on price erosion; Constellation and LG are not competitors; and neither party has a policy of increasing prices to match the general rate of inflation. *Minnesota* does not support Sullivan's inflation uplift.

Nor does Sullivan's uplift have any real-world factual support. Appx20285 (64:16-21). His uplift relied on the ***general*** CPI, which covers a "large basket of goods and services" including food, housing, apparel, transportation, and medical care. Appx20279 (39:9-40:8); Appx20286 (66:8-67:20). But he ignored the ***television-specific*** CPI, which shows that television prices have markedly declined since the 2000s. Appx20286-20287 (68:3-70:14); Appx20290 (83:2-4). Thus, the only evidence about pricing in the relevant market shows that prices for televisions have ***dropped***.

Sullivan also pointed to a 1997 PriceWaterhouseCoopers ("PwC") report. Appx20278 (34:4-17). But PwC's recommendation that Zenith adjust for inflation never came to fruition, as no one paid Zenith any royalty rates adjusted for inflation, Appx20278 (35:3-19); Appx20327 (230:20-23), and the significant price drop from 2002 to 2010 shows that inflation was not a pricing factor for televisions, Appx20278 (36:6-21).

Finally, Sullivan relied on three "professional" Zenith agreements (distinct from the other agreements used to derive a starting $5 rate) covering broadcast equipment, not consumer products like televisions. Appx20287-20288 (72:6-73:24, 74:1-13). But neither he nor Constellation's technical expert ever opined that these "professional" agreements are comparable to the hypothetical negotiation. So, these agreements are irrelevant.

Because the jury's damages award lacks any adequate legal or factual basis as set forth above, this Court should vacate the damages award.

CONCLUSION

The Court should reverse the District Court's patent eligibility judgment, liability judgment, and damages judgment.

Dated: August 19, 2024

Respectfully submitted,

/s/ Michael J. McKeon

Michael J. McKeon
Christian A. Chu
R. Andrew Schwentker
Michael J. Ballanco
Benjamin J. Christoff
FISH & RICHARDSON P.C.
1000 Maine Avenue, S.W.
Suite 1000
Washington, DC 20024
Telephone: (202) 783-5070

Ashley A. Bolt
FISH & RICHARDSON P.C.
1180 Peachtree Street NE
21st Floor
Atlanta, GA 30309
Telephone: (404) 892-5008

Attorneys for Defendants-Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc.

ADDENDUM

2:21-cv-00448 Docket Entry	**Date**	**Description**	**Appendix Page Numbers**
267	06/27/2023	Order re Pretrial Motions and Motions *In Limine*	Appx1-Appx6
303	08/23/2023	Final Judgment	Appx40-Appx42
364	04/23/2024	Order Denying LG's Motion for JMOL of No Damages	Appx43-Appx52
366 (redacted)	04/23/2024	Order Denying LG's Motion for JMOL of No Liability	Appx53-Appx77
371	04/26/2024	Amended Final Judgment	Appx83-Appx86
JTX-001		U.S. Patent No. 8,842,761 [CD-0001032-0001086]	Appx87-Appx141
JTX-002		U.S. Patent No. 10,693,700 [CD-0001290-0001348]	Appx142-Appx200
JTX-003		U.S. Patent No. 11,019,509 [CD-0001537-0001597]	Appx201-Appx261
JTX-004		U.S. Patent No. 11,018,922 [CD-0001349-0001536]	Appx262-Appx449

CONFIDENTIAL MATERIAL REDACTED

The material redacted at Appx66 is confidential technical information, including third-party information.

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CONSTELLATION DESIGNS, LLC, *Plaintiff*, v. LG ELECTRONICS, INC., LG ELECTRONICS USA, INC., LG ELECTRONICS ALABAMA INC, *Defendants*.	§ § § § § § § § § § § §	CIVIL ACTION NO. 2:21-CV-00448-JRG

MEMORANDUM OPINION AND ORDER

The Court held a Pretrial Conference in the above-captioned matter on Thursday, June 15, 2023 regarding pending pretrial motions and motions *in limine* ("MILs") filed by Plaintiff Constellation Designs, LLC ("Plaintiff" or "Constellation") and Defendants LG Electronics, Inc., LG Electronics USA, Inc., and LG Electronics Alabama Inc. ("Defendants" or "LG"). (Dkt. Nos. 130, 131, 129, 133, 137, 134, 135, 136, 132, 128, 127, 200, 201, and 241.) This Order memorializes the Court's rulings on the aforementioned pretrial motions and MILs as announced into the record, including additional instructions that were given to the Parties. Although this Order summarizes the Court's rulings as announced into the record during the Pretrial Conference, this Order in no way limits or constrains such rulings from the bench. Accordingly, it is hereby **ORDERED** as follows:

PRETRIAL MOTIONS

Defendants' Motion for Partial Summary Judgment of Noninfringement for Products Containing a Realtek Chip (Dkt. No. 130)

The motion was **DENIED**. (Dkt. No. 257 at 47:10–25.) The Court held that there are questions of fact that preclude the entry of summary judgment.

Defendants' Motion to Strike Portions of the Opening Report of Dr. Mark Jones (Dkt. No. 131)

The motion was **GRANTED-IN-PART and DENIED-IN-PART**. (Dkt. No. 257 at 49:7–24; 59:9–61:12.) The Court struck "and/or knew and specifically intended infringement of the asserted patents" from Paragraph 97 of Dr. Jones' report. The balance of Defendants' motion was denied.

Plaintiff's Motion for Summary Judgment on LG's Ineligibility Defenses (Dkt. No. 129)

The motion was **GRANTED**. (Dkt. No. 257 at 93:23–95:1.) The Court, after considering the claims at issue and the patents-in-suit and the invention as a whole, was persuaded that the patents-in-suit are not directed primarily to an abstract concept. The claims are focused on improvements of systems and are directed to a patent-eligible subject matter.

Plaintiff's Motion to Strike Certain Portions of the Expert Report of Dr. Robert Akl Relating to His Patent Eligibility Analysis (Dkt. No. 133)

The motion was **GRANTED**. (Dkt. No. 257 at 95:2–11.). In light of the Court's ruling regarding Dkt. No. 129, the Court struck the sections of Dr. Akl's report relating to his patent eligibility analysis.

Plaintiff's Motion to Strike Certain Portions of the Expert Report of Mr. Robert Akl (Dkt. No. 137)

The motion was **GRANTED-IN-PART and DENIED-IN-PART**. (Dkt. No. 257 at 122:4–126:8; 129:23-130:8). The Court struck Paragraphs 58, 59, 65, 68, 325–332, 1318–19, 1359–60, 1822, 1823, 1824, and 1849 of Dr. Akl's Invalidity Report as such paragraphs concern references outside of LG's final election of prior art references and combinations. The parties further agreed (and the Court accepted their agreement) that LG would not contend that the claims are product-by-process. The balance of Plaintiff's motion was denied. The Court intends to enforce its Motions *in Limine*. To the extent Dr. Akl or any other witness violates the Court's Motions *in Limine*, the Court expects the parties to raise an objection to such violation during the trial.

Plaintiff's Motion for Summary Judgment on LG's Improper Inventorship Defense (Dkt. No. 134)

The motion was **GRANTED**. (Dkt. No. 257 at 158:3–159:5.)

Plaintiff's Partial Motion for Summary Judgment as to Written Description and Priority and Motion to Strike Dr. Akl's Opinions in Violation of the Court's Claim Construction Order (Dkt. No. 135)

The motion was **GRANTED**. (Dkt. No. 257 at 191:24–193:8.) The Court struck Defendants' written description defense and the paragraphs in Dr. Akl's report related thereto. The Court further held that the applicable priority date for U.S. Patent No. 11,018,922 is December 30th, 2008.

Plaintiff's Motion to Strike and Exclude Portions of the Expert Report of Brian Napper (Dkt. No. 136)

The motion was **DENIED**. (Dkt. No. 257 at 205:8–206:13.)

Defendants' Motion to Strike the Expert Report of Ryan Sullivan (Dkt. No. 132)

The motion was **GRANTED-IN-PART and DENIED-IN-PART**. (Dkt. No. 257 at 225:1–228:9.) The Court struck paragraphs 450 through 462 of Dr. Sullivan's report as the report fails to adequately account for the patents' essentiality in analyzing *Georgia-Pacific* factors 8-10. The remainder of the motion was denied.

Plaintiff's Motion for Summary Judgment and Motion to Strike on or, in the alternative, to Strike LG's Equitable Defenses (Dkt. No. 128)

The motion was **DENIED**. (Dkt. No. 257 at 232:24)

Defendants' Motion to Strike the Supplement and Errata of the Expert Reports of Dr. Mark Jones (Dkt. No. 127)

The motion was **DENIED-AS-MOOT**. (Dkt. No. 257 at 61:13–62:3.)

MOTIONS *IN LIMINE*

It is **ORDERED** that the Parties, their witnesses, and counsel shall not raise, discuss, or argue the following before the venire panel or the jury without prior leave of the Court:

PLAINTIFF'S OPPOSED MOTIONS *IN LIMINE* (Dkt. No. 201)

Plaintiff's MIL 1 **Preclude LG from introducing any argument, evidence, or suggestion concerning specific patents beyond the asserted patents, prior art, or patents in the Zenith comparable licenses, including suggesting LG or ATSC 3.0 is practicing specific LG patents or other patents from participants in ATSC 3.0.**

The MIL was **DENIED**. (Dkt. No. 257 at 244:6–245:22.)

Plaintiff's MIL 2 **Any argument, evidence, testimony, reference, or suggestion that Constellation Designs has not asserted its patents against other entities, including Samsung or Sony, and associated settlement discussions with those third parties.**

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 246:20–248:6; Dkt. No. 252 at 2.)

Plaintiff's MIL 3 **Preclude LG from introducing any argument, evidence, suggestion that there is an obligation to participate in a standard setting organization or that participation in a SSO is necessary to have a patent that covers products related to that standard.**

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 248:13–249:17; Dkt. No. 249 at 2, 3.)

Plaintiff's MIL 4 **Any argument, testimony, evidence, reference to, or suggestion about lump sum damages calculations or implying in any way that Dr. Sullivan's or Mr. Napper's damages calculations result in, or that the jury may award, a "lump sum."**

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 249:22–251:3; Dkt. 249 at 3.)

Plaintiff's MIL 5 **Preclude LG from introducing any argument, evidence, or suggestion**

regarding Fortress or Constellation Designs, LLC receiving funding from Fortress.

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 251:11–252:7.)

DEFENDANTS' OPPOSED MOTIONS *IN LIMINE* (Dkt. No. 200)

Defendants' MIL 1 **To Exclude the IEEE Magazine and Articles within the Magazine, And Testimony Relating Thereto.**

The MIL was **WITHDRAWN**. (Dkt. No. 257 at 253:4–7.)

Defendants' MIL 2 **To Exclude Evidence Suggesting a Failure to Seek Opinion of Counsel after being Allegedly Notified about CD's Asserted Patents.**

The MIL was **WITHDRAWN**. (Dkt. No. 257 at 253:4–7.)

Defendants' MIL 3 **Preclude Any Argument, Evidence, or Testimony that LG Has Improper Influence over the Development of the ATSC Standard.**

The MIL was **DENIED**. (Dkt. No. 257 at 255:22–257:21.)

Defendants' MIL 4 **To Preclude Any Argument, Document, or Testimony Presenting the Patent Pool Members in Derogatory Terms or Implying the Patent Pool Agreements Are Illegal.**

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 259:6–260:2; Dkt. No. 252 at 3.)

Defendants' MIL 5 **To Preclude Any Argument, Document, or Testimony Regarding Zenith's Bankruptcy and LG's Ownership of Zenith.**

The MIL was **GRANTED-AS-MODIFIED** per the parties' agreement as annotated in the record. (Dkt. No. 257 at 260:8–261:15; Dkt. No. 252 at 3.)

COURT MOTIONS *IN LIMINE*

Refer to the Court's Standing Order on Motions *in Limine*.

PLAINTIFF'S MOTION TO DISMISS UNDER RULE 41 (Dkt. No. 241)

The Court **GRANTED** the motion. (Dkt. No. 257 at 22:19–24:12.)

So ORDERED and SIGNED this 27th day of June, 2023.

RODNEY GILSTRAP
UNITED STATES DISTRICT JUDGE

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CONSTELLATION DESIGNS, LLC, *Plaintiff*, v. LG ELECTRONICS, INC., LG ELECTRONICS USA, INC., LG ELECTRONICS ALABAMA INC, *Defendants*.	§ § § § § § § § § §	CIVIL ACTION NO. 2:21-CV-00448-JRG

FINAL JUDGMENT

A jury trial commenced in the above-captioned case on July 5, 2023, and on July 11, 2023, the jury reached and returned its unanimous verdict finding that Defendants LG Electronics Inc., LG Electronics USA, Inc., and LG Electronics Alabama, Inc. (together "LG") infringed at least one of Claims 17, 21, 24, and 28 of U.S. Patent No. 8,842,761 (the "'761 Patent), at least one of Claims 21 and 23 of U.S. Patent No. 11,019,509 (the "'509 Patent"), at least one of Claims 24 and 44 of U.S. Patent No. 11,018,992 (the "'992 Patent"), and Claim 5 of U.S. Patent No. 10,693,700 (the "'700 Patent") (collectively, the "Asserted Claims"); that none of the Asserted Claims were invalid; that LG willfully infringed at least one of the Asserted Claims; and that Plaintiff Constellation Designs LLC ("CD") is owed $1,684,469.00 in the form of a running royalty for LG's infringement. (Dkt. No. 277).

Pursuant to Rule 58 of the Federal Rules of Civil Procedure, and in accordance with the jury's unanimous verdict and the entirety of the record, the Court hereby **ORDERS** and **ENTERS JUDGMENT** as follows:

1. LG has infringed at least one Asserted Claim of each of the ’761 Patent, ’509 Patent, ’992 Patent, and the ’700 Patent;

2. The Asserted Claims are not invalid;

3. LG’s infringement was willful;

4. CD is hereby awarded damages from and against LG and shall accordingly have and recover from LG the sum of $1,684,469.00 U.S. Dollars for past infringement by LG and as a running royalty;

5. Notwithstanding the jury’s finding of willfulness, the Court having considered the totality of the circumstances together with the material benefit of having presided throughout the jury trial and having seen the same evidence and heard the same arguments as the jury, and mindful that enhancement is generally reserved for “egregious cases of culpable behavior,”[1] concludes that enhancement of the compensatory award herein is not warranted under 35 U.S.C. § 284 and consequently, the Court elects not to enhance the damages awarded herein;

6. Pursuant to 35 U.S.C. § 284 and Supreme Court guidance that “prejudgment interest shall ordinarily be awarded absent some justification for withholding such an award,”[2] the Court awards to CD from LG pre-judgment interest applicable to all sums awarded herein, calculated at the 5-year U.S. Treasury Bill rate, compounded quarterly, from the date of infringement through the date of entry of this Judgment;[3]

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[1] *Halo Elecs., Inc. v. Pulse Elecs., Inc.*, 579 U.S. 93, 106 (2016).
[2] *General Motors Corp. v. Devex Corp.*, 461 U.S. 648, 657 (1983).
[3] *See Nickson Indus., Inc. v. Rol Mfg. Co., Ltd.*, 847 F.2d 795, 800–801 (Fed. Cir. 1988).

7. Pursuant to 28 U.S.C. § 1961, the Court awards to CD from LG post-judgment interest applicable to all sums awarded herein, at the statutory rate, from the date of entry of this Judgment until paid; and

8. Pursuant to Federal Rule of Civil Procedure 54(d), Local Rule CV-54, and 28 U.S.C. § 1920, CD is the prevailing party in this case and shall recover its costs from LG. CD is directed to file its proposed Bill of Costs.

All other requests for relief now pending and requested by either party but not specifically addressed herein are **DENIED**.

So ORDERED and SIGNED this 22nd day of August, 2023.

RODNEY GILSTRAP
UNITED STATES DISTRICT JUDGE

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CONSTELLATION DESIGNS, LLC, *Plaintiff*, v. LG ELECTRONICS, INC., LG ELECTRONICS USA, INC., and LG ELECTRONICS ALABAMA, INC., *Defendants*.	§ § § § § § § § § § §	CIVIL ACTION NO. 2:21-CV-00448-JRG **FILED UNDER SEAL**

MEMORANDUM OPINION AND ORDER

I. INTRODUCTION

Before the Court is Defendants LG Electronics, Inc., LG Electronics USA, Inc., and LG Electronics Alabama, Inc.'s (collectively, "LG") Motion for Judgment as a Matter of Law ("JMOL") of No Damages (the "Motion"). (Dkt. No. 310.) Plaintiff Constellation Designs, LLC ("Constellation") opposes the Motion. (*See* Dkt. No. 326.) For the following reasons, the Court finds that the Motion should be **DENIED**.

II. BACKGROUND

Constellation filed a Complaint on December 9, 2021, alleging that LG infringed several of its United States Patents related to digital communications technology, including U.S. Patent Nos. 8,842,761 (the "'761 Patent), 10,693,700 (the "'700 Patent"), 11,018,922 (the "'922 Patent"), and 11,019,509 (the "'059 Patent") (collectively, the "Asserted Patents"). (Dkt. No. 1.) On May 1, 2023, LG moved to strike the expert report of Dr. Sullivan, Constellation's damages expert. (Dkt. No. 132.) The Court denied LG's motion on all grounds. (Dkt. No. 257 at 225:4–226:22, 227:11–20.) A jury trial was held on July 5–7 and 10–11, 2023. At the close of evidence, LG moved for

JMOL under Rule 50(a) of no damages on three bases: (1) "CD failed to prove that the Zenith agreements are technically and economically comparable;" (2) "CD failed to apportion the value of ATSC 3.0;" and (3) "CD's inflation adjustment … lacks an evidentiary basis." (Dkt. No. 293 at 275:9–276:16.) The Court denied LG's Rule 50(a) motion on all grounds. (*Id.* at 277:24–278:3.)

On July 11, 2023, the jury returned a verdict finding that LG infringed all Asserted Patents and that LG's infringement was willful. (Dkt. No. 277 at 4, 6.) The jury also found that LG had failed to prove that any of the asserted claims were invalid. (*Id.* at 5.) Accordingly, the jury awarded damages of $1,684,469.00 in the form of a reasonable royalty for past damages. (*Id.* at 7.)

III. LEGAL STANDARD

"Judgment as a matter of law is proper when 'a reasonable jury would not have a legally sufficient evidentiary basis to find for the party on that issue.'" *Abraham v. Alpha Chi Omega*, 708 F.3d 614, 620 (5th Cir. 2013) (quoting Fed. R. Civ. P. 50(a)). The non-moving party must identify "substantial evidence" to support its positions. *TGIP, Inc. v. AT&T Corp.*, 527 F. Supp. 2d 561, 569 (E.D. Tex. 2007). "Substantial evidence is more than a mere scintilla. It means such relevant evidence as a reasonable mind might accept as adequate to support a conclusion." *Eli Lilly & Co. v. Aradigm Corp.*, 376 F.3d 1352, 1363 (Fed. Cir. 2004).

"The Fifth Circuit views all evidence in a light most favorable to the verdict and will reverse a jury's verdict only if the evidence points so overwhelmingly in favor of one party that reasonable jurors could not arrive at any contrary conclusion." *Core Wireless Licensing S.A.R.L. v. LG Elecs., Inc.*, 880 F.3d 1356, 1361 (Fed. Cir. 2018) (citing *Bagby Elevator Co. v. Schindler Elevator Corp.*, 609 F.3d 768, 773 (5th Cir. 2010)). A court must "resolve all conflicting evidence in favor of [the verdict] and refrain from weighing the evidence or making credibility determinations." *Gomez v. St. Jude Med. Daig. Div. Inc.*, 442 F.3d 919, 937–38 (5th Cir. 2006).

IV. ANALYSIS

A. Apportionment

LG argues that Constellation's damages expert, Dr. Sullivan, did not apportion. (Dkt. No. 310 at 3.) More specifically, LG argues that Dr. Sullivan simply ported over the $5 per-unit from the Zenith license without adjusting the per-unit rate to be "based on the incremental value that the patented invention adds to the end product." (*Id.* at 3–4 (quoting *Ericsson, Inc. v. D-Link Sys., Inc.*, 773 F.3d 1201, 1226 (Fed. Cir. 2014).)

Next, LG contends that there are only two exceptions to the apportionment rule and that Constellation cannot show either of them apply. (*Id.* at 4–9.) First, LG argues that Constellation cannot show that the "patented technology drove demand for the entire product," which would excuse Constellation from the apportionment requirement. (*Id.* at 4 (quoting *VirnetX, Inc. v. Cisco Sys., Inc.*, 767 F.3d 1308, 1329 (Fed. Cir. 2014)).) Second, LG contends that Constellation cannot show that apportionment is "built in" to the comparable Zenith licenses, which would also excuse Constellation from the apportionment requirement. (*Id.* at 5–9 (quoting *Vectura Ltd. v. Glaxosmithkline LLC*, 981 F.3d 1030, 1041 (Fed. Cir. 2020)).) Samsung contends that Dr. Sullivan never asserted that his opinions relied on built-in apportionment. (*Id.* at 5.) Next, Samsung contends that the Zenith licenses covered 13 different patents, none of which are asserted here, which distinguishes this case from two cases in which the Federal Circuit has permitted built-in apportionment. (*Id.* at 5–6 (citing *Pavo Sols. LLC v. Kingston Tech. Co., Inc.*, 35 F.4th 1367, 1380 (Fed. Cir. 2022), *Commonwealth Sci. & Indus. Rsch. Org. v. Cisco Sys., Inc.*, 809 F.3d 1295, 1303 (Fed. Cir. 2015) ("*CSIRO*")).) Also, Samsung contends that the technologies at issue in the allegedly comparable licenses are different from that contained in the Asserted Patents. (*Id.* at 6–7.) Samsung argues that this distinguishes the other two cases in which the Federal Circuit has permitted built-in apportionment. (*Id.* (citing *Vectura*, 981 F.3d at 1041, *Bio-Rad Lab., Inc. v. 10X*

Genomics Inc., 967 F.3d 1353, 1376–77 (Fed. Cir. 2020)).) Further, Samsung argues that the scope of the licenses are different. (*Id.* at 7–8.) Finally, Samsung argues that there are "additional economic differences between the Zenith licenses and the hypothetical negotiation." (*Id.* at 8.)

In response, Constellation first argues that LG's arguments are, in fact, *Daubert* arguments, which are inappropriate at JMOL. (Dkt. No. 326 at 5 (citing *Rembrandt Wireless Techs., LP v. Samsung Elecs. Co.*, No. 2:13-CV-213-JRG, 2016 WL 362540, at *3–4 (E.D. Tex. Jan. 29, 2016), *aff'd*, 853 F.3d 1370 (Fed. Cir. 2017), *Versata Software, Inc. v. SAP Am., Inc.*, 717 F.3d 1255, 1261 (Fed. Cir. 2013). Next, Constellation argues that Dr. Sullivan appropriately "relied on Zenith licenses that capture only the value of the patented technology and are comparable to the license that would result from a hypothetical negotiation over the asserted patents." (*Id.* at 5–6.) Constellation contends that the Federal Circuit permits parties to rely on prior licenses as long as the license is sufficiently comparable. (*Id.* at 6 (citing, among others, *Lucent Techs., Inc.* v. *Gateway, Inc.*, 580 F.3d 1301, 1329 (Fed. Cir. 2009)).) According to Constellation, "[t]he Federal Circuit has also explained that 'when a sufficiently comparable license is used as the basis for determining the appropriate royalty, further apportionment may not necessarily be required' because apportionment is 'built-in.'" (*Id.* (quoting *Omega Pats., LLC v. CalAmp Corp.*, 13 F.4th 1361, 1376–77 (Fed. Cir. 2021)).) Constellation urges that Dr. Sullivan "followed that law to the letter." (*Id.* at 7–8.) Further, Constellation contends, comparability is a question of fact for the jury. (*Id.* at 8 (citing *Bio-Rad Labs*, 967 F.3d at 1373–74).) Also, Constellation contends Dr. Sullivan addressed each of the complaints LG identifies. (*Id.* at 9.) Constellation then acknowledges that there are differences between the hypothetical negotiation and the Zenith licenses, but argues that "LG cites no case law supporting its position that adjustments must be made based on alleged

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technological or economic differences." (*Id.* at 9–10.) Instead, Constellation contends, the Federal Circuit has said the opposite. (*Id.* at 10 (citing *Bio-Rad*, 967 F.3d at 1376).)

In reply, LG first argues that its motion is not a re-urged *Daubert* attack on methodology, and that the cases cited by Constellation to support that assertion are distinguishable. (Dkt. No. 336 at 1 (citing *Rembrandt*, 2016 WL 362540, at *4, *Versata*, 717 F.3d at 1264).) Rather, LG asserts that it is challenging the sufficiency Constellation's damages theory. (*Id.*) LG then notes that Constellation does not dispute that its expert failed to perform a separate apportionment analysis and failed to rely on the other exception to apportionment. (*Id.*)

LG then argues that Constellation cannot show built-in apportionment. (*Id.* at 1–3.) LG first argues that comparability is not the same as built-in apportionment, and that Constellation has conflated the two. (*Id.* at 2 (quoting *Omega Pats.*, 13 F.4th at 1377 ("For built-in apportionment to apply the license must be 'sufficiently comparable' in that 'principles of apportionment were effectively baked into' the purportedly comparable license."')).) Constellation contends that "[c]omparability is necessary, but not sufficient, for built-in apportionment." (*Id.*) Next, LG argues that Constellation has not shown comparability, only "loose similarities." (*Id.* (quotations omitted).) LG then re-urges its argument that the complete lack of overlap of patents means that LG's "built-in" theory fails. (*Id.* at 2–3.)

In sur-reply, Constellation re-urges that LG's arguments are *Daubert* arguments that are inappropriate at this stage. (Dkt. No. 355 at 1.) Constellation also re-urges that Dr. Sullivan baked in apportionment to his analysis. (*Id.* at 1–3.) According to Constellation, it is "textbook Federal Circuit law" that if agreements are sufficiently comparable, apportionment is built-in. (*Id.* at 1 (citing *CSIRO*, 809 F.3d at 1303).) Further, Constellation asserts that "Dr. Sullivan testified at length that his comparable-license analysis appropriately accounted for the value of the patented technology." (*Id.* at 1–2.) Finally, Constellation contends that Dr. Sullivan was not required to make adjustments based on every difference between the Zenith license and the hypothetical negotiation, and that if he were, "Dr. Sullivan explained that many of those differences would have pushed the reasonable royalty higher." (*Id.* at 2–3.)

First, the Court finds that all of LG's arguments regarding apportionment are challenges to the admissibility of Dr. Sullivan's testimony under the guise of challenging the sufficiency of the evidence. *See Versata*, 717 F.3d at 1264. LG argues that *Versata* is distinguishable because the defendant in that case argued that an expert's opinions should have been excluded while LG is not making that argument here. (*See* Dkt. No. 336 at 1 (citing *Versata*, 717 F.3d at 1264).) The Court disagrees—*Versata* is on point. The Federal Circuit in *Versata* rejected an argument as "improperly raised" because "[u]nder the guise of sufficiency of the evidence, [defendant] questions the admissibility of [defendant]'s expert testimony and whether his damages model is properly tied to the facts of the case." *Versata*, 717 F.3d at 1264. So too here, the Court finds that all of LG's arguments regarding apportionment are challenges to the admissibility of Dr. Sullivan's testimony under the guise of challenging the sufficiency of the evidence. *See id.* "Such questions should be resolved under the framework of the Federal Rules of Evidence and through a challenge under *Daubert*." *Id.* All arguments that LG has raised regarding apportionment are therefore improper.

Even so, the Court finds that there is substantial evidence that Dr. Sullivan relied on sufficiently comparable licenses such that he did not need to perform a separate apportionment analysis. The Federal Circuit was clear that "when a sufficiently comparable license is used as the basis for determining the appropriate royalty, further apportionment may not necessarily be required." *Omega Pats.*, 13 F.4th at 1376–77 (quoting *Vectura*, 981 F.3d at 1040). Further, "[f]or

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built-in apportionment to apply the license must be 'sufficiently comparable' in that 'principles of apportionment were effectively baked into' the purportedly comparable license." *Id.* at 1377 (quoting *Vectura*, 981 F.3d at 1041).)

LG argues that the lack of overlap between the patents of the comparable licenses and the patents of the hypothetical negotiation and that the differences in the technologies between the same means that the "baked-in" rule cannot apply. (*See* Dkt. No. 310 at 5–7.) However, the Federal Circuit has never held this. As spelled out above, all that is required is that the license(s) be "sufficiently comparable." *Omega Pats.*, 13 F.4th at 1377 (quoting *Vectura*, 981 F.3d at 1041).)

There is substantial evidence in the record that the licenses are comparable, which is a fact intensive inquiry. *See Bio-Rad Labs*, 967 F.3d at 1373–74. The evidence showed that the agreements (1) relate to similar patented technology—namely, (a) an ATSC physical layer technology (b) incorporated into receivers (c) including demodulators, demappers, symbols, and decoders (d) used in commercial televisions (Dkt. No. 292 at 15:20– 21; 20:22–21:4, 26:7–22; 28:17–19 (Sullivan)); (2) were structured as a running royalty (*Id.* at 28:2–9; 29:10–18; 29:22–25; Dkt. No. 290 at 27:4–7 (Marino) (testifying to proposing an ongoing per-unit royalty in negotiations with LG); (3) included commercial televisions and specifically not semiconductor chips as the licensed product (Dkt. No. 292 at 21:3–4, 28:24–29:4, 31:8 (Sullivan)); (4) extended through the life of the patents (*Id.* at 28:1–23); (5) were nonexclusive (*Id.* at 31:8–13); (6) were entered into by licensors that do not manufacture commercial products but rather license technology (*Id.* at 25:9–10; 27:18–24; 43:15–21; Dkt. No. 290 at 64:21–23 (Marino)); (7) and in some instances, were even entered into by the same licensee, LG (Dkt. No. 292 at 27:15–21 (Sullivan)).

LG contends that this evidence does not show sufficient comparability, and that these "similarities" are "loose." (Dkt. No. 336 at 2.) The Court disagrees. Further, these criticisms invite the Court to weigh the evidence itself, which is impermissible. *See Gomez*, 442 F.3d at 937–38. There is clearly more than a "mere scintilla" of evidence that the licenses are sufficiently comparable. *Eli Lilly*, 376 F.3d at 1363.

LG also argues (1) that Dr. Sullivan did not take adequate account of the differences in the licenses, and (2) that when he did, he did not adjust his rate appropriately. (*See* Dkt. No. 336 at 2–3.) These arguments are squarely challenges to Dr. Sullivan's methodology and are therefore not appropriate at the Rule 50(b) stage. *See Versata*, 717 F.3d at 1264.

B. Whether any Failure to Apportion Caused the Damages Award to Capture More Than the Value of Constellation's Patented Contribution

LG argues that since Dr. Sullivan improperly failed to apportion, he (1) captured the entire value of a relevant standard, the ATSC 3.0 standard, (2) captured the value of unaccused features and components in the accused products, and (3) captured the value of non-asserted patents. (Dkt. No. 310 at 9–14.)

The Court finds that these arguments fail because they depend on Dr. Sullivan failing to properly apportion and, as discussed above, Dr. Sullivan did not fail to properly apportion.

C. Dr. Sullivan's Inflationary Adjustment

Dr. Sullivan adjusted the $5 per unit from the Zenith licenses, the earliest of which was signed in 2005, upwards to $6.75 per unit. (*See* Dkt. No. 326 at 10.)

LG argues that this upward adjustment is improper because (1) none of the Zenith licenses permit an upward adjustment for inflation, and (2) Dr. Sullivan relied on non-comparable licenses to justify the increase. (Dkt. No. 310 at 14–15.) In response, Constellation notes that the Zenith licenses were issued after a report from Price Waterhouse Coopers ("PWC"). (Dkt. No. 326 at 14.)

Constellation also notes that this same report recommends that royalty rates be adjusted for inflation year-over-year. (*Id.* at 14–15.) Further, Constellation argues that the Federal Circuit has approved adjustment for inflation. (*Id.* at 15 (citing *Minnesota Min. & Mfg. Co. v. Johnson & Johnson Orthopaedics, Inc.*, 976 F.2d 1559, 1579 (Fed. Cir. 1992)).) In reply, LG argues that the Federal Circuit did not approve an adjustment for inflation in *Minnesota Min.* but rejected a 4% annual increase in the price of goods for lost profits. (Dkt. No. 336 at 5 (citing *Minnesota Min.*, 976 F.2d at 1579).) Additionally, LG contends that Constellation's evidence is insufficient because (1) the PWC report was not adopted in any of the licenses that Dr. Sullivan considered economically comparable and (2) the licenses that did contain an adjustment were not shown to be comparable. (*Id.*) LG also notes that the price of the accused products has decreased while inflation rose. (*Id.*) In sur-reply, Constellation largely re-urges the same points it raised in its response. (Dkt. No. 355 at 5.)

The Court is not persuaded by LG's arguments. LG cites no authority stating that an inflation adjustment is impermissible. On the other hand, the Federal Circuit in *Minnesota Min.* approved inflationary adjustments in the context of a lost profits analysis. 976 F.2d at 1579. There, plaintiff's expert testified that a 4% raise in price per year would "match the rate of inflation," while defendant's expert testified that there would have been "zero inflation." *Id.* The Federal Circuit concluded that a Special Master's determination that defendant would have raised its prices 2% per year was not clearly erroneous. *Id.* This indicates that inflationary adjustments are permissible.

Further, the Court finds that there is a factual basis for an inflationary adjustment. The PWC report, commissioned by Zenith, recommended inflationary adjustments. JTX-032. Additionally, other licenses showed that Zenith, a subsidiary of LG, included inflationary

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adjustments. (Dkt. No. 292 at 34:18–35:19, 39:9–40:8 (Sullivan).) LG's counterarguments in this regard go to weight of the evidence, which is not appropriate for the Court to consider at the JMOL stage. *See Gomez*, 442 F.3d at 937–38.

V. CONCLUSION

For the foregoing reasons, the Court finds that the Motion (Dkt. No. 310) should be and hereby is **DENIED**.

The parties are directed to jointly prepare a redacted version of this Order for public viewing and to file the same on the Court's docket as an attachment to a Notice of Redaction within five (5) business days of this Order.

So ORDERED and SIGNED this 23rd day of April, 2024.

RODNEY GILSTRAP
UNITED STATES DISTRICT JUDGE

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CONSTELLATION DESIGNS, LLC,	§	
	§	
Plaintiff,	§	
	§	
v.	§	CIVIL ACTION NO. 2:21-CV-00448-JRG
	§	
LG ELECTRONICS, INC., LG ELECTRONICS USA, INC., and LG ELECTRONICS ALABAMA, INC.,	§	**FILED UNDER SEAL**
	§	
	§	
	§	
Defendants.	§	

MEMORANDUM OPINION AND ORDER

I. INTRODUCTION

Before the Court is Defendants LG Electronics, Inc., LG Electronics USA, Inc., and LG Electronics Alabama, Inc.'s (collectively, "LG") Motion for Judgment as a Matter of Law ("JMOL") of No Liability (the "Motion"). (Dkt. No. 314.) Plaintiff Constellation Designs, LLC ("Constellation") opposes the Motion. (*See* Dkt. No. 329.) For the following reasons, the Court finds that the Motion should be **DENIED**.

II. BACKGROUND

Constellation filed a Complaint on December 9, 2021, alleging that LG infringed several of its United States Patents related to digital communications technology, including U.S. Patent Nos. 8,842,761 (the "'761 Patent), 10,693,700 (the "'700 Patent"), 11,018,922 (the "'922 Patent"), and 11,019,509 (the "'059 Patent") (collectively, the "Asserted Patents"). (Dkt. No. 1.) A jury trial was held on July 5–7 and 10–11, 2023.

On July 11, 2023, the jury returned a verdict finding that LG infringed all Asserted Patents and that LG's infringement was willful. (Dkt. No. 277 at 4, 6.) The jury also found that LG had failed to prove that any of the asserted claims were invalid. (*Id.* at 5.) Accordingly, the jury awarded damages of $1,684,469.00 in the form of a reasonable royalty for past damages. (*Id.* at 7.)

III. LEGAL STANDARD

"Judgment as a matter of law is proper when 'a reasonable jury would not have a legally sufficient evidentiary basis to find for the party on that issue.'" *Abraham v. Alpha Chi Omega*, 708 F.3d 614, 620 (5th Cir. 2013) (quoting Fed. R. Civ. P. 50(a)). The non-moving party must identify "substantial evidence" to support its positions. *TGIP, Inc. v. AT&T Corp.*, 527 F. Supp. 2d 561, 569 (E.D. Tex. 2007). "Substantial evidence is more than a mere scintilla. It means such relevant evidence as a reasonable mind might accept as adequate to support a conclusion." *Eli Lilly & Co. v. Aradigm Corp.*, 376 F.3d 1352, 1363 (Fed. Cir. 2004).

"The Fifth Circuit views all evidence in a light most favorable to the verdict and will reverse a jury's verdict only if the evidence points so overwhelmingly in favor of one party that reasonable jurors could not arrive at any contrary conclusion." *Core Wireless Licensing S.A.R.L. v. LG Elecs., Inc.*, 880 F.3d 1356, 1361 (Fed. Cir. 2018) (citing *Bagby Elevator Co. v. Schindler Elevator Corp.*, 609 F.3d 768, 773 (5th Cir. 2010)). A court must "resolve all conflicting evidence in favor of [the verdict] and refrain from weighing the evidence or making credibility determinations." *Gomez v. St. Jude Med. Daig. Div. Inc.*, 442 F.3d 919, 937–38 (5th Cir. 2006).

IV. ANALYSIS

A. Reliance on the A/322 Standard to Show Infringement

The parties agree that Constellation relied on the ATSC 3.0 and A/322 standards in some form to show infringement. (*See* Dkt. No. 314 at 3; Dkt. No. 329 at 5.) However, the parties disagree on the extent to which this is permissible. LG contends that standards may only be relied upon to show infringement "where the claim covers all devices practicing the standard," relying primarily on *Fujitsu Ltd. v. Netgear Inc.* and *INVT SPE LLC v. Int'l Trade Comm'n*, and

Constellation contends that no such showing is required, arguing that *Toshiba Corp. v. Imation Corp* demonstrates that *Fujitsu* and *INVT* do not require the claim to cover all devices practicing the standard to show infringement. (*See* Dkt. No. 314 at 3–5 (citing *Fujitsu Ltd. v. Netgear Inc.*, 620 F.3d 1321 (Fed. Cir. 2010), *Godo Kaisha IP Bridge 1 v. TCL Commc'n Tech. Holdings Ltd.*, 967 F.3d 1380 (Fed. Cir. 2020), *INVT SPE LLC v. Int'l Trade Comm'n*, 46 F.4th 1361 (Fed. Cir. 2022)); Dkt. No. 329 at 5–7 (citing *Fujitsu*, 620 F.3d 1321, *INVT*, 46 F. 4th 1361, *Toshiba Corp. v. Imation Corp.*, 681 F.3d 1358 (Fed. Cir. 2012)).)

For the reasons detailed below, the Court does not agree with LG: a plaintiff need not always show that a ***claim*** covers all devices practicing the standard to rely on the standard for infringement purposes. For example, a patent owner may rely solely on a standard to show that a product practices a limitation of a claim if (1) the relevant portion of the standard is sufficiently specific to show that practicing it would always result in practicing that limitation, and (2) the relevant portion of the standard is mandatory, or, if it is optional, there is evidence showing that the accused device implements that portion of the standard. Further, nothing in *Fujitsu* or its progeny prevents a plaintiff from performing both a standard-based infringement read and a direct comparison of a limitation to an accused product.

In *Fujitsu* the defendant disputed whether infringement may be assessed via a standard. 620 F.3d at 1326–27. The Federal Circuit held as follows:

> We hold that a district court may rely on an industry standard in analyzing infringement. If a district court construes the claims and finds that the reach of the claims includes any device that practices a standard, then this can be sufficient for a finding of infringement. We agree that claims should be compared to the accused product to determine infringement. However, if an accused product operates in accordance with a standard, then comparing the claims to that standard is the same as comparing the claims to the accused product. We accepted this approach in [*Dynacore Holdings Corp. v. U.S. Philips Corp.*, 363 F.3d 1263 (Fed. Cir. 2004)] where the court held a claim not infringed by comparing it to an industry standard rather than an accused product. An accused infringer is free to either prove that the

claims do not cover all implementations of the standard or to prove that it does not practice the standard.

Public policy weighs in favor of this approach. If a court determines that all implementations of a standard infringe the claims of a patent, then it would be a waste of judicial resources to separately analyze every accused product that undisputedly practices the standard. This is not prejudicial to present or future litigants. If two products undisputedly operate in the same manner, a finding of infringement against one will create a persuasive case against the other. In such a case, there will be no prejudice.

We acknowledge, however, that in many instances, an industry standard does not provide the level of specificity required to establish that practicing that standard would always result in infringement. Or, as with the [relevant] patent, the relevant section of the standard is optional, and standards compliance alone would not establish that the accused infringer chooses to implement the optional section. In these instances, it is not sufficient for the patent owner to establish infringement by arguing that the product admittedly practices the standard, therefore it infringes. In these cases, the patent owner must compare the claims to the accused products or, if appropriate, prove that the accused products implement any relevant optional sections of the standard. This should alleviate any concern about the use of standard compliance in assessing patent infringement. Only in the situation where a patent covers every possible implementation of a standard will it be enough to prove infringement by showing standard compliance.

Id. at 1327–28.

LG relies on this last sentence to argue that a very strict requirement must be met—the claim covers all devices practicing a standard—if a patent owner is to rely on a standard in any way to show infringement. (Dkt. No. 314 at 3–4.) The Court disagrees. *Fujitsu* was addressing a situation where a standard was relied upon to show infringement of an entire claim, not a particular limitation. *See* 620 F.3d at 1326 ("[Defendant] asks us to find no evidence of direct infringement because the district court relied on the [standard], rather than the accused products, in assessing infringement.") Thus, when the court refers to "prov[ing] infringement" in the last quoted sentence above, it is discussing the requirements for proving an entire claim is infringed via a standard, and not forbidding standards from being used unless "the claim covers all devices practicing the standard." *See id.* at 1327–28. (*See* Dkt. No. 314 at 3–4.)

The Court holds that the reasoning of *Fujitsu* also applies on a limitation-by-limitation basis. First, public policy weighs in favor of this approach. Judicial resources may be conserved by showing that a class of products practices a limitation. *See Fujitsu*, 620 F.3d at 1327. It would be a waste of judicial resources to separately analyze a limitation for each individual product that practices a standard when it can be shown that all products practice that limitation because they practice a standard. *See id.* As in *Fujitsu*, this is not prejudicial to future litigants. *See id.*

Additionally, though the same concerns noted by the *Fujitsu* court are present in a limitation analysis, they can be resolved by requiring the same evidentiary showings that the *Fujitsu* court required. The *Fujitsu* court noted that "in many instances, an industry standard does not provide the level of specificity required to establish that practicing that standard would always result in infringement." *Id.* Thus, the Court held that infringement (of a claim) cannot be shown by showing compliance with the standard. *Id.* at 1328. This same requirement can be imported to an analysis done on a limitation level. The *Fujitsu* court also noted that some standards contain optional portions and so required either that any portion of the standard be mandatory or that there be evidence showing the accused product implements the optional portion of the standard. *See id.* These requirements can also be implemented on a limitation level. Additionally, allowing patent owners to rely on standards to show infringement on a limitation basis will reduce discovery costs for both the patent owner and the alleged infringer. Finally, nothing in *Fujitsu* prevents the same reasoning that the court applied to a claim from being applied to a limitation. *See* 620 F.3d at 1327–28.

LG contends that *INVT* demonstrates "the Federal Circuit's strict application of the *Fujitsu* requirement," but LG is mistaken that *INVT* precludes a patent from using a standard to show that a product practices a limitation unless that plaintiff can use the standard to show that the product practices all limitations of a claim. (*See* Dkt. No. 314 at 4–5 (citing *INVT*, 46 F.4th at 1361).) LG first points to the Federal Circuit's statement that "[i]nfringement can be proven based on an accused product's use of an industry standard if the asserted claim is standard essential." (*Id.* at 5 (quoting *INVT*, 46 F.4th at 1377).) This statement simply shows that if every limitation of a claim reads on a standard, then infringement can be proven by showing compliance with a standard. This statement does not show that standards may not be used to show a product practices a limitation.

Next, LG points to the Federal Circuit's statement that "[c]laims are standard essential if 'the reach of the claims includes any device that practices the standard.'" (*Id.* (quoting *INVT*, 46 F.4th at 1377).) Thus, according to LG, a claim is only essential if "'all implementations of a standard infringe the claim' and the 'patent covers every possible implementation of a standard.'" (*Id.* (quoting *INVT*, 46 F.4th at 1377).) Again, this statement concerns infringement reads where the patent owner asserts infringement of a claim based on a standard, not practice of a limitation.

Finally, LG notes that the Federal Circuit in *INVT* held that the patent owner had to prove infringement by comparing the claims to the products because the patent owner had failed to prove standard essentiality. (*Id.* (quoting *INVT*, 46 F.4th at 1380).) Specifically, the Federal Circuit found that "[b]ecause the … claims [a]re not essential to the [] standard … [the patent owner] was required to prove infringement in the ordinary manner, which involves 'comparing the claims to the accused products.'" *Id.* (quoting *Fujitsu*, 620 F.3d at 1328) (brackets removed). This does not support LG's argument for two reasons. First, these statements are a natural consequence of the fact that the patent owner "asserted two infringement theories: (1) the [asserted] claims are essential to the practice of the standard, and (2) the accused products practice the asserted claims." *INVT*, 46 F.4th at 1368 (citation omitted). If the standard-based read fails, then the only other read the plaintiff has is the direct comparison. The plaintiff did not attempt to use the standard on a limitation basis in combination with direct evidence for other limitations. Second, a showing that a product operates in accordance with a portion of a standard and that a limitation reads on that portion of the standard is a comparison of the limitations to the accused products. *See Fujitsu*, 620 F.3d at 1327 ("[I]f an accused product operates in accordance with a standard, then comparing the claims to that standard is the same as comparing the claims to the accused product.")

Nothing in *INVT* precludes or counsels against extending the reasoning the Federal Circuit laid out in *Fujitsu* from a claim-by-claim basis to a limitation-by-limitation basis. Accordingly, the Court holds that a patent owner may rely solely on a standard to show that a product practices a limitation of a claim if (1) the relevant portion of the standard is sufficiently specific to show that practicing it would always result in practicing that limitation, and (2) the relevant portion of the standard is mandatory, or, if it is optional, there is evidence showing that the accused device implements that portion of the standard.

The Court also holds that nothing in *Fujitsu* or *INVT* precludes a party from relying on a standard in combination with direct comparison for a particular limitation. Indeed, there are some statements in these cases that, if unexamined, might appear to support such a division, as LG argues. *See INVT*, 46 F.4th at 1380 ("Because the … claims [a]re not essential to the [] standard … [the patent owner] was required to prove infringement in the ordinary manner, which involves comparing the claims to the accused products." (quotations omitted)); *Fujitsu*, 620 F.3d at 1328 (holding that in circumstances where the standard is either insufficiently specific or optional and there is no evidence that the accused products implement that portion of the standard, then "[i]n these instances it is not sufficient for the patent owner to establish infringement by arguing that the product admittedly practices the standard" but, instead, "the patent owner must compare the claims to the accused products"). However, these statements concern what the patent owner must do to show infringement, which is analyzed on a claim-by-claim basis, when the patent owner asserts that the claims are essential to a standard. *See Fujitsu*, 620 F.3d at 1326–28; *INVT*, 46 F.4th at 1368. Again, it is a natural consequence of a theory of infringement based on standard essentiality that, if one limitation is not shown, then the theory as a whole fails. Since these are not scenarios where the patent owner argued that both a standard read and a direct comparison may be used at the same time as evidence of infringement, it follows that if the standard read fails infringement "must" be shown by direct comparison. Indeed, allowing both at the same time will somewhat undercut the efficiency identified above, but (1) there is still no prejudice to alleged infringers and (2) the Federal Circuit has specifically allowed plaintiffs to pursue one standards-based infringement theory and a direct comparison theory at the same time. *See INVT*, 46 F.4th at 1380; *Fujitsu*, 620 F.3d at 1328. Finally, neither LG nor the Federal Circuit has stated a reason why both a direct comparison and a standard-read cannot be undertaken at the same time on a limitation-by-limitation basis. Accordingly, the Court holds that a standard-read may be used in addition to a direct comparison on a limitation level.

The Court will now address the parties' substantive arguments. LG argues that Dr. Mark Jones, Constellation's technical expert, "did not compare the [] standard to most of the independent claims' limitations." (Dkt. No. 314 at 5–6.) The Court finds that, for the reasons stated above, Dr. Jones was not required to do so. LG additionally argues that Constellation cannot "mix-and-match" evidence of infringement by relying on the standard for some limitations and on a direct comparison for others. (*Id.* at 9–10.) Again, the Court finds that, for the reasons explained above, Constellation may mix and match evidence of standard compliance with a direct comparison.

LG argues that Dr. Mark Jones failed to show that "the section of the standard on which he relied was mandatory." (*Id.* at 6–7.) LG also contends that "because there is no standard governing

TVs, CD assumed without support that the accused TVs 'reverse' the operations of ATSC 3.0 transmitters and thus use their constellations." (*Id.* at 8–9.)

In response, Constellation argues that the record shows that the accused products implement the A/322 standard. (Dkt. No. 329 at 7–9.) Constellation then argues that "LG's own corporate witness testified that the A/322 standard has been incorporated into LG chipsets and that the standard defines what it takes to receive and demodulate the ATSC 3.0 signal." (*Id.* at 9 (citing Dkt. No. 292 at 192:3–25).) Thus, Constellation urges that "because the evidence demonstrates that LG's accused products operate in accordance with A/322, there is no need to show that A/322 is mandatory for all ATSC 3.0 TVs." (*Id.*) Nonetheless, Constellation also argues that Dr. Jones testified that the FCC made A/322 mandatory for ATSC 3.0 TVs in November 2017. (*Id.* at 9–10 (citing Dkt. No. 290 at 93:15–94:1).) Finally, Constellation argues that there was substantial evidence that the accused TVs must use the same constellations as the transmitters in order to operate. (*Id.* at 10 (citing Dkt. No. 289 at 225:6–15).)

LG and Constellation do not meaningfully develop these arguments in reply and sur-reply. (*See* Dkt. No. 339 at 1–4; Dkt. No. 354 at 1–4.)

The Court is not persuaded by LG's arguments. There was substantial evidence at trial that a transmitter and a receiver need to use the same constellation. (Dkt. No. 289 at 225:6–15 (Dr. Chris Jones) "Q. So I'm not talking about this specific point here for the moment. I want to talk about the overall constellation, the overall 16 options. It looks the same as the one we had on the transmitter side, and I want to know is that on purpose? A. It's absolutely on purpose. You need to know the constellation that the transmitter used in order to successfully demap a receive symbol. So that is what's called prior knowledge that has to be provided in the receiver. So the receiver has to know what constellation the transmitter used.").)

Further, there was substantial evidence that the accused TVs comply with the A/322 component of the ATSC 3.0 standard. Dr. Mark Jones testified that LG itself identifies the accused TVs as compatible with ATSC 3.0, (Dkt. No. 290 at 86:21–87:11), as LG's expert, Dr. Akl, confirmed this (Dkt. No. 293 at 61:17–19). Dr. Mark Jones testified that the FCC mandated the use of the A/322 standard for ATSC 3.0 compatible televisions. (Dkt. No. 290 at 93:15–94:10.) Additionally, LG's corporate representative admitted that LG told the FCC that "A/322 is the component of ATSC 3.0 that ensures that receivers in televisions and other consumer reception devices are able to demodulate an ATSC 3.0 signal." (Dkt. No. 292 at 189:9–13.)

B. The "Communication Channel" Limitation

All of the asserted independent claims include a "receiver" limitation. (*See* Dkt. No. 314 at 10.) At trial, Dr. Mark Jones presented his infringement evidence for the '922 Patent's "receiver" element and referred back to this evidence for the "receiver" limitations in the other three patents. (*Id.* at 10–11.) The '922 Patent's "receiver" limitation recites: "a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR)." (*Id.* at 11.) At trial, Dr. Mark Jones testified that, in his view, the "communication channel extends from the transmitter over the air through an antenna and into the back of the television." (Dkt. No. 290 at 133:13–23.)

LG argues that since Dr. Mark Jones relied on an antenna, and since the accused products are not sold with antennae, there can be no infringement as a matter of law. (Dkt. No. 314 at 11–12 (citing *Omega Pats., LLC v. CalAmp Corp.*, 920 F.3d 1337, 1345 (Fed. Cir. 2019)).) Further, LG argues that the claim language of the '509 Patent requires a structure and does not permit capability. (Dkt. No. 314 at 12 (quoting '509 Patent ("a receiver *that receives signals* via a communication channel having a channel signal-to-noise ratio (SNR)")).) LG also contends that, regardless of this, Dr. Mark Jones simply relied on the ability to receive signals rather than the capability of the TVs to receive signals for all the Asserted Patents. (*Id.* at 12–13.)

In response, Constellation first argues that it is immaterial to infringement that the accused products are not sold with the "communication channel" because the products are capable of receiving signals via a "communication channel." (Dkt. No. 329 at 12–13.) Constellation also argues that the claims of Asserted Patents, including the '509 Patent, are directed to capability. (*Id.* at 13–14.) Finally, Constellation argues that the record reflects that the accused products were capable of receiving signals via a communication channel. (*Id.* at 14–15.)

In reply, LG argues that even though it might not be possible to sell a channel with the accused products, Constellation crafted its claims to require this structure and cannot now show infringement without it. (Dkt. No 339 at 4–5 (citing *Becton, Dickinson & Co. v. Tyco Healthcare Grp., LP*, 616 F.3d 1249, 1255 (Fed. Cir. 2010)).) LG also argues that Constellation's capability argument suffers from three flaws: (1) the interpretation of the ''509 Patent's claim is waived, (2) even for claims actually reciting capability, Constellation's argument contradicts the testimony of its expert, who told the jury that the receiver's capability is just to receive signals, and (3) judicial estoppel prevents Constellation from advancing this new position—that capability includes channel—contrary to its infringement theory—that capability does not include channel. (*Id.* at 5–6.)

In sur-reply, Constellation first argues that LG is advancing a new argument in reply—that the asserted claims require that the communication channel be sold with a receiver—and so it is waived. (Dkt. No. 354 at 4.) Constellation also contends that this argument is wrong: "[b]ecause the communication channel is not a component of the claimed receiver, it need not be sold with the accused products to show infringement." (*Id.*) Constellation then urges that (1) it has

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maintained the same stance with respect to the ’509 Patent throughout the case, (2) the record establishes that a receiver takes in signals through a communication channel, and (3) Constellation is not presenting a new position by arguing that the claimed receivers are capable of receiving signals via a communication channel. (*Id.* at 4–5.)

The Court is not persuaded by LG’s arguments. First, it is undisputed that the “receiver” limitation for all asserted claims of the Asserted Patents, except the ’509 Patent, are capability limitations: “a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR).”

Second, the Court finds that the ’509 Patent’s “receiver” limitation is also a capability limitation. Claim 21 of the ’509 Patent recites “a receiver that receives signals via a communication channel.” (Dkt. No. 329 at 13.) In *MasterMine Software, Inc. v. Microsoft Corp.*, the Federal Circuit held that the following claim was drawn to capability: “wherein the reporting module … receives from the user a selection.” 874 F.3d 1307, 1315 (Fed. Cir. 2017). The court held that though the language includes the “active verb[] … receives,” the “verb[] represent[s] permissible functional language used to described the capabilities of the ‘reporting module.’” *Id.* The phrase a “[structure] that receives” is not meaningfully different from “wherein the [structure] receives.” Both terms denote a structure that has a function: receiving. Accordingly, in line with *MasterMine*, the Court holds that the ’509 Patent’s “receiver” limitation is a capability limitation.

Third, the Court finds that LG has not adequately shown that the dispute about the ’509 Patent is a claim construction that has been waived because LG has not shown that this is a new position. (*See* Dkt. No. 339 at 5.)

Fourth, in light of the fact that these are capability limitations it is clear that the products did not need to include a “communication channel” when sold, under any of the Asserted Patents.

Fifth, the Court finds that there was substantial evidence to support a finding that the accused products are capable of receiving signals via a "communication channel." (*See, e.g.*, Dkt. No. 290 at 133:15–134:6 (Dr. Mark Jones).) LG argues that the capability theory conflicts with LG's rendition of Constellation's infringement theory. (*See* Dkt. No. 339 at 5.) Even if this were the case, it would not negate the substantial evidence that the accused TVs are capable of receiving a signal via a "communication channel."

Sixth, the Court finds that LG has not shown why judicial estoppel applies here. LG even failed to cite the elements of judicial estoppel as part of its request. (*See id.* at 5–6.) *Reed v. City of Arlington*, 650 F.3d 571, 574 (5th Cir. 2011) (*en banc*) (Courts look to the following elements when applying judicial estoppel: "(1) the party against whom judicial estoppel is sought has asserted a legal position which is plainly inconsistent with a prior position; (2) a court accepted the prior position; and (3) the party did not act inadvertently."). Judicial estoppel does not apply here.

C. Realtek Chip

Constellation accused a group of LG TVs that incorporate the Realtek-made K8Hp chip. (Dkt. No. 314 at 14.) LG argues that Constellation did not show that the Realtek chip meets all limitations of the asserted claims. (*Id.* at 14–21.)

i. Dr. Mark Jones' Testimony

LG argues that Dr. Mark Jones' testimony must be corroborated, but that the A/322 standard cannot provide the necessary corroboration because he never established that any claim was essential for that standard. (*Id.* at 14–15.) LG also contends that since Dr. Mark Jones cannot use the standard, he must make a direct comparison but he could not have done so because he did not have any evidence from Realtek about how its chip operates since Realtek did not produce any evidence. (*Id.* at 16 (citing Dkt. No. 290 at 219:18–22, 228:9–24).)

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In response, the Court simply notes that Dr. Mark Jones' testimony can be corroborated with the A/322 standard.

ii. "Demapper" and "Decoder" Limitations

Dr. Mark Jones referred to the '922 Patent to assert that the "demodulator," "demapper," and "decoder" limitations from the independent claims of the other Asserted Patents were met. (*Id.*) The claimed "demodulator" is a structure coupled to the "demapper," which is in turn coupled to the "decoder." (*See id.* at 17.) For the Realtek chip, Dr. Mark Jones relied on the below demonstrative slide to testify that the "demapper" and "decoder" limitations were present:

[redacted]

LG argues that this slide (and Dr. Mark Jones' testimony about this slide) is insufficient to show infringement because it is from 2018, four years before any LG TV with a K8Hp chip was sold. (Dkt. No. 314 at 17.) Next, LG argues that if the Realtek chips operate in accordance with the slide, then the demapper and decoder would be inside the demodulator, not coupled to it. (*Id.*) LG also points out that the words "demapper" and "decoder" do not appear anywhere on this slide. (*Id.* at 18.) LG acknowledges that at trial Dr. Mark Jones relied on the last bullet point, the "BICM" bullet point, as both his demapper and decoder even though the block diagram does not show a "BICM" within it. (*Id.*) LG asserts that Dr. Mark Jones "never reconciled these inconsistencies and contradictions." Finally, LG argues that any testing LG did of the Realtek chip is irrelevant on this point because the testing does not show how the Realtek chip operates. (*Id.* at 18–19.)

In response, Constellation argues that Dr. Mark Jones testified that the A/322 standard requires ATSC 3.0 compliant TVs to have a demapper and decoder. (Dkt. No. 329 at 16 (citing Dkt. No. 290 at 98:8–99:10; 99:21–102:2).) Constellation also contends that Dr. Mark Jones explained that the BICM-labeled block diagrams of PTX-107.005 indicate demapping the demodulated signal coming from the demodulator. (*Id.*) LG contends that the 2018 date of PTX-107.005 is irrelevant because LG represented that the block diagram was for the K8Hp chip. (*Id.* at 16.) Finally, Constellation contends that Dr. Mark Jones explained that the testing he completed "showed that the accused ATSC 3.0 TVs really performed demodulation, demapping, and decoding. (*Id.* at 17.)

In reply, LG repeats its argument that Dr. Mark Jones cannot rely on the A/322 standard because he failed to show standard essentiality. (Dkt. No. 339 at 6.) LG also again argues that the compatibility tests only show that Realtek chips can process ATSC 3.0 signals, not what structures are in those chips. (*Id.*at 6–7.) Finally, LG argues that Constellation did not address any factual inconsistencies that LG noted in its opening brief. (*Id.* at 7.)

In sur-reply, Constellation notes that "[b]oth Dr. Chris Jones[, Constllation's corporate representative,] and Dr. Akl explained that digital communication systems use demappers and decoders to process digital signals and that those components are common to digital systems." (Dkt. No. 354 at 5.) Thus, Constellation argues, "[t]he jury was therefore free to infer that the LG TVs with Realtek chips had demappers and decoders, particularly when no party presented any alternative means of processing signals."

The Court is not persuaded by LG's arguments. Dr. Mark Jones explained that the A/322 standard required a decoder and demapper (Dkt. No. 290 at 98:8–99:10), and LG only challenges the sufficiency of this evidence by arguing that it should not be considered at all (s*ee* Dkt. No. 339 at 7). LG's objections to this evidence are unavailing for the same reasons stated above. This evidence, in combination with Dr. Mark Jones' testimony regarding PTX-107.005 constitutes substantial evidence in support of the verdict. (*See id.* at 137:10–18.)

iii. "Likelihoods"

Dr. Mark Jones referred to the '922 Patent to show that the other Asserted Patents were infringed and the '922 Patent recites "likelihoods." (Dkt. No. 314 at 19, n. 3.)

LG argues that the slide depicted above, PTX-107.005 says nothing about likelihoods (sometimes called "LLRs"). (*Id.* at 19–20.) Further, LG notes that Constellation's own corporate witness testified that there are two relevant operations: "soft decoding," where likelihoods are computed, and "hard decoding" where likelihoods are not used. (*Id.* at 20.) LG asserts that there was no evidence that the Realtek chips did either. (*Id.*)

In response, Constellation argues that "Dr. Chris Jones and Dr. Akl explained that digital communications systems use likelihoods in the demapper and decoder to deal with errors in the symbols." (Dkt. No. 329 at 17.) Further, Constellation contends that Dr. Mark Jones testified that ATSC 3.0 televisions utilize likelihoods in the demappers and decoders. (*Id.* at 18.) Constellation also argues that the A/327 standard recommends using likelihoods and that testing of the Realtek chips shows that they match the performance of chips implementing the A/327 recommended practices. (*Id.*)

In reply, LG asserts that "Dr. Jones' general testimony on ATSC 3.0 receivers does not help [Constellation] either, as it is not specific to Realtek chips." (Dkt. No. 339 at 7.) LG also argues that the A/327 standard permits both hard and soft decoding, so any test data is inconclusive. (*Id.*)

In sur-reply, Constellation argues that LG is reweighing the evidence, which is impermissible. (Dkt. No. 354 at 6.)

The Court is not persuaded by LG's arguments. LG does not dispute that "Dr. Chris Jones and Dr. Akl explained that digital communications systems use likelihoods in the demapper and decoder to deal with errors in the symbols." (Dkt. No. 329 at 17 (citing Dkt. No. 289 at 223:13–227:10 (Dr. Chris Jones); Dkt. No. 293 at 59:11–61:1 (Dr. Akl)).) LG also does not dispute that "Dr. Mark Jones [] testified that ATSC 3.0 televisions use likelihoods in the demappers and decoders." (*Id.* at 18 (citing Dkt. No. 298 at 98:9–99:10).) Instead, LG simply argues that "Dr. Jones' general testimony on ATSC 3.0 receivers does not help [Constellation] [], as it is not specific to Realtek chips." (Dkt. No. 339 at 7.) LG is mistaken. As mentioned above, there is substantial evidence that all accused TVs are compatible with ATSC 3.0. (Dkt. No. 290 at 86:21–87:11 (Dr. Chris Jones); Dkt. No. 293 at 61:17–19 (Dr. Akl) ("Q. Are the accused TVs limited to those who are compatible with ATSC 3.0 broadcast signals? A. Yes.")). This includes the accused TVs with Realtek chips. Thus, there is, in fact, substantial evidence that the ATSC 3.0 TVs with Realtek chips utilize "likelihoods." The Court declines to address the arguments about A/327 as unnecessary.

iv. "Symbol Constellations"

Dr. Mark Jones referred to the '922 Patent to show that the other Asserted Patents were infringed and the '922 Patent certain "symbol constellations." (Dkt. No. 314 at 19, n. 3, 20.)

LG argues that Dr. Mark Jones did not show evidence of infringement of this limitation for the Realtek chips outside of reliance on the A/322 standard and testing. (Dkt. No. 314 at 20–21) LG contends Dr. Mark Jones may not rely on the standard because he did not show that the claim was standard essential and contends that the testing was insufficient because it did not show the inner workings of the accused products. (*Id.* at 20–21.) In response, Constellation argues that the A/322 standard supports the jury's finding. (Dkt. No. 329 at 18–19.) Constellation also argues that the testing shows that the accused TVs could process constellations generated by transmitters, and that there was testimony showing that the TVs use the same constellations transmitted by the transmitters. (*Id.* at 19.) Thus, Constellation argues, "LG provides no reason why a jury could not have agreed with Dr. Mark Jones' interpretation over that of LG['s]." (*Id.*) In reply, LG argues that there was no testimony that the TVs use the same constellation, only that the TVs "need to know the constellation that the transmitter used." (Dkt. No. 339 at 7–8 (quoting Dkt. No. 289 at 225:6–15 (Dr. Chris Jones)).) In sur-reply, Constellation argues that Dr. Chris Jones did testify that the TVs use the constellation used by the transmitter. (Dkt. No. 354 at 7 (citing Dkt. No. 289 at 225:6–15 (Dr. Chris Jones)).)

The Court is not persuaded by LG's arguments. Dr. Chris Jones may rely on the A/322 standard, as discussed. Moreover, there is no doubt that Dr. Chris Jones' testimony in connection with the standard constitutes substantial evidence. (*See* Dkt. No. 290 at 96:13–18, 98:18–99:1, 102:3–16; 104:15–105:2.) The testing data in connection with the testimony that the receiver "need[s] to know the constellation used in order to successfully demap" the received signal also constitutes substantial evidence. (Dkt. No. 289 at 225:6–15; Dkt. No. 290 at 105:14–108:16.) The testimony regarding testing data shows that accused TVs can process signals produced using the accused constellations, and the testimony establishes that the TVs use the same constellations to demap—the only reason the TVs need to know the constellations is so the TVs can use them (to demap).

D. The "Wherein" Clause of the '761 Patent

The "wherein" clause of the ''761 Patent recites:

> wherein the QAM symbol constellation is a geometrically spaced symbol constellation optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a QAM signal constellation that maximizes d_{min}.

(Dkt. No. 314 at 21.) For this clause, Dr. Mark Jones relied on 2014 LG, Harris, and Zenith proposal to ATSC (JTX-010), a 2013 Samsung-Sony proposal to ATSC (PTX-086), and his description of the non-admitted 2016 IEEE article (Dkt. No. 290 at 148:5–150:7). (See Dkt. No. 314 at 21.)

Regarding the 2016 IEEE article, Dr. Mark Jones testified that it was an article titled "[n]on-uniform constellations for ATSC 3.0" from "the March 2016 IEEE special issue journal … that experts would rely on in the field of electrical engineering." (Dkt. No. 290 at 85:15–23, 148:5–149:20.) Regarding this article, Dr. Mark Jones testified as follows (note that "BICM capacity" means parallel decode capacity (Dkt. No. 329 at 2)):

> Q. All right. For this last limitation beginning with "wherein" and going all the way down to "Dmin", [Dmin], has your analysis shown that this limitation is met?
>
> A. Yes, it has.
>
> Q. What are we seeing here?

A. This is an -- the -- an article from the special issue -- the IEEE special issue on ATSC 3.0. It's the article [n]on-uniform Constellations for ATSC 3.0. This article indicates that BICM capacity will be used as an optimization criteria for non-uniform constellations.

It indicates that when optimizing non-uniform constellations of a given size M for a transmission system using a BICM chain, we need to maximize the BICM capacity CB.

It goes on to indicate that ATSC 3.0, the constellations for 16 QAM, 64 QAM, and 256 QAM have been optimized as 2-D NUC, and that's non-uniform constellations, but for 1K and 4K constellations lower complexity 1-D NUCs have been proposed. It further indicates lower down the 1-D NUC with 1024 constellation points, 1K NUC, optimized for an LDPC rate of7/15ths.

…

Q. Do the ATSC 3.0 non-uniform constellations provide gains over constellations that maximize Dmin as it says in the end of the claim?

A. Yes. Maximize Dmin, that is a mathematical way of saying at a high level that these are – it's comparing to uniform constellations. So it's saying that the optimized constellations provide a gain over the uniform constellations.

And so this figure from that figure 9 from that same paper is plotting on the left side the improvement or gain from using the non-uniform constellations over the uniform constellations. And the gain goes -- you know, there are a wide range of gains that are accomplished with this, some of them as high as 1.8.

Q. Was that considered a significant improvement over uniform constellations?

A. Yes, that was a very significant improvement.

(Dkt. No. 290 at 148:5–149:20.)

Regarding the two proposals, Dr. Mark Jones testified as followed.

Q. Have you also considered any confidential information from the ATSC standardization body, the internal documents for this limitation?

A. Yes, I have. What I'm showing here are JTX 10 and PTX 86. JTX 10 is describing the LG Harris and Zenith proposal to ATSC which indicates that it is -- the constellations that they're described in that proposal are optimized to get maximum BICM capacity. Similarly, in the Sony proposal, it's indicating that the constellations are optimized for each constellation order and LDPC code rate.

(*Id.* at 149:21–150:5.)

Dr. Byeongkook Jeong, an LG witness, testified that what he was "trying to achieve in developing non-uniform constellations for this standard for transmitters" was to "maximize BICM capacity." (Dkt. No. 293 at 16:5–8.) Dr. Byeongkook Jeong also testified that he submitted proposals to ATSC in March 2014, which is around the same time the LG, Harris, and Zenith proposals were submitted. (*Id.* at 17:9–14.) He further testified that his proposals were adopted. (*Id.* at 20:4–8.)

LG argues that the evidence presented and relied upon by Dr. Mark Jones falls short because it requires the jury to make too many inferences and because it is conclusory. (Dkt. No. 314 at 21–24.) LG also argues that the Federal Circuit upheld a grant of JMOL that required the jury to make a smaller inferential leap. (*Id.* at 23–24 (citing *Brigham & Women's Hosp., Inc. v. Perrigo Co.*, 761 F. App'x 995, 1003–05 (Fed. Cir. 2019)).)

In response, Constellation argues that there is substantial evidence supporting the jury's decision. (Dkt. No. 329 at 19–21.) Next, Constellation urges that LG did not raise the "too many and too big inferences" ground in its 50(a) motion and so it is waived. (Dkt. No. 329 at 21.) Constellation then argues that evidence showed that the accused products implement the constellations in the A/322 standard and that the evidence described above constitutes substantial evidence that "LG's accused products use constellations optimized for parallel decode capacity." (Dkt. No. 329 at 22.)

In reply, LG argues that it did not need to raise this specific argument at the 50(a) hearing, only the specific defense. (Dkt. No. 339 at 1.) Next, LG contends that the documents and the testimony were published years before LG sold any accused TVs and do not describe either LG's TVs or the chips in those TVs. (*Id.* at 8.) LG also asserts that Constellation does not dispute that the jury had to "pile four or five unsupported assumptions, resulting in pure speculation." (*Id.* at 8–9 (citing *Brigham*, 761 F. App'x at 1003).)

In sur-reply, Constellation contends that LG's standard for waiver is too low. (Dkt. No. 354 at 1.) Constellation additionally re-argues the evidence discussed above. (*Id.* at 7–8.)

The Court finds that LG has not waived this argument. Rule 50(a)(2) requires the moving party, when moving for JMOL before the case is submitted to the jury, to "specify the judgment sought and the law and the facts that entitle the movant to the judgment." Further, this Court has previously recognized that individual issues need to be raised and not the specific grounds. *Whirlpool Corp. v. TST Water, LLC*, 2018 WL 1536875, at *11-12 (E.D. Tex. Mar. 29, 2018).

The Court is not persuaded by LG's remaining arguments. First, the Court does not find that the evidence and testimony cited above is conclusory. Second, the evidence cited above is far more than a "mere scintilla." *Eli Lilly & Co.*, 376 F.3d at 1363. Third, *Brigham* does not compel an opposite result. 761 Fed. App'x at 1004–05. The inference at issue in *Brigham*—requiring the jury to find that evidence of relief at 15 minutes "necessarily showed" onset of relief within 5–10 minutes—is much larger than any inference here. *See id.* There, "only speculation" supported the inference, and the same is not true here. *See id.* As described above, there is testimony that the proposals submitted to ATSC were optimized for parallel decode capacity, and that ATSC was looking to implement constellations that were optimized for the same. (Dkt. No. 290 at 148:5–150:5; Dkt. No. 293 at 16:5–8, 17:9–14, 20:4–8.) The jury's verdict in this regard is supported by substantial evidence.

E. "The Demodulated Signal" – The '761, '700, and '922 Patents

Independent claim 17 of the '761 Patent calls for a "demodulator configured to demodulate the signal received" and a "demapper configured to estimate likelihoods . . . from the demodulated signal." (Dkt. No. 314 at 24.) Similarly, the '700 Patent's independent claim 1 and the '922 Patent's independent claim 24 both require "a demodulator capable of demodulating a received signal into a demodulated signal" and "a demapper . . . capable of determining likelihoods using the demodulated signal." (*Id.*) The experts agree that these claims require that the demodulated signal used by the demapper to be the signal output by the demodulator. (*See id.*) Further, the parties agree that the signal output by the demodulator is modified, converted, and partitioned before reaching the demapper in the TVs containing chips other than the Realtek chips. (*See id.* at 24–25.)

LG argues that the changes done to the signal exiting the demodulator "negates the claimed requirement that the demodulator's output signal must be the signal used by the demapper." (*Id.* at 25–26.) In response, Constellation argues that the jury could have viewed the "demodulator" as all the steps that occurred prior to the "demapper." (Dkt. No. 329 at 22–23.) Constellation also argues that Dr. Akl's view was that the entirety of the signal be used, and since the jury rejected this view, so should the Court. (*Id.* at 24.) In reply, LG argues that there is no support in the record for Constellation's view that the "demodulator" ends just before the "demapper" begins. (Dkt. No. 339 at 9.) Next, LG contends that the jury was not entitled to "ignore Dr. Akl's uncontradicted and unimpeached testimony" and that Constellation's "arguments on this issue rest solely on attorney argument and lack evidentiary support." (*Id.*) In sur-reply, Constellation argues that LG has failed to show why the jury was required to take such a strict view of the claim limitation. (Dkt. No. 354 at 8.) Constellation's remaining arguments in sur-reply do not meaningfully contribute to this discussion. (*Id.* at 8–9.)

The Court is not persuaded by LG's arguments. Indeed, as Constellation argues, LG has not pointed to any reason why the jury was required to accept Dr. Akl's view of the claim language. (*See id.* at 8.) Moreover, the jury could have found that the signal exiting the demodulator was used by the demapper, notwithstanding that it had been changed. An egg is used in an omelet notwithstanding that the whole egg (hopefully) does not make it into the omelet—since the shell is discarded. (*See* Dkt. No. 293 at 156:6–12.)

F. JMOL of Obviousness – '509 Patent

LG presented evidence at trial that the Bauch reference, either alone or in combination with the Zhang reference, renders obvious claims 21 and 23 of the '509 Patent. (Dkt. No. 314 at 26.) There is no dispute that these references predate the priority date of the '509 Patent. (*See* Dkt. No. 329 at 24–29; Dkt. No. 339 at 10.) Constellation did not offer any rebuttal testimony, but did cross the expert sponsoring this theory, Dr. Akl. (Dkt. No. 339 at 10.)

LG argues that Dr. Akl demonstrated how Bauch discloses every limitation of the dependent claim, except two limitations, and how both are disclosed by Zhang. (Dkt. No. 314 at 26–27.) LG then argues that Dr. Akl disclosed a motivation to combine. (*Id.* at 27.) LG contends that the points raised by counsel for Constellation during Dr. Akl's cross examination do not undermine his testimony. (*Id.* at 27–30.)

In response, Constellation argues that granting JMOL of invalidity should be "reserved for extreme cases." (Dkt. No. 329 at 24–25 (quoting *Core Wireless*, 880 F.3d at 1364).) Constellation then argues that Dr. Akl's testimony was confusing and contradictory. (*Id.* at 25–27.) Finally, Constellation argues that secondary considerations of non-obviousness support the jury's decision to not find the '509 Patent invalid. (*Id.* at 28–29.)

In reply, LG argues that Dr. Akl did not offer contradictory testimony about understanding the claims and that Constellation's impeachments were ineffective. (Dkt. No. 339 at 10.) LG also argues that there is no nexus on secondary considerations. (*Id.*)

In sur-reply, Constellation notes that LG does not even argue that this is an extreme case. (Dkt. No. 354 at 9.) Constellation asserts that "LG tries to limit the cited secondary consideration to non-uniform constellations. But the record refutes that point and LG's arguments are not a substitute for the jury's findings." (*Id.* at 10.) Otherwise, Constellation largely re-urges the same or similar points it made in its response. (*Id.* at 9–10.)

The Court is not persuaded that it should grant JMOL of obviousness. "[Since] the burden rests with the alleged infringer to present clear and convincing evidence supporting a finding of invalidity, granting judgment as a matter of law for the party carrying the burden of proof is generally 'reserved for extreme cases,' such as when the opposing party's witness makes a key admission." *Core Wireless*, 880 F.3d at 1364 (first citing 9 Charles Alan Wright & Arthur R. Miller, Federal Practice and Procedure § 2535 (3d ed.); then citing *Grey v. First Nat'l Bank in Dall.*, 393 F.2d 371, 380 (5th Cir. 1968) ("[W]hen the party moving for a directed verdict has such a burden, the evidence to support the granting of the motion must be so one-sided as to be of over-whelming effect.")). As Constellation correctly noted, LG does explain why this is a such an "extreme case." *Id.*

V. CONCLUSION

For the foregoing reasons, the Court finds that the Motion (Dkt. No. 314) should be and hereby is **DENIED**. JMOL is not warranted under these facts and precedents.

The parties are directed to jointly prepare a redacted version of this Order for public viewing and to file the same on the Court's docket as an attachment to a Notice of Redaction within five (5) business days of this Order.

So ORDERED and SIGNED this 23rd day of April, 2024.

RODNEY GILSTRAP
UNITED STATES DISTRICT JUDGE

IN THE UNITED STATES DISTRICT COURT
FOR THE EASTERN DISTRICT OF TEXAS
MARSHALL DIVISION

CONSTELLATION DESIGNS, LLC, *Plaintiff*, v. LG ELECTRONICS, INC., LG ELECTRONICS USA, INC., LG ELECTRONICS ALABAMA INC, *Defendants*.	§ § § § § § § § § §	CIVIL ACTION NO. 2:21-CV-00448-JRG

AMENDED FINAL JUDGMENT

A jury trial commenced in the above-captioned case on July 5, 2023. On July 11, 2023, the jury returned its unanimous verdict finding that Defendants LG Electronics Inc., LG Electronics USA, Inc., and LG Electronics Alabama, Inc. (together, "LG") infringed at least one of Claims 17, 21, 24, and 28 of U.S. Patent No. 8,842,761 (the "'761 Patent), at least one of Claims 21 and 23 of U.S. Patent No. 11,019,509 (the "'509 Patent"), at least one of Claims 24 and 44 of U.S. Patent No. 11,018,922 (the "'922 Patent"), and Claim 5 of U.S. Patent No. 10,693,700 (the "'700 Patent") (collectively, the "Asserted Claims"); that none of the Asserted Claims were invalid; that LG willfully infringed at least one of the Asserted Claims; and that Plaintiff Constellation Designs LLC ("CD") should recover from LG $1,684,469.00 in the form of a running royalty as a damages award for LG's infringement. (Dkt. No. 277).

The Court entered Final Judgment based on the jury's verdict on August 23, 2023. (Dkt. No. 303.) Following entry of the Final Judgment, CD filed a Motion for Supplemental Damages, Ongoing Royalties, and Interest ("Supplemental Damages Motion") (Dkt. No. 315), a Motion for Attorney's Fees and Costs ("Fees Motion") (Dkt. No. 316), and an Unopposed Motion for Bill of

Costs ("Motion for Bill of Costs") (Dkt. No. 311). Additionally, LG filed a Motion for Judgment as a Matter of Law of No Liability ("JMOL of No Liability") (Dkt. No. 314), a Motion for Judgment as a Matter of Law of No Damages ("JMOL of No Damages") (Dkt. No. 310), a Motion for Judgment as a Matter of Law of No Willfulness ("JMOL of No Willfulness") (Dkt. No. 313), and a Motion for a New Trial ("New Trial Motion") (Dkt. No. 312) (collectively, the "Post-Judgment Motions").

The Court having now ruled on each of the Post-Judgment Motions, and pursuant to Rule 58 of the Federal Rules of Civil Procedure, and in accordance with the jury's unanimous verdict and the entirety of the record, the Court hereby **ORDERS** and **ENTERS** its **AMENDED FINAL JUDGMENT** as follows:

1. LG has infringed at least one Asserted Claim from each of the '761 Patent, '509 Patent, '922 Patent, and the '700 Patent;
2. The Asserted Claims are not invalid;
3. LG's infringement was willful;
4. CD is awarded damages from and against LG and shall accordingly have and recover from LG the sum of $1,684,469.00 U.S. Dollars for past infringement and as a running royalty;
5. CD is awarded supplemental damages against LG and shall accordingly have and recover from LG the sum of $157,241 U.S. Dollars;
6. CD is awarded an ongoing forward-looking royalty from LG at the rate of $6.75 per unit to be paid on a quarterly basis, such royalty being limited to the accused products litigated in this case; and LG must submit to CD by the 15th day of each month the total sold infringing units of the previous calendar month, with the first

such submission from LG to CD taking place on June 15, 2024 and continuing monthly thereafter during the life of the patents-in-suit;

7. Notwithstanding the jury's finding of willfulness, the Court having considered the totality of the circumstances together with the material benefit of having presided throughout the jury trial and having seen the same evidence and heard the same arguments as the jury, and mindful that enhancement is generally reserved for "egregious cases of culpable behavior,"[1] concludes that enhancement of the compensatory award herein is not warranted under 35 U.S.C. § 284 and consequently, the Court elects not to enhance the damages awarded herein or the ongoing royalty rate for future sales;

8. Pursuant to 35 U.S.C. § 284 and Supreme Court guidance that "prejudgment interest shall ordinarily be awarded absent some justification for withholding such an award,"[2] the Court awards to CD from LG pre-judgment interest applicable to all sums awarded herein, calculated at the 5-year U.S. Treasury Bill rate, compounded quarterly, from the date of infringement through the date of entry of this Judgment;[3]

9. Pursuant to 28 U.S.C. § 1961, the Court awards to CD from LG post-judgment interest applicable to all sums awarded herein, at the statutory rate, from the date of entry of this Judgment until paid; and

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[1] *Halo Elecs., Inc. v. Pulse Elecs., Inc.*, 579 U.S. 93, 106 (2016).
[2] *General Motors Corp. v. Devex Corp.*, 461 U.S. 648, 657 (1983).
[3] *See Nickson Indus., Inc. v. Rol Mfg. Co., Ltd.*, 847 F.2d 795, 800–801 (Fed. Cir. 1988).

10. Pursuant to Federal Rule of Civil Procedure 54(d), Local Rule CV-54, and 28 U.S.C. § 1920, CD is the prevailing party in this case and shall recover its costs from LG;

11. This Amended Final Judgment shall be and is effective for all purposes as of August 23, 2023, being the date of entry of the original Final Judgment herein.

All other requests for relief now pending and requested by either party but not specifically addressed herein are **DENIED**. The Clerk of Court is directed to **CLOSE** this case.

So ORDERED and SIGNED this 26th day of April, 2024.

RODNEY GILSTRAP
UNITED STATES DISTRICT JUDGE

U 8189233

THE UNITED STATES OF AMERICA

TO ALL TO WHOM THESE PRESENTS SHALL COME:

UNITED STATES DEPARTMENT OF COMMERCE
United States Patent and Trademark Office

December 13, 2021

THIS IS TO CERTIFY THAT ANNEXED HERETO IS A TRUE COPY FROM THE RECORDS OF THIS OFFICE OF:

U.S. PATENT: *8,842,761*
ISSUE DATE: *September 23, 2014*

By Authority of the
Under Secretary of Commerce for Intellectual Property
and Director of the United States Patent and Trademark Office

SYLVIA HOLLEY
Certifying Officer

Joint Exhibit
Constellation v. LG
JTX-001
No. 2:21-cv-00448-JRG

US008842761B2

(12) **United States Patent**
Barsoum et al.

(10) **Patent No.: US 8,842,761 B2**
(45) **Date of Patent: Sep. 23, 2014**

(54) **METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS**

(75) Inventors: **Maged F. Barsoum**, Saratoga, CA (US); **Christopher R. Jones**, Pacific Palisades, CA (US)

(73) Assignee: **Constellation Designs, Inc.**, Pacific Palisades, CA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **13/618,630**

(22) Filed: **Sep. 14, 2012**

(65) **Prior Publication Data**

US 2013/0083862 A1 Apr. 4, 2013

Related U.S. Application Data

(63) Continuation of application No. 13/118,921, filed on May 31, 2011, now Pat. No. 8,270,511, which is a continuation of application No. 12/156,989, filed on Jun. 5, 2008, now Pat. No. 7,978,777.

(60) Provisional application No. 60/933,319, filed on Jun. 5, 2007.

(51) **Int. Cl.**

H04L 23/02	(2006.01)
H04L 27/36	(2006.01)
H04L 27/38	(2006.01)
H04L 27/04	(2006.01)
H03C 3/00	(2006.01)
H04L 1/00	(2006.01)
H04L 27/34	(2006.01)
H04B 15/00	(2006.01)

(52) **U.S. Cl.**
CPC ***H04B 15/00*** (2013.01); ***H04L 1/0003*** (2013.01); *H04L 1/0009* (2013.01); ***H04L 27/3405*** (2013.01)
USPC **375/261**; 375/298; 329/304; 332/103

(58) **Field of Classification Search**
USPC 375/259–262, 268, 269, 295, 298, 279, 375/280, 329, 308, 316, 340, 341, 332; 332/103; 329/304
See application file for complete search history.

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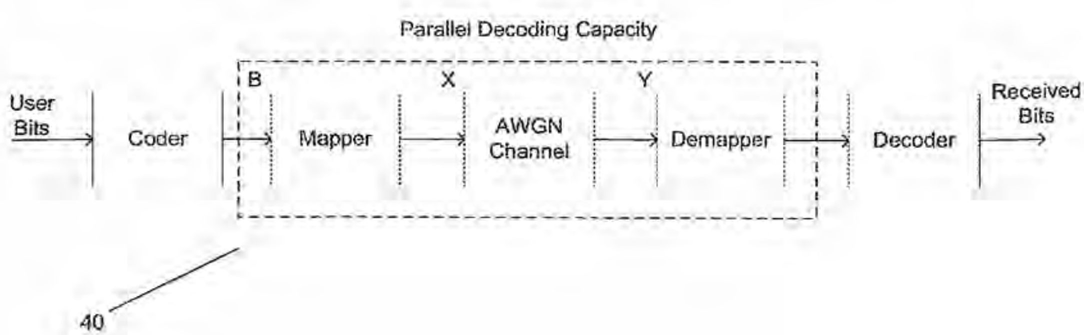
Primary Examiner — Tesfaldet Bocure
(74) *Attorney, Agent, or Firm* — KPPB LLP

(57) **ABSTRACT**

Communication systems are described that use geometrically shaped constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. In several embodiments, the geometrically shaped is optimized based upon a capacity measure such as parallel decoding capacity or joint capacity. In many embodiments, a capacity optimized geometrically shaped constellation can be used to replace a conventional constellation as part of a firmware upgrade to transmitters and receivers within a communication system. In a number of embodiments, the geometrically shaped constellation is optimized for an Additive White Gaussian Noise channel or a fading channel. In numerous embodiments, the communication uses adaptive rate encoding and the location of points within the geometrically shaped constellation changes as the code rate changes.

32 Claims, 44 Drawing Sheets



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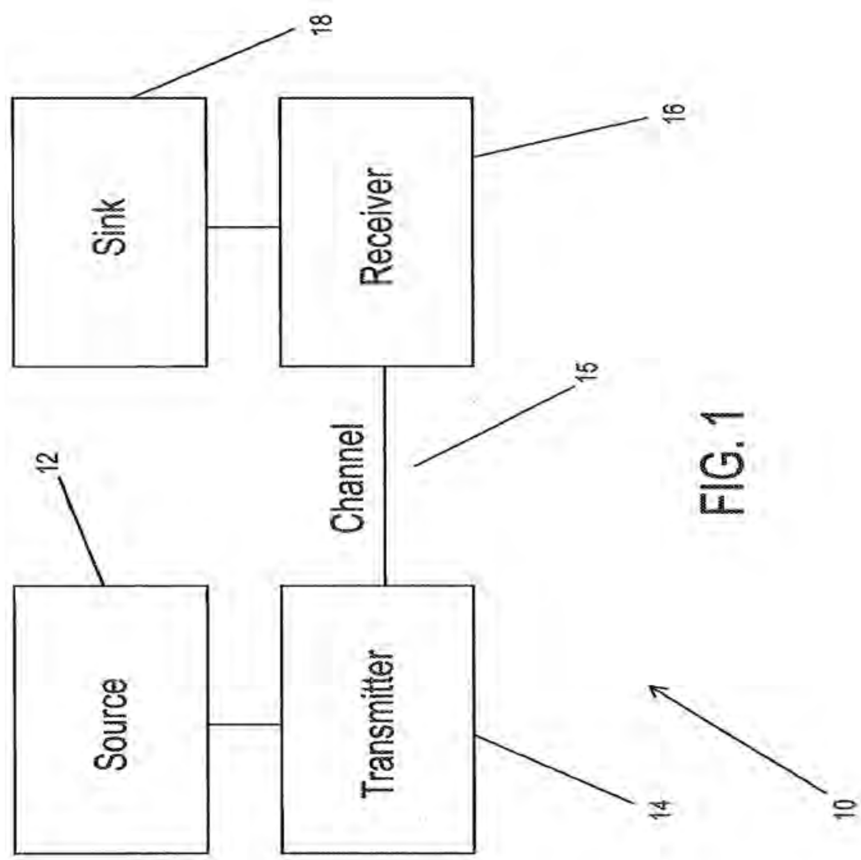


FIG. 1

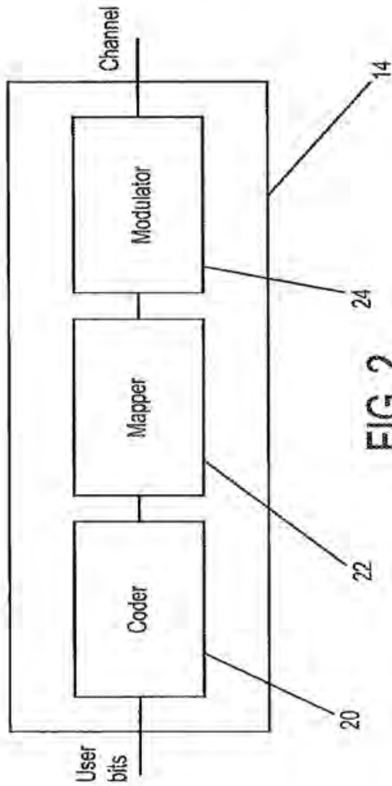
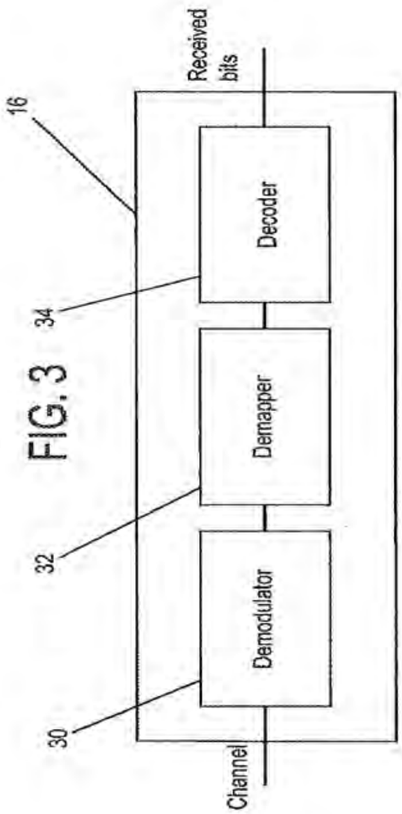

Channel
Modulator
Mapper
Coder
User bits
14
24
22
20

FIG. 2

FIG. 3


Received bits
Decoder
Demapper
Demodulator
Channel
16
34
32
30

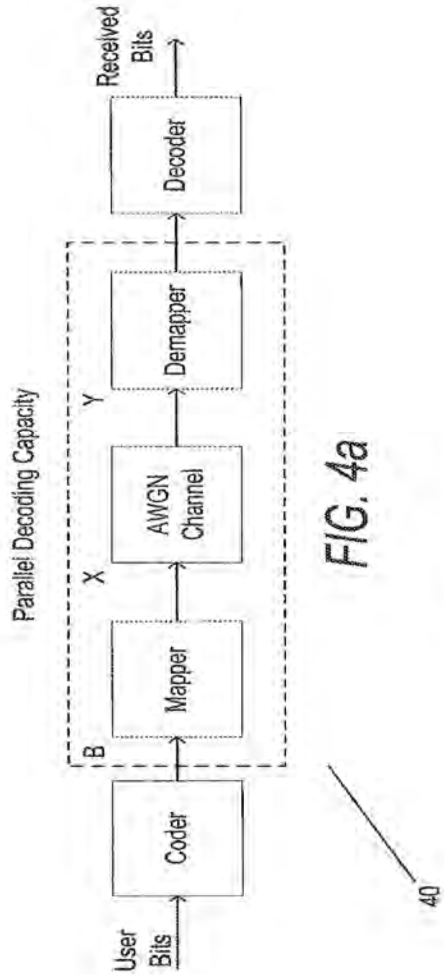

Parallel Decoding Capacity
User Bits
Coder
B
Mapper
X
AWGN Channel
Y
Demapper
Decoder
Received Bits
40

FIG. 4a

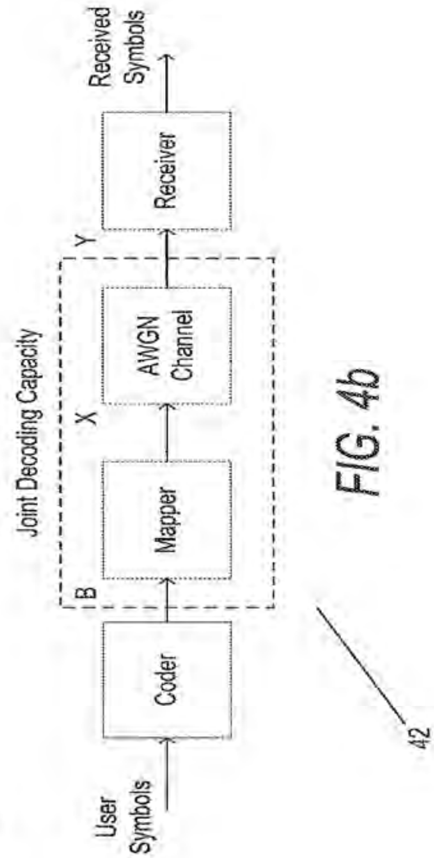

Joint Decoding Capacity
User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
42

FIG. 4b

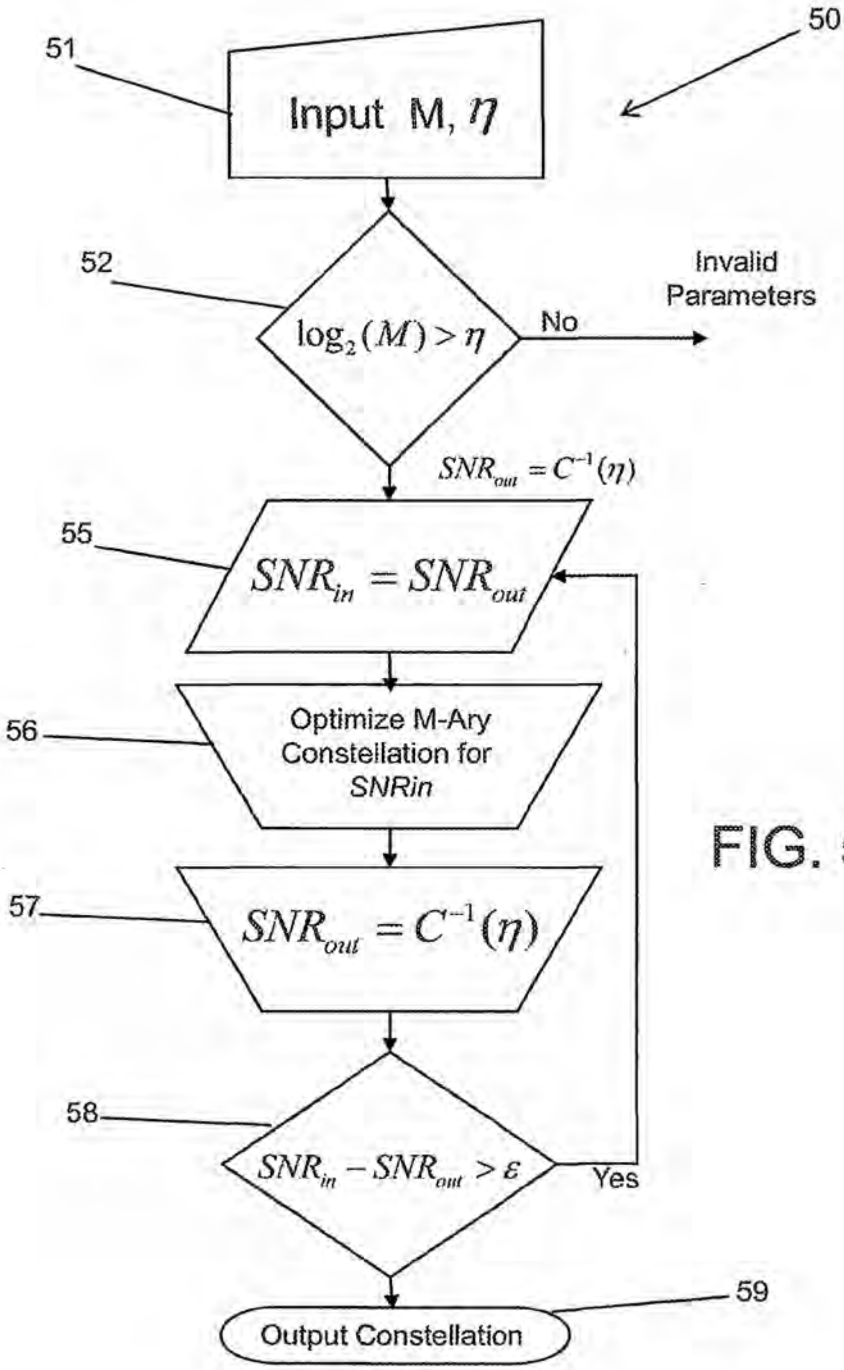

50
51
Input M, η
52
log₂(M) > η
No
Invalid Parameters
SNR_out = C⁻¹(η)
55
SNR_in = SNR_out
56
Optimize M-Ary Constellation for SNRin
57
SNR_out = C⁻¹(η)
58
SNR_in − SNR_out > ε
Yes
59
Output Constellation

FIG. 5

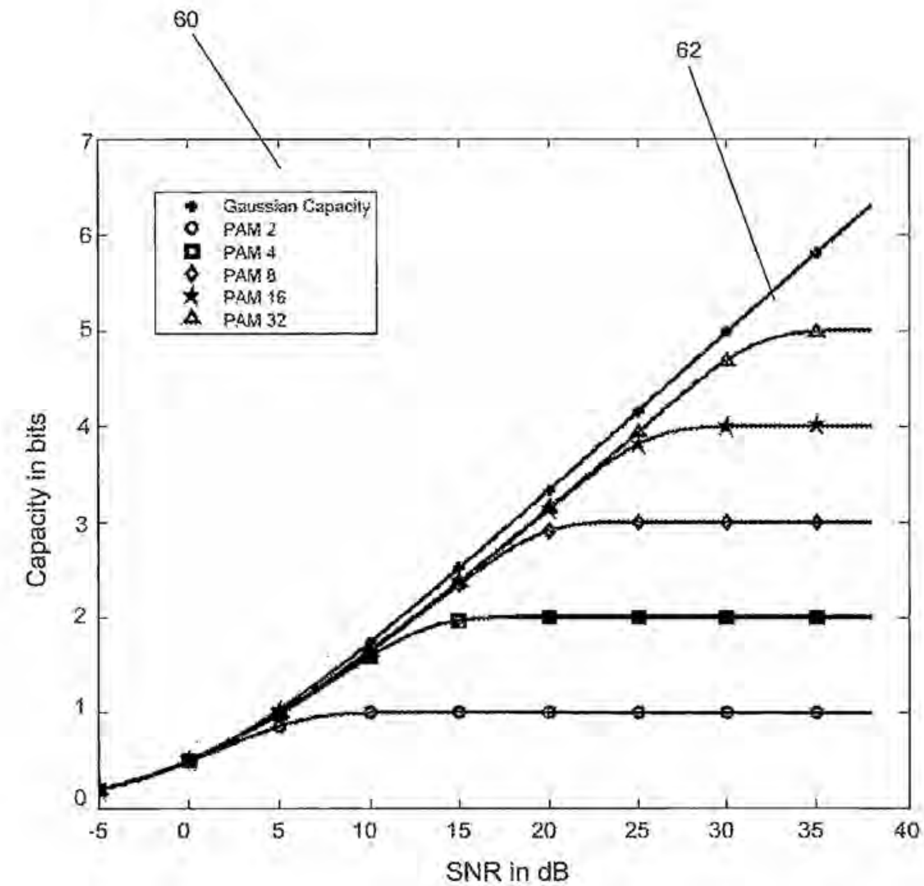
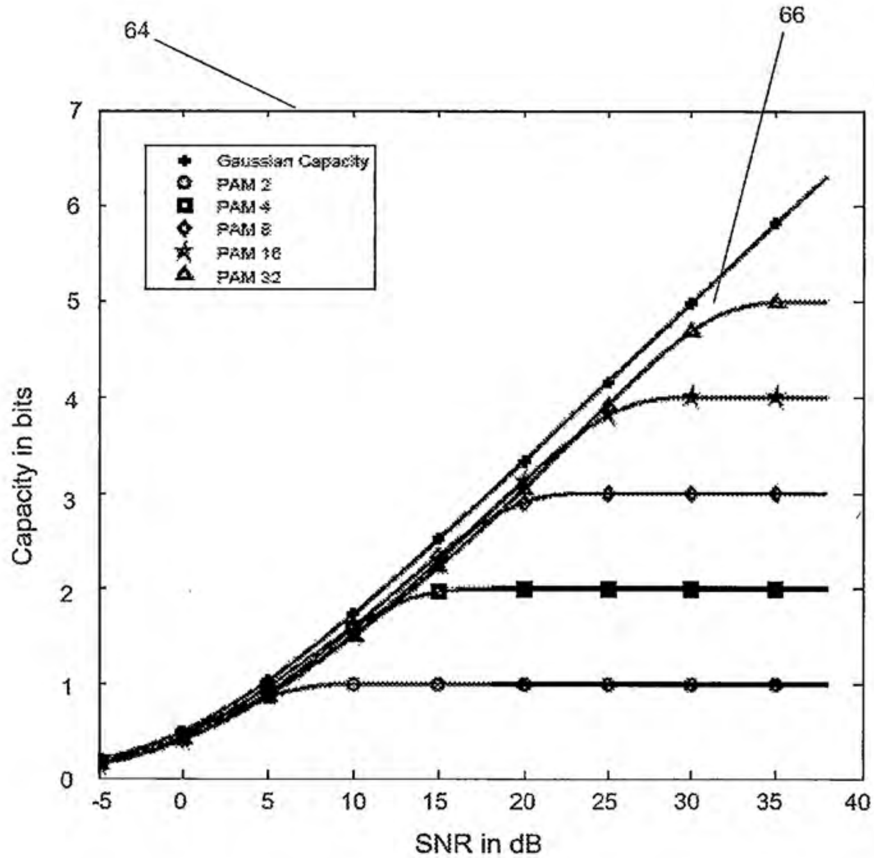

60
62
Gaussian Capacity
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Capacity in bits
SNR in dB
0
1
2
3
4
5
6
7
-5
0
5
10
15
20
25
30
35
40

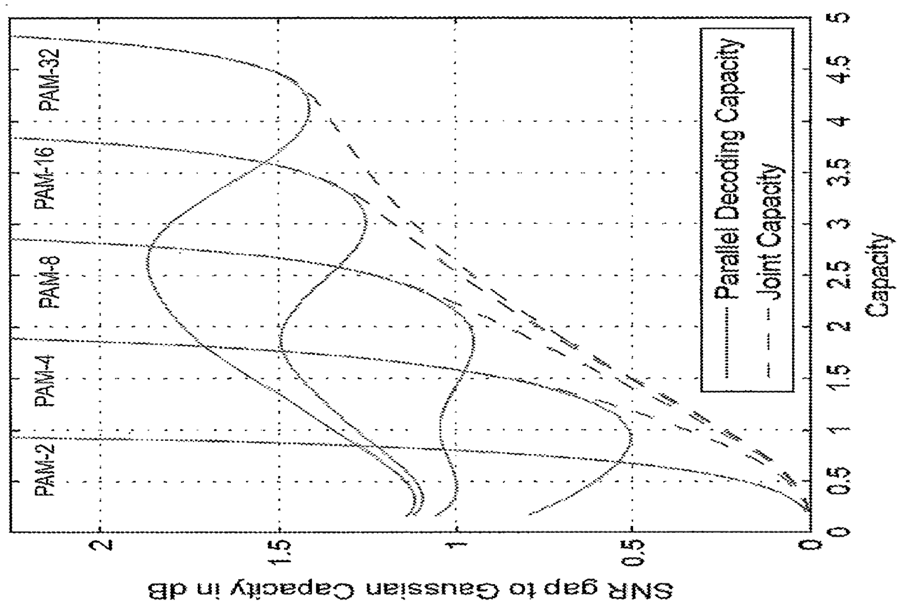
FIG. 6a



64
66
Gaussian Capacity
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Capacity in bits
SNR in dB
0
1
2
3
4
5
6
7
-5
0
5
10
15
20
25
30
35
40

FIG. 6b

FIG. 7


PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
Parallel Decoding Capacity
Joint Capacity
Capacity
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

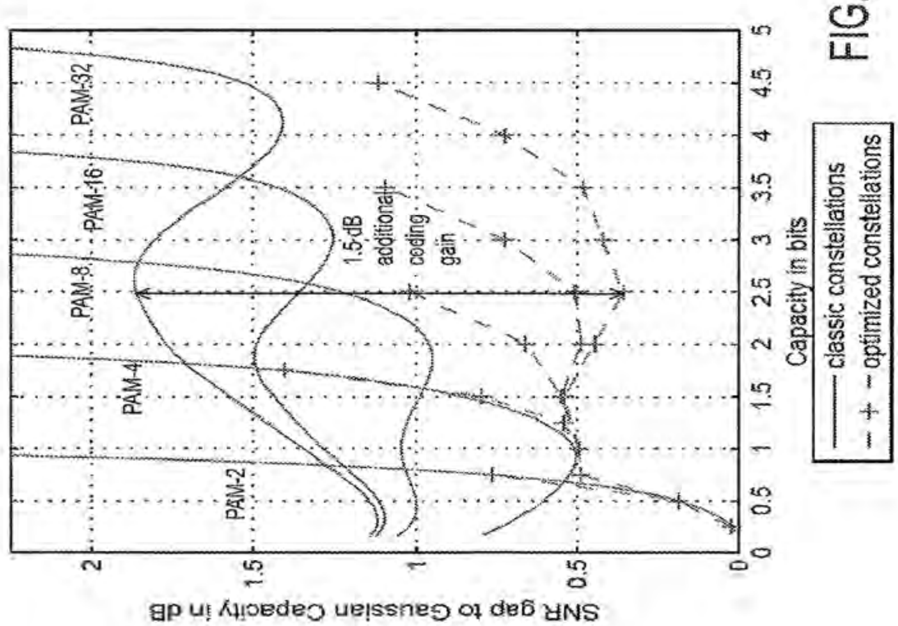

SNR gap to Gaussian Capacity in dB
Capacity in bits
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5dB
additional
coding
gain
classic constellations
optimized constellations
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 8a

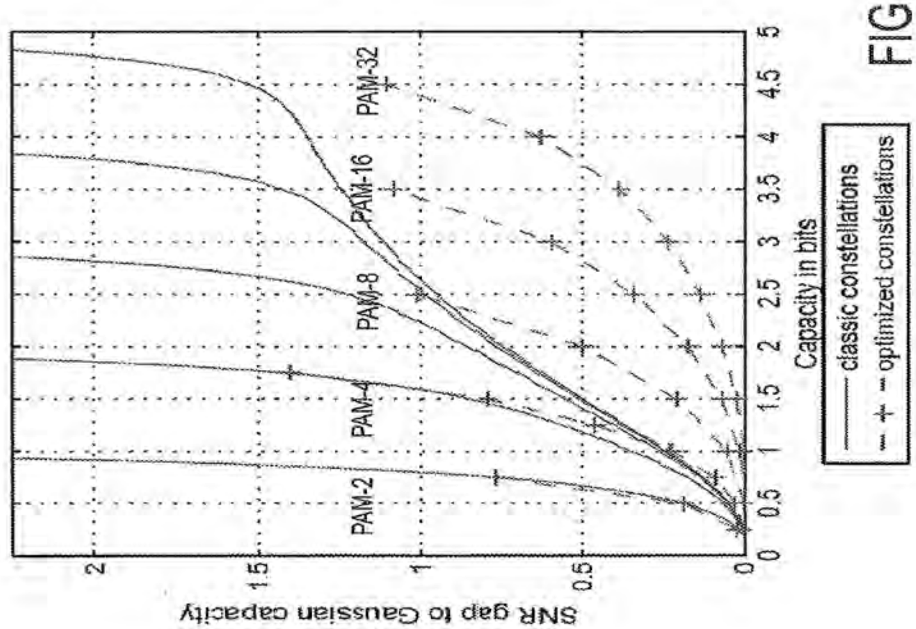

SNR gap to Gaussian capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
classic constellations
optimized constellations

FIG. 8b

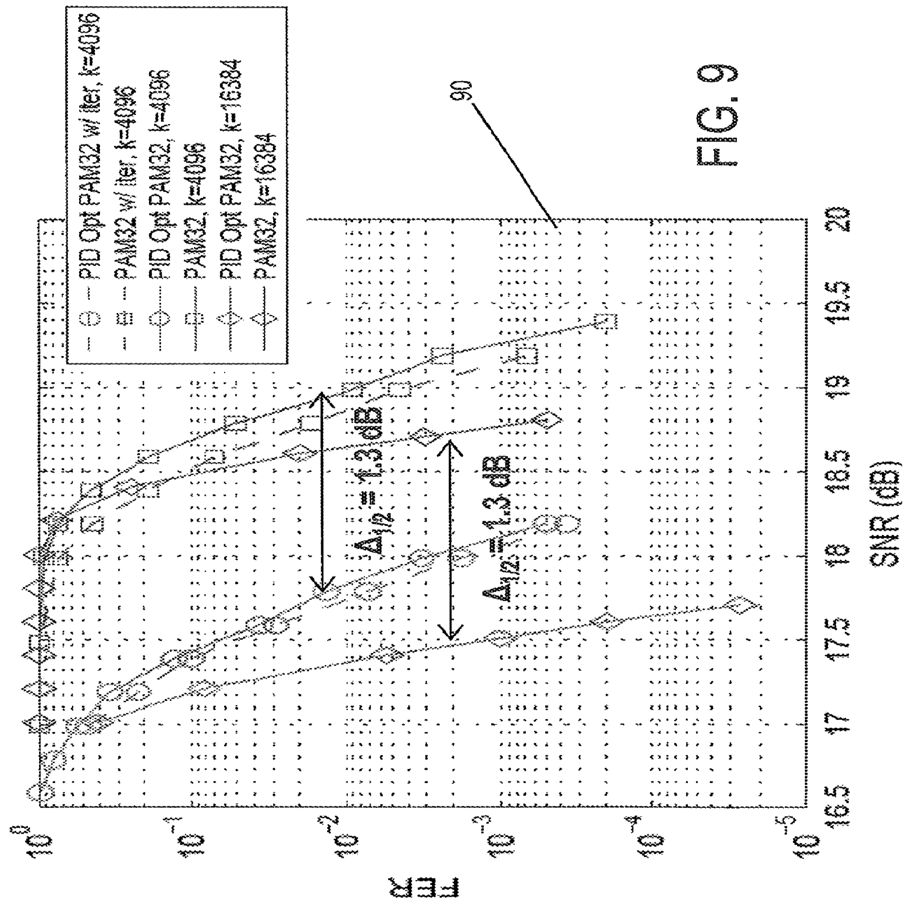

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

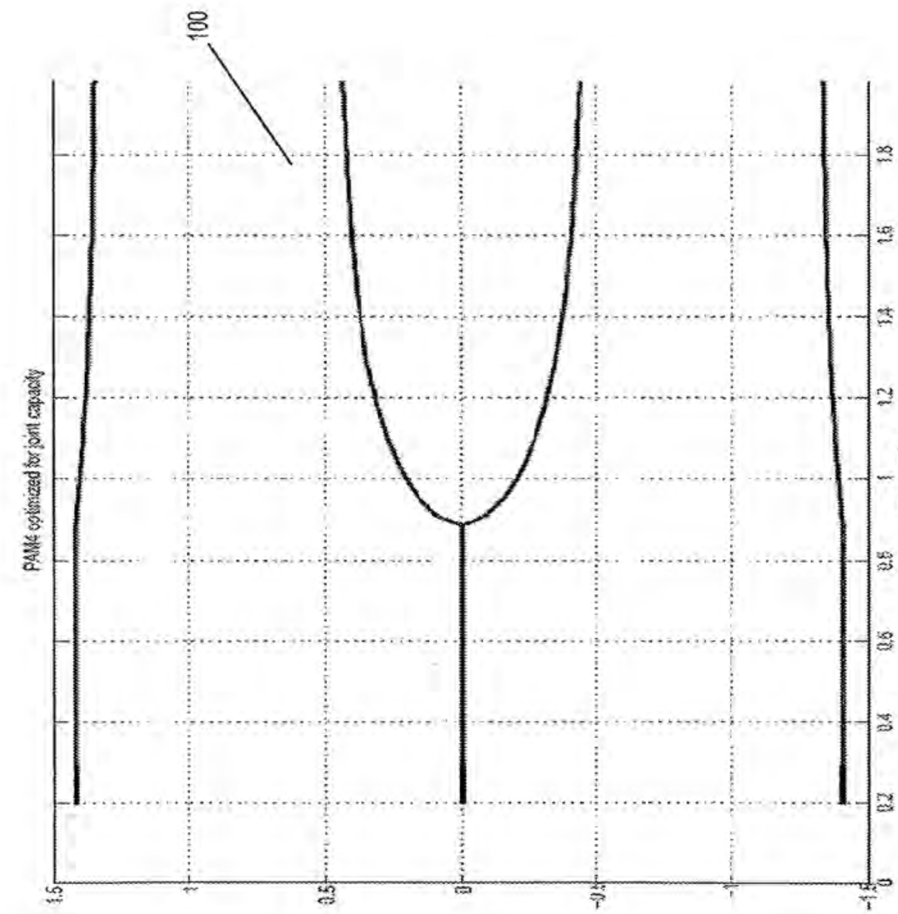


FIG. 10a

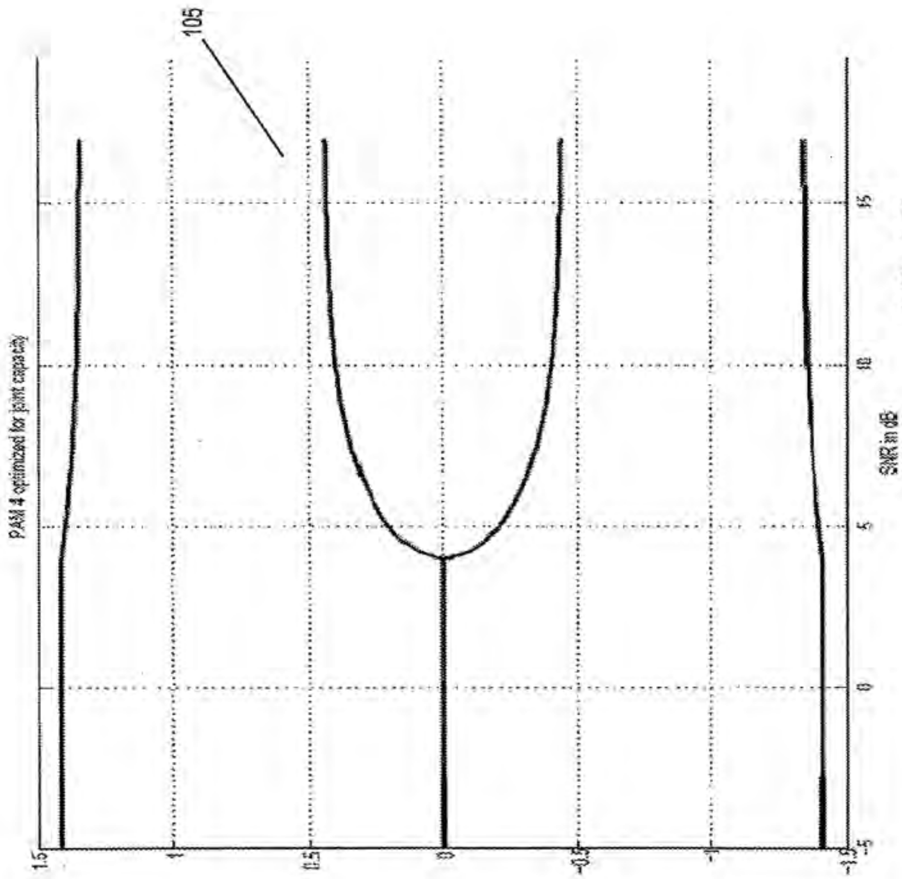


FIG. 10b

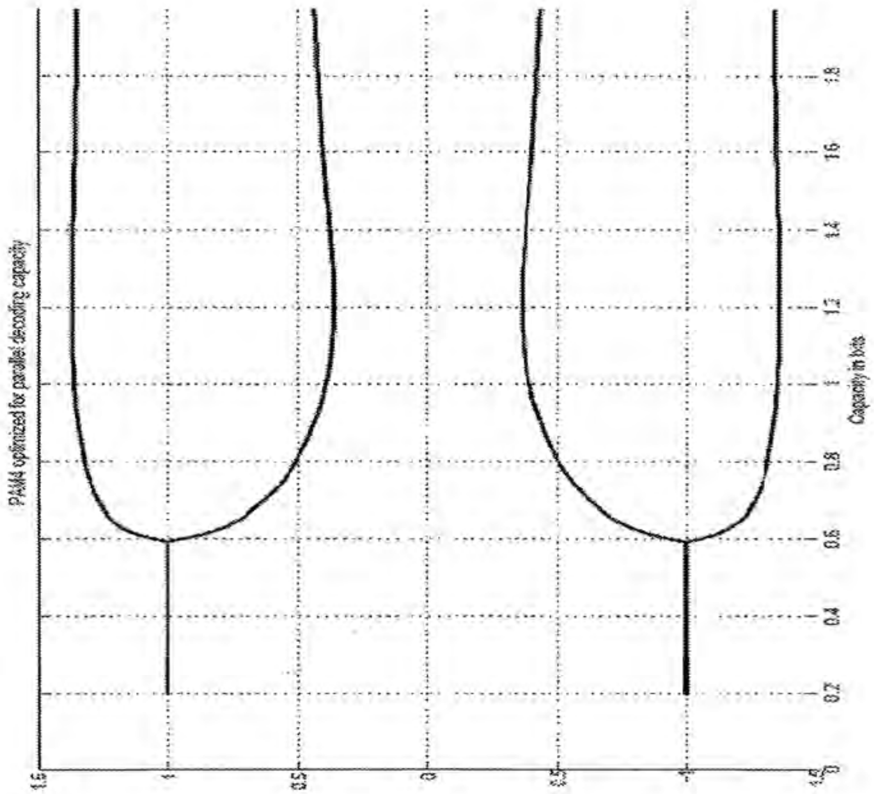

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

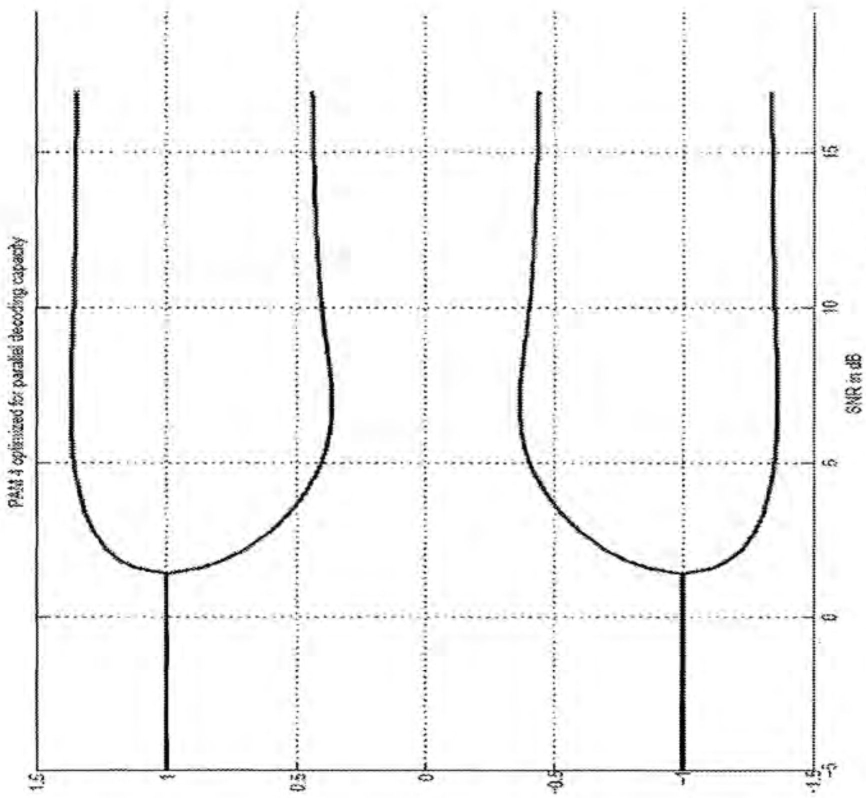

PAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

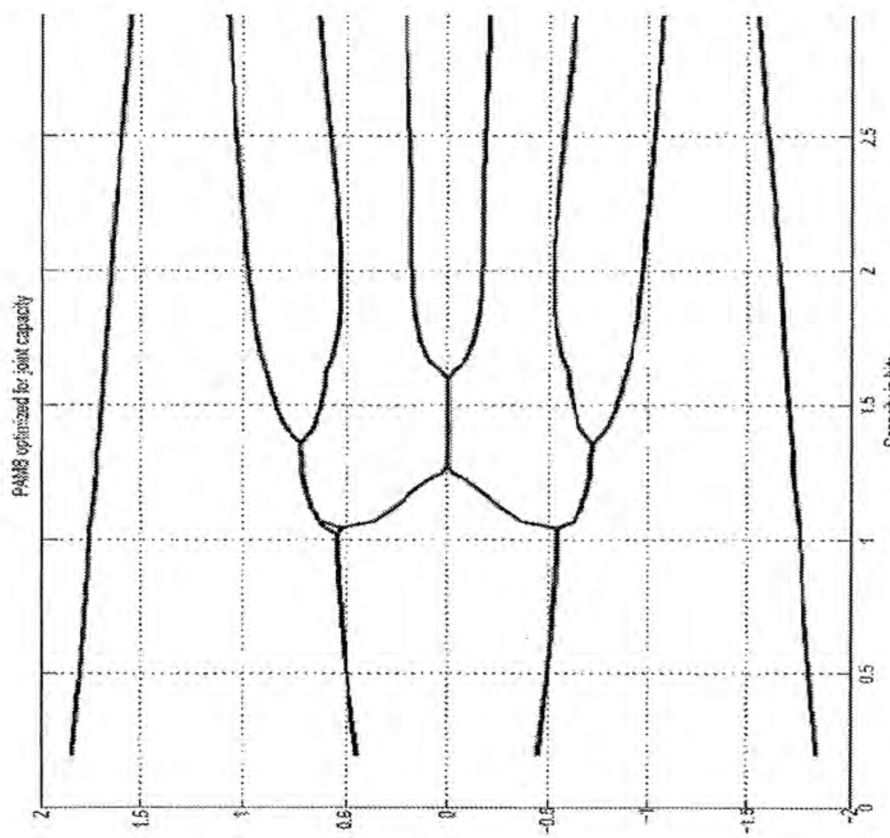

PAM8 optimized for joint capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12a

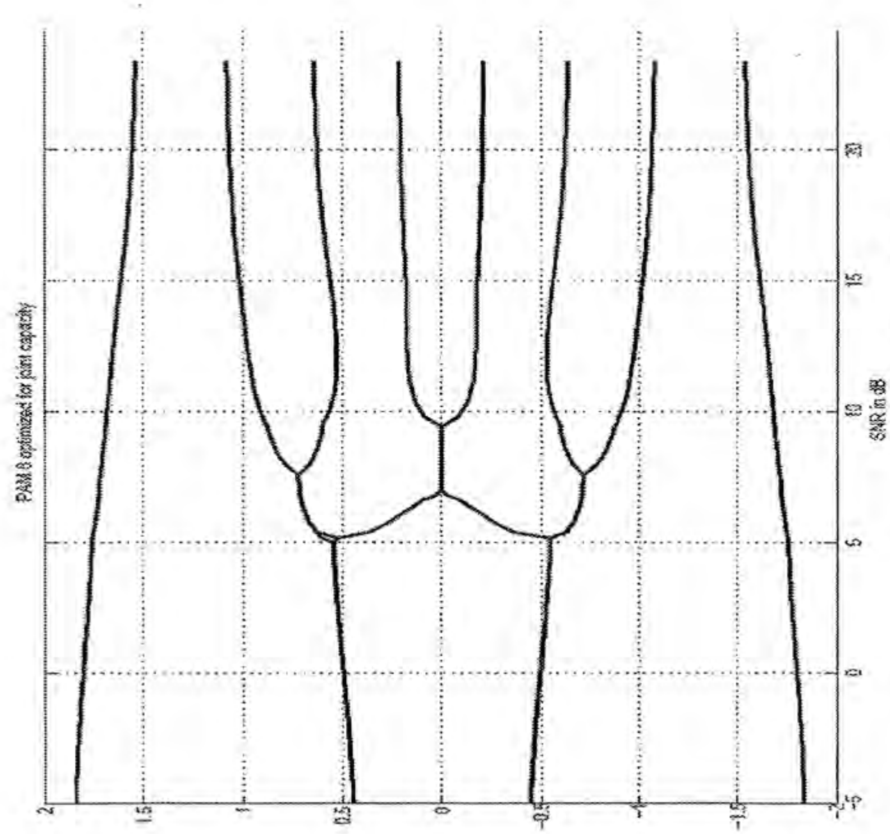

PAM 8 optimized for joint capacity
SNR in dB
-5
0
5
10
15
20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12b

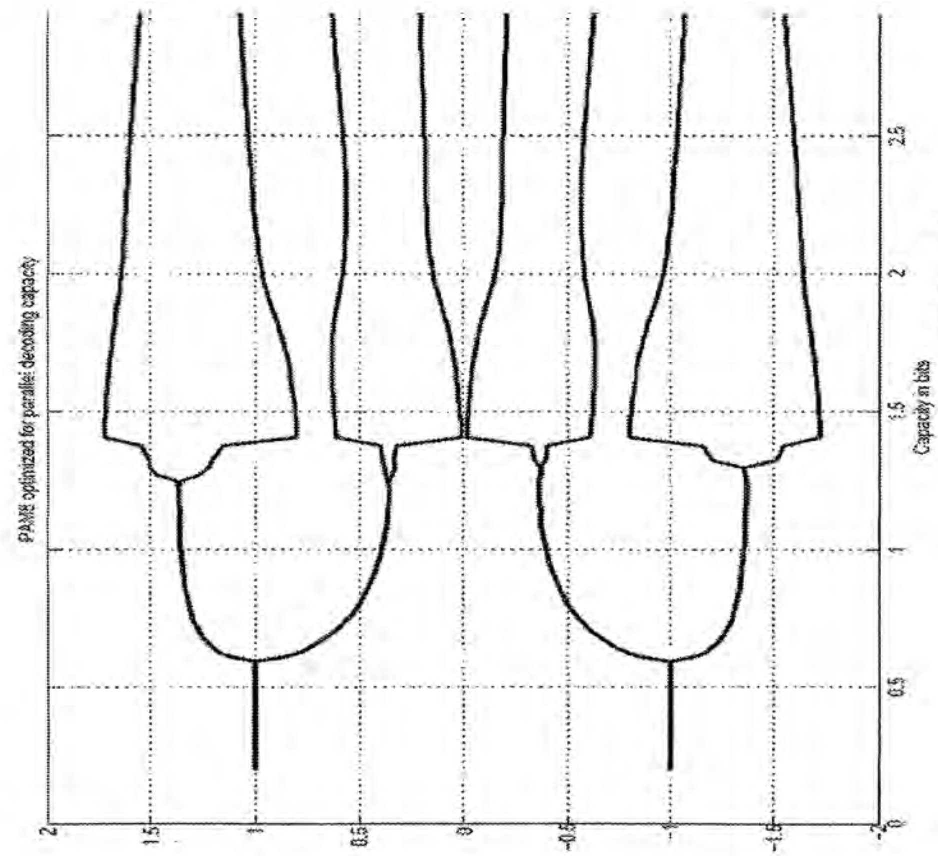

PAME optimized for parallel decoding capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12c

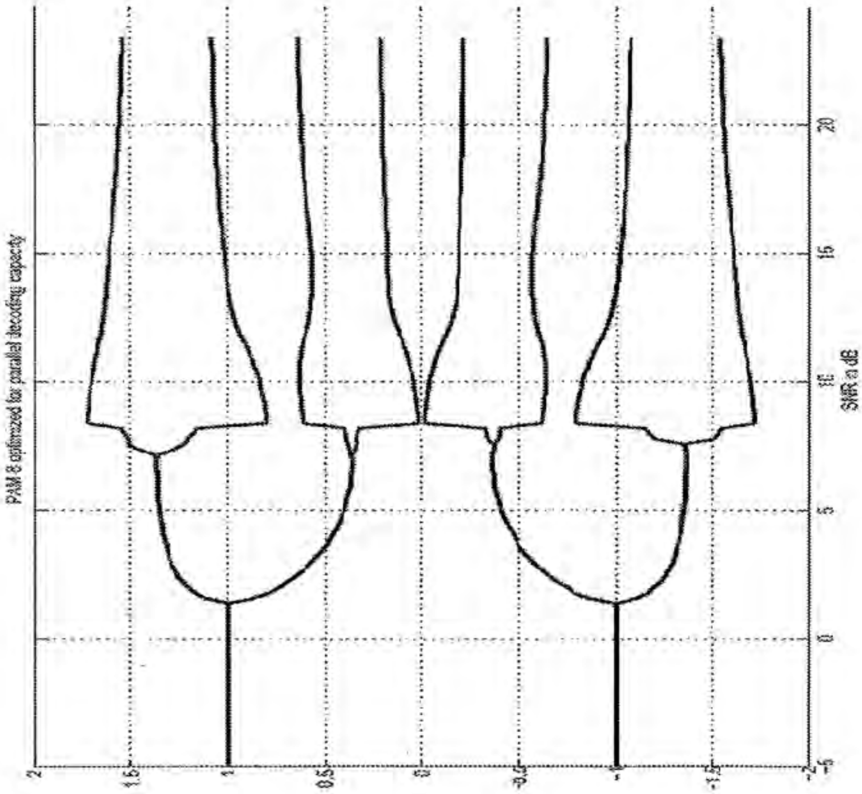

PAM 8 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5,27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

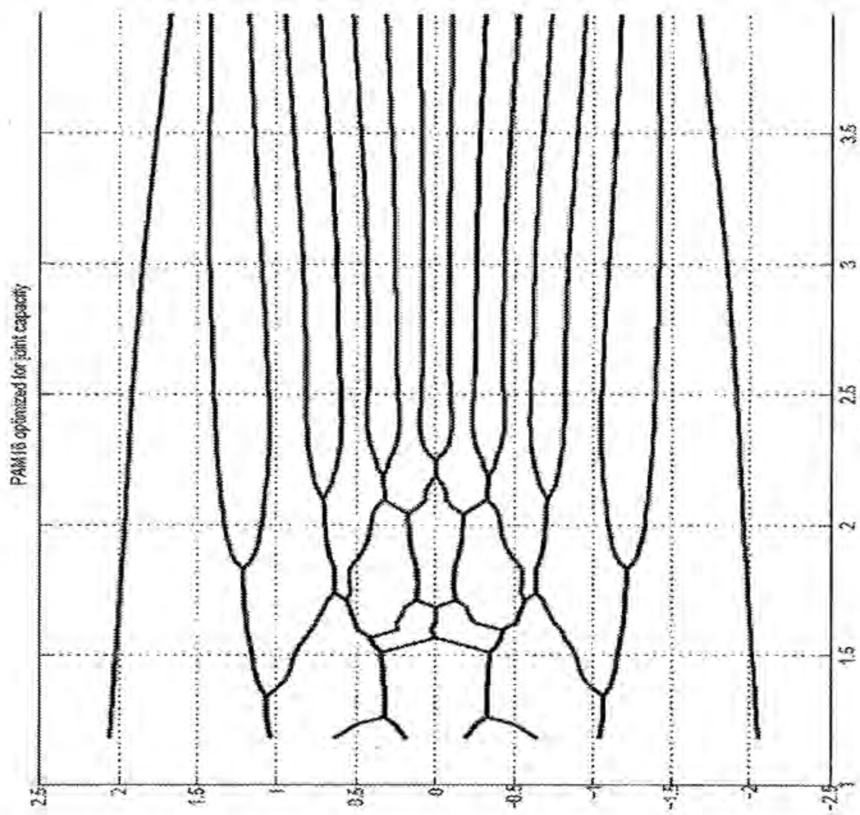

PAM 16 optimized for joint capacity
Capacity in bits
1.5
2
2.5
3
3.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14a

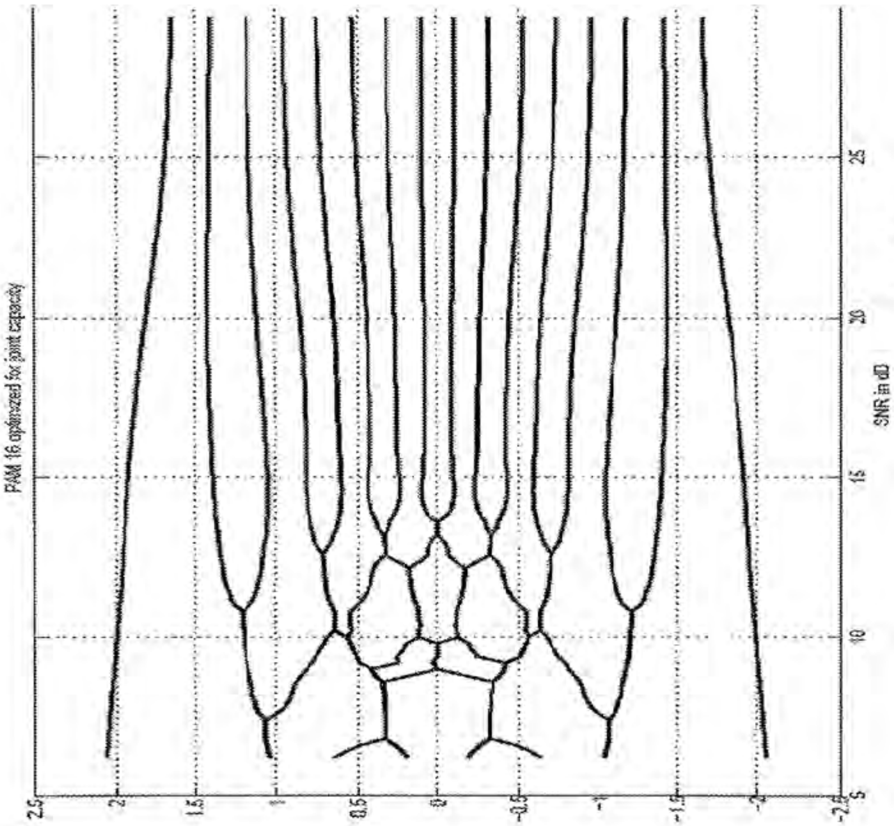

PAM 16 optimized for joint capacity
SNR in dB
5
10
15
20
25
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14b

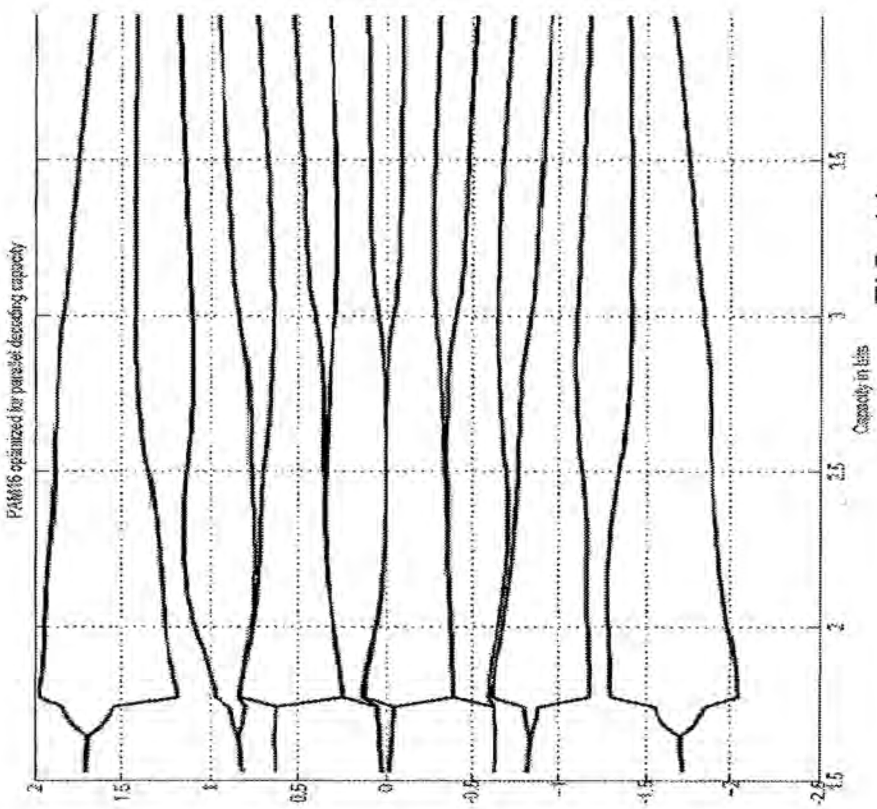

PAM16 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5

FIG. 14c

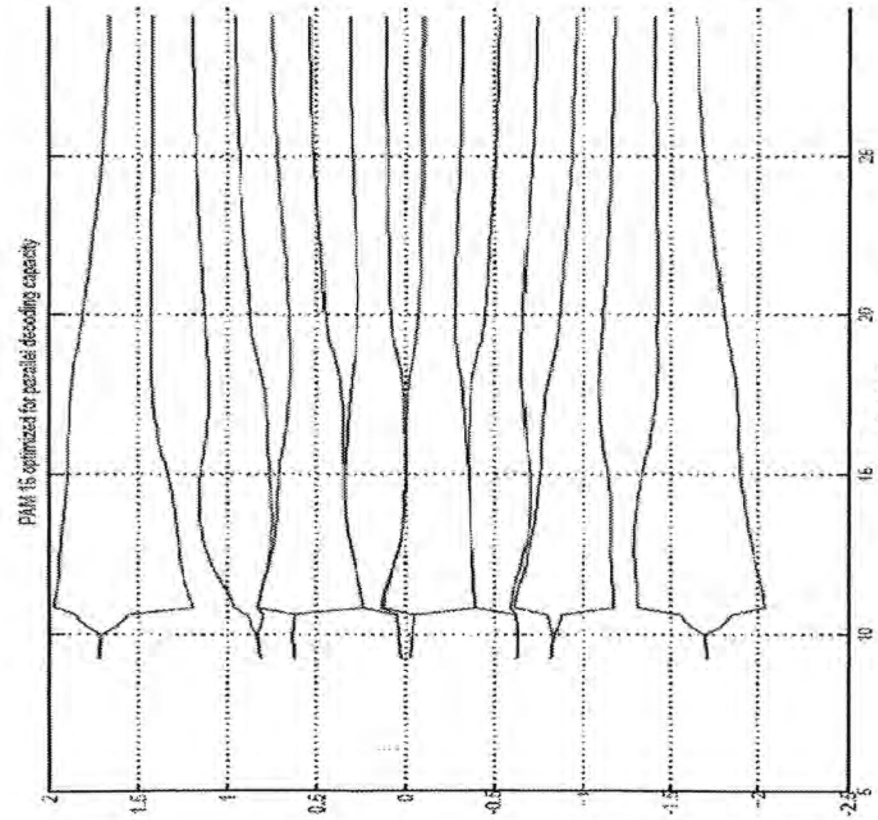


FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

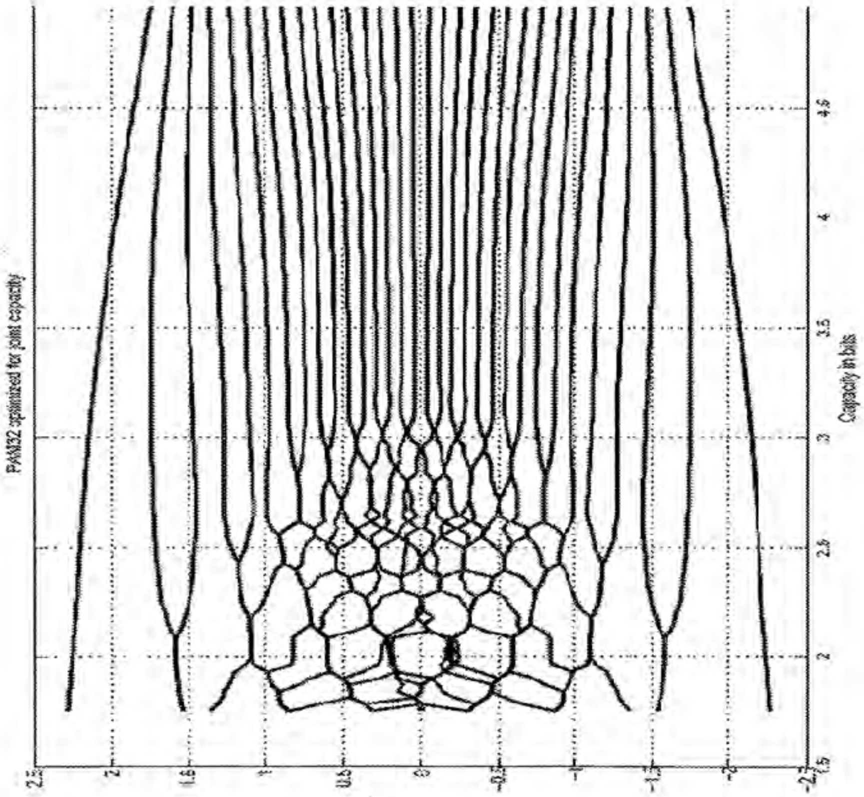


FIG. 16a

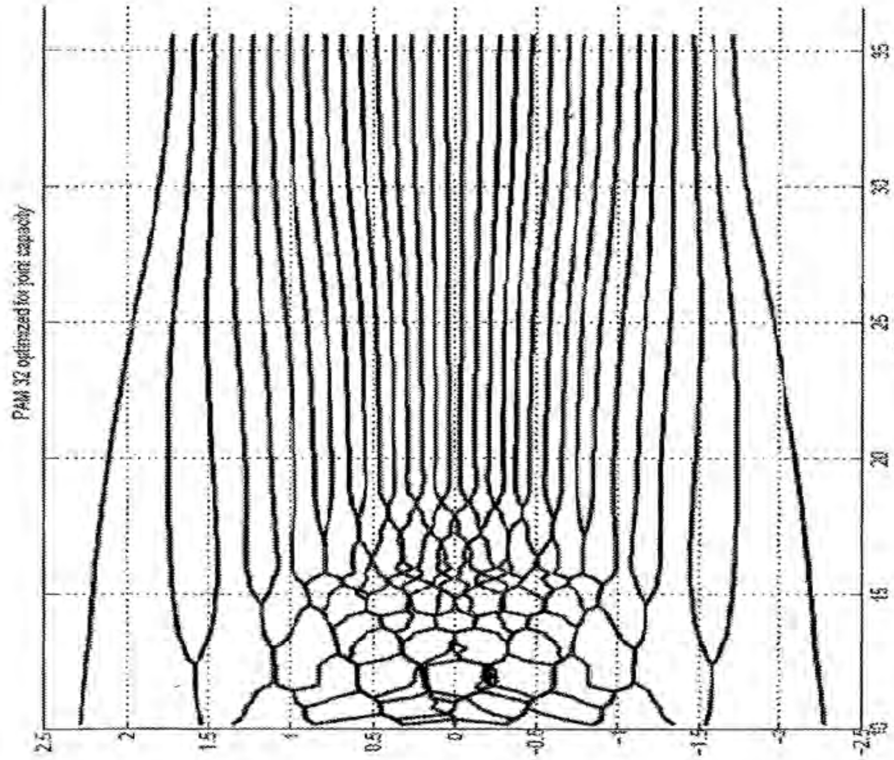

PAM 32 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35

FIG. 16b

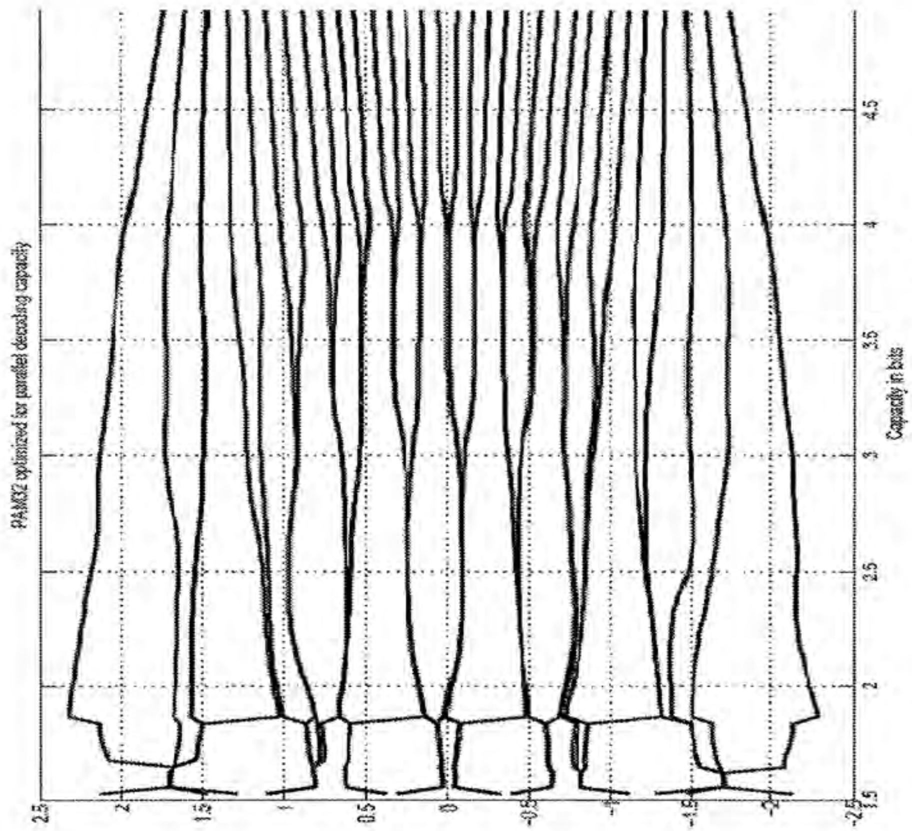

PAM32 optimized for parallel decoding capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16c

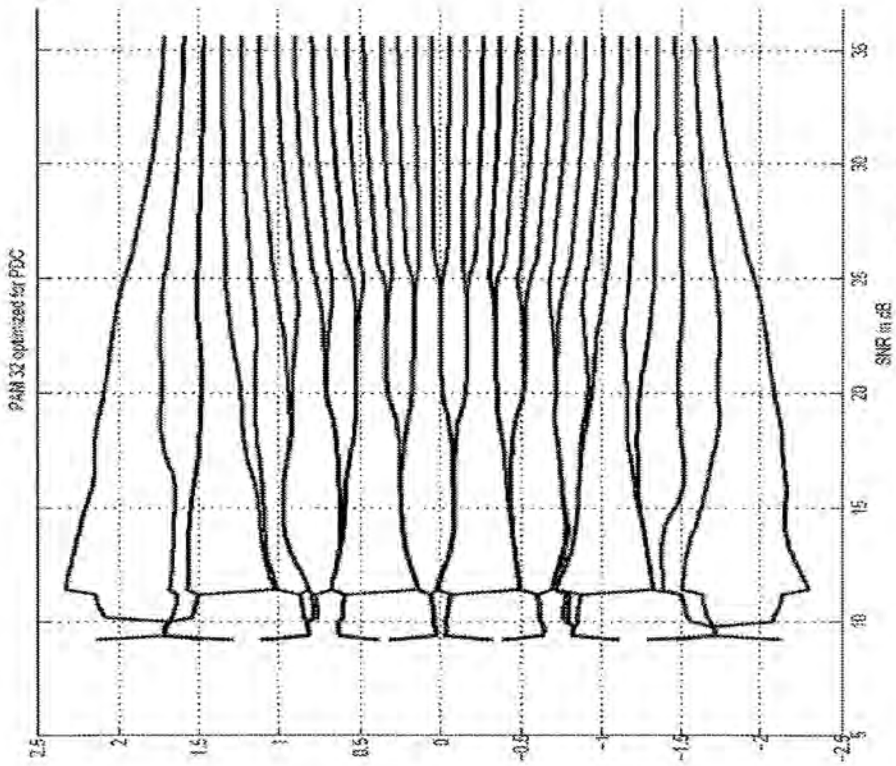

PAM 32 optimized for PDC
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25
30
35

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

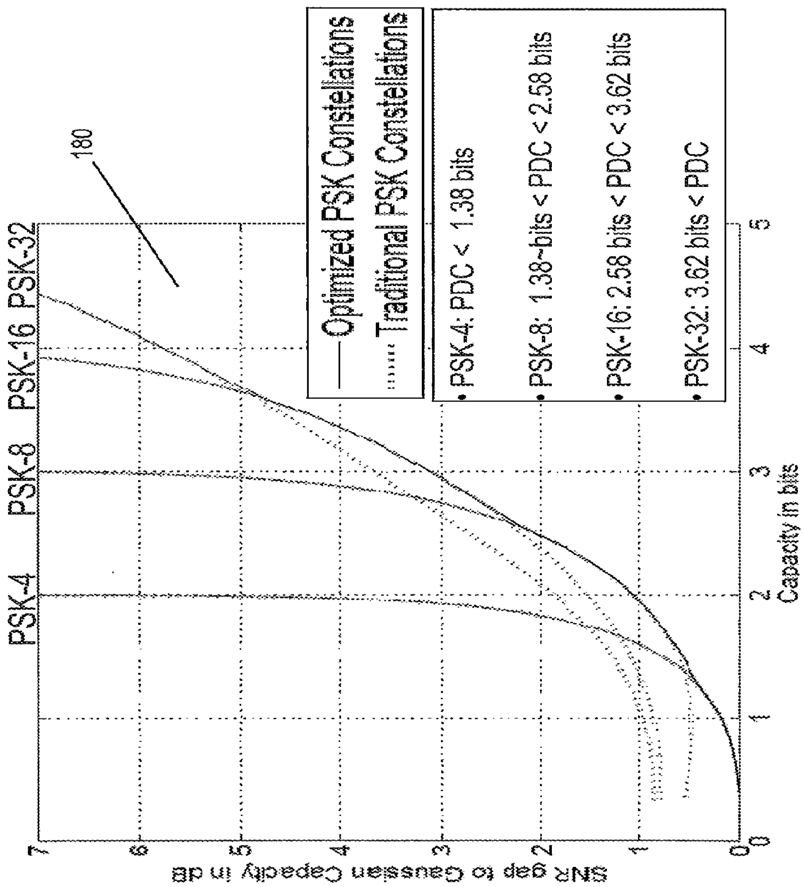

PSK-4
PSK-8
PSK-16
PSK-32
180
Optimized PSK Constellations
Traditional PSK Constellations
• PSK-4: PDC < 1.38 bits
• PSK-8: 1.38~bits < PDC < 2.58 bits
• PSK-16: 2.58 bits < PDC < 3.62 bits
• PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

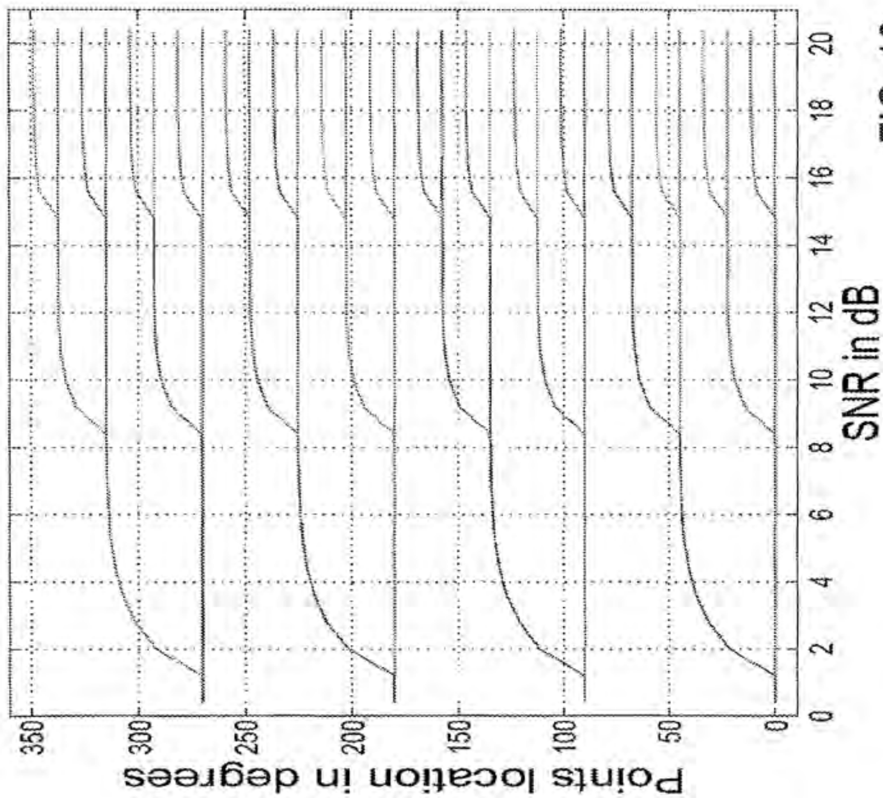

SNR in dB
0 2 4 6 8 10 12 14 16 18 20
Points location in degrees
0 50 100 150 200 250 300 350

FIG. 19

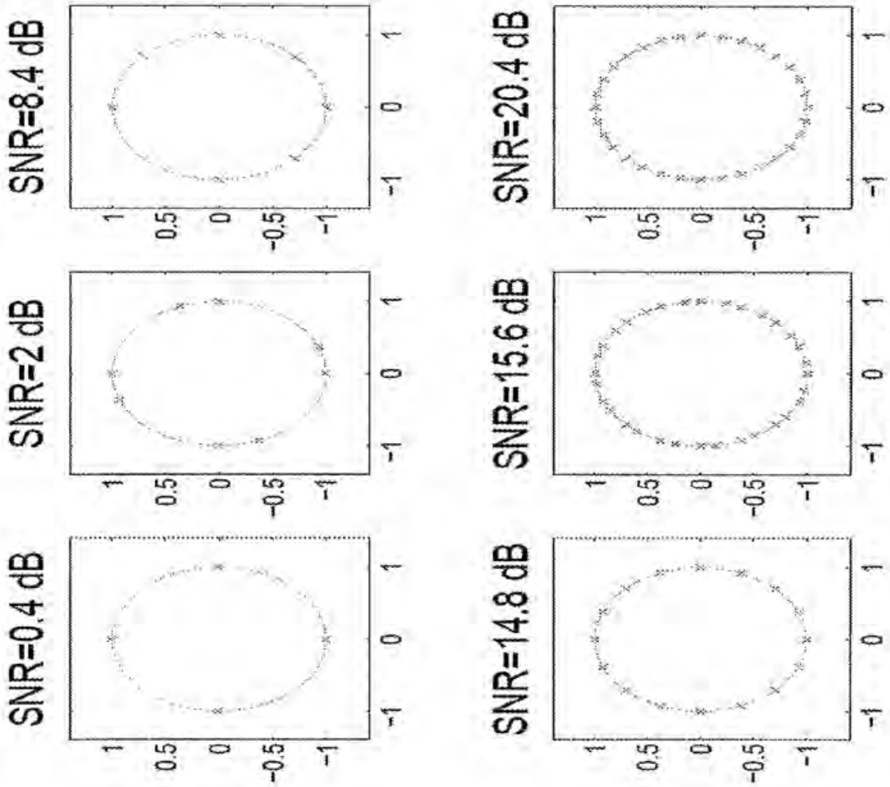


FIG. 20

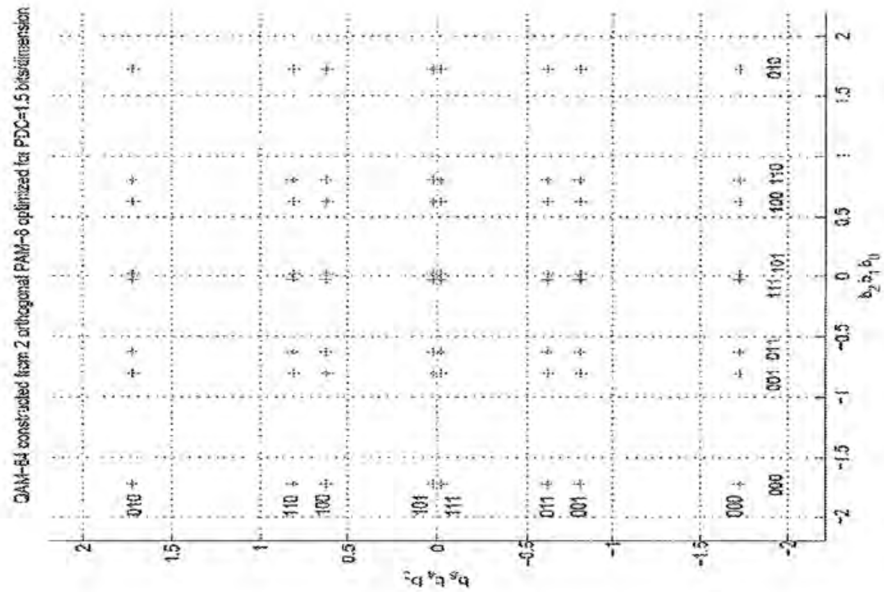

QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
b_2, b_1, b_0
b_5, b_4, b_3

FIG. 21

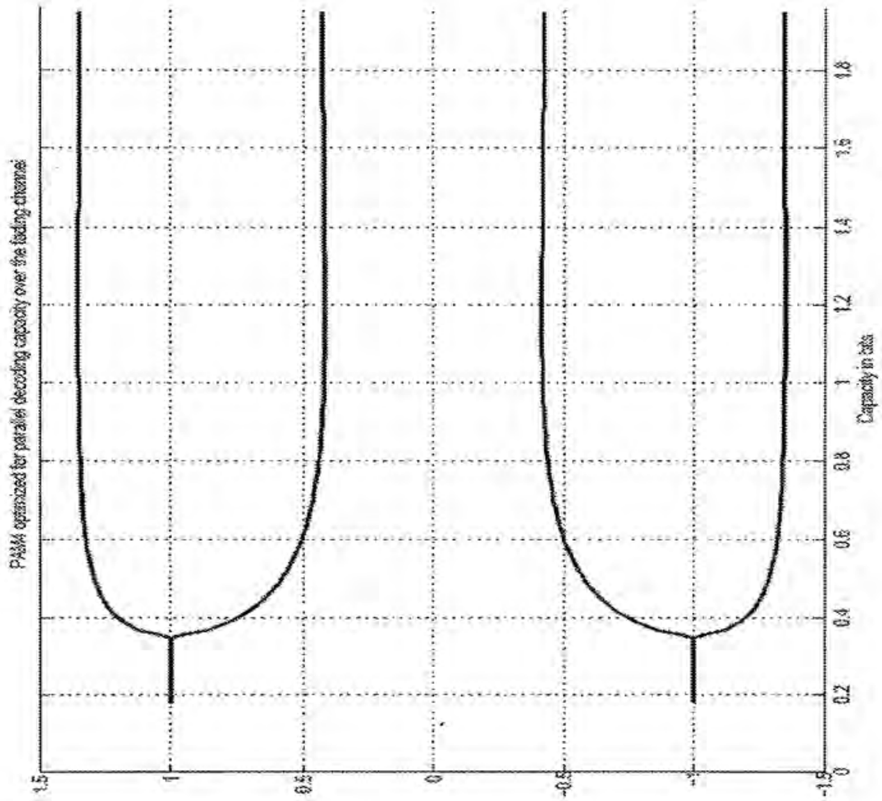

PAM optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 22a

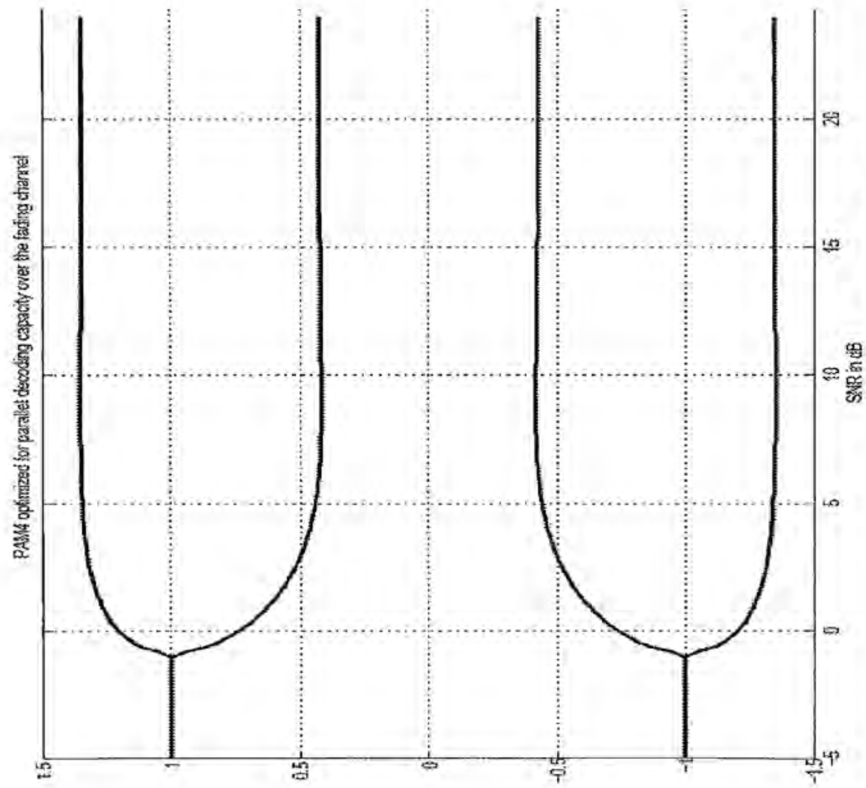

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15
20

FIG. 22b

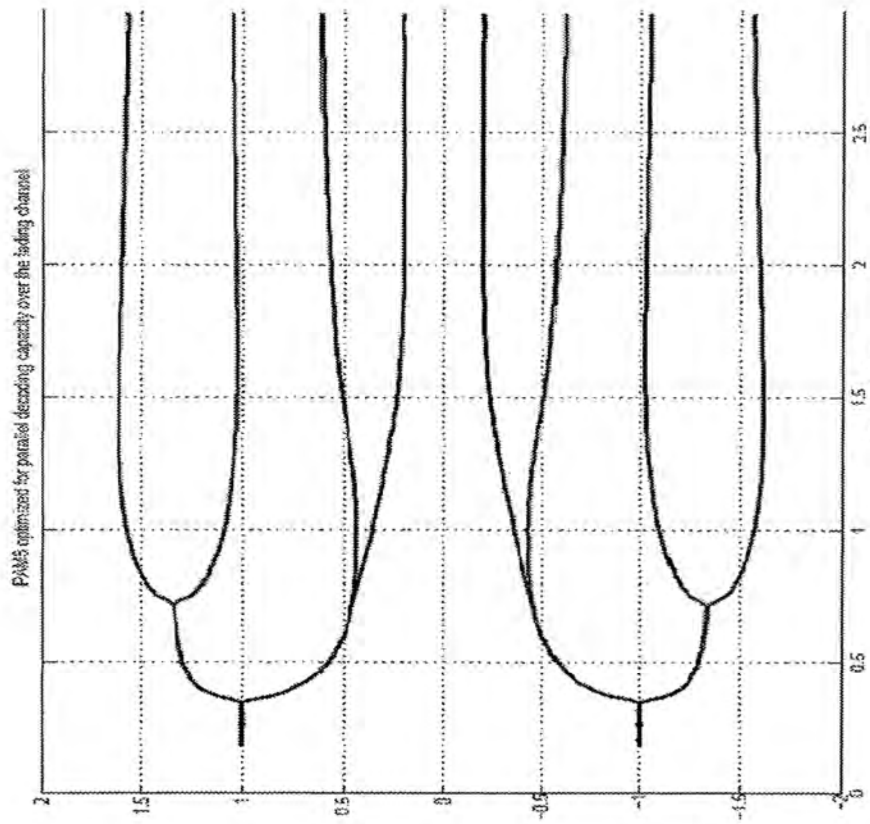

PAM5 optimized for parallel decoding capacity over the fading channel
Capacity in bits

FIG. 23a

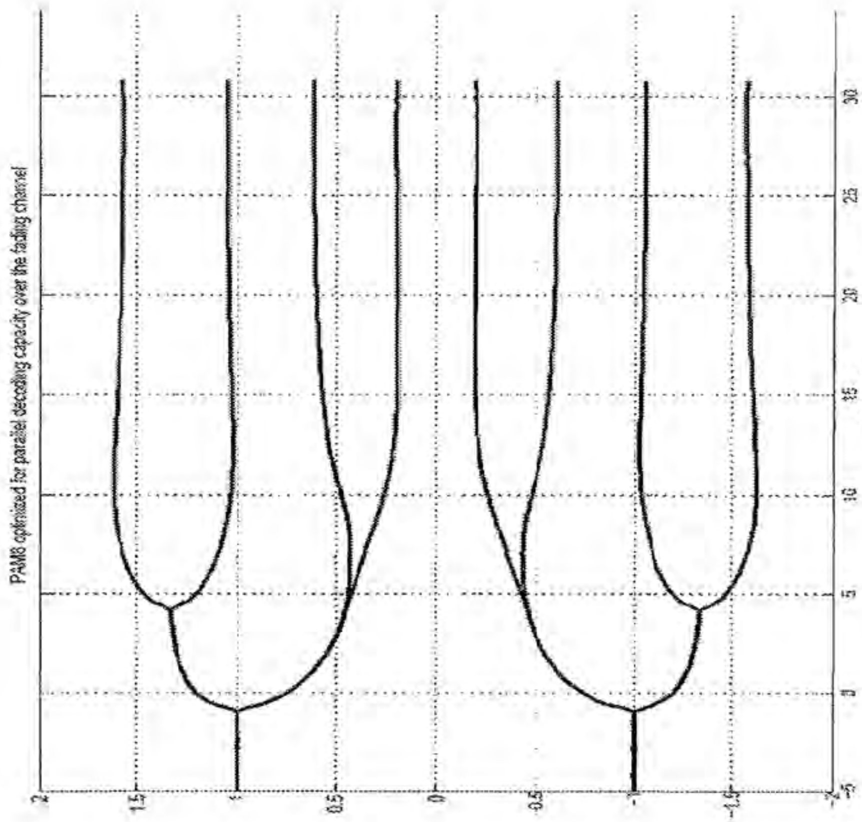

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23b

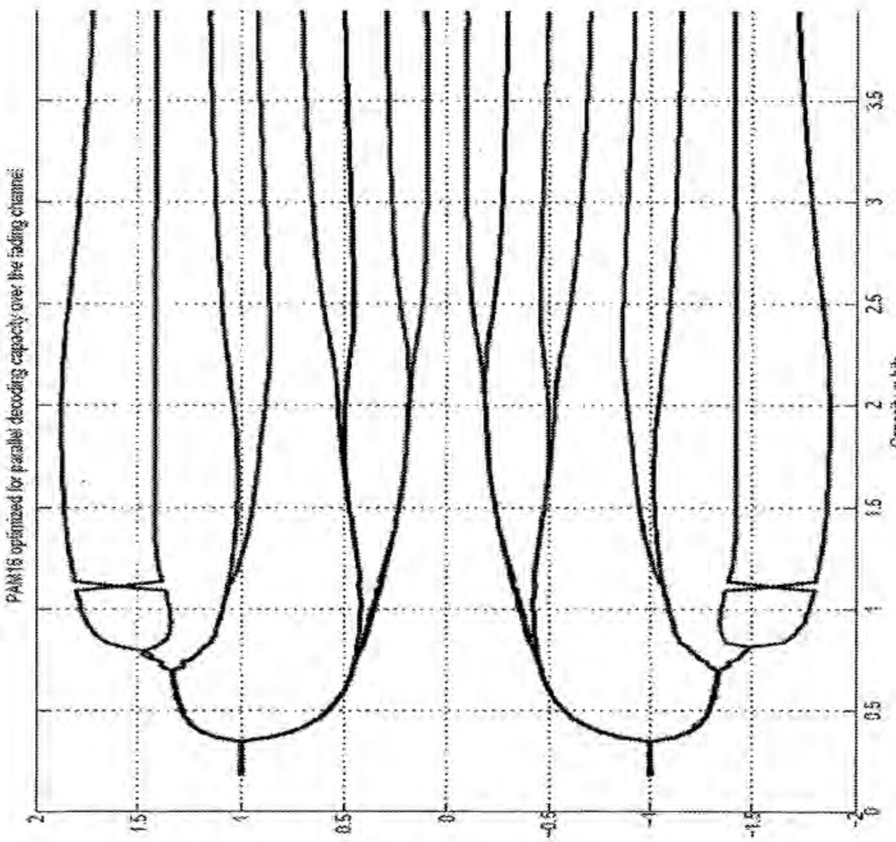

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5

FIG. 24a

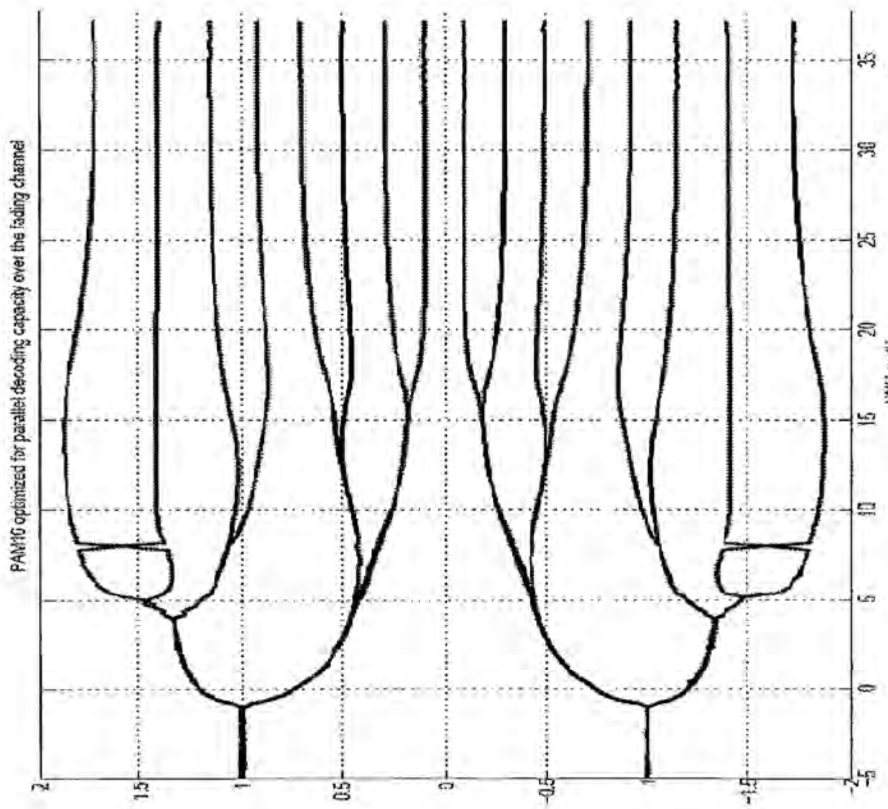

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
35
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24b

METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS

RELATED APPLICATIONS

This application is a continuation of application Ser. No. 13/118,921 filed May 31, 2011, issued on Sep. 18, 2012 as U.S. Pat. No. 8,270,511, which application is a continuation of application Ser. No. 12/156,989 filed Jun. 5, 2008, issued on Jul. 12, 2011 as U.S. Pat. No. 7,978,777, which application claimed priority to U.S. Provisional Application 60/933319 filed Jun. 5, 2007, the disclosures of which are incorporated herein by reference.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

FIG. **1** is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

FIG. **2** is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

FIG. **3** is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

FIG. **4*a*** is a conceptual illustration of the joint capacity of a channel.

FIG. **4*b*** is a conceptual illustration of the parallel decoding capacity of a channel.

FIG. **5** is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

FIG. **6*a*** is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

FIG. **6*b*** is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8,16,32.

FIG. **7** is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2, 4,8,16,32 constellations.

FIG. **8*a*** is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **8*b*** is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **9** is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

FIGS. **10*a*-10*d*** are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **11*a*** and **11*b*** are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

FIGS. **12*a*-12*d*** are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **13*a*** and **13*b*** are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

FIGS. **14*a*-14*d*** are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **15*a*** and **15*b*** are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

FIGS. **16*a*-16*d*** are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **17*a*** and **17*b*** are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

FIG. **18** is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

FIG. **19** is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

FIG. **20** is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

FIG. **21** illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

FIGS. **22*a*** and **22*b*** are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **23*a*** and **23*b*** are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **24*a*** and **24*b*** are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

Communication Systems

A communication system in accordance with an embodiment of the invention is shown in FIG. **1**. The communication system **10** includes a source **12** that provides user bits to a transmitter **14**. The transmitter transmits symbols over a channel to a receiver **16** using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

A transmitter in accordance with an embodiment of the invention is shown in FIG. **2**. The transmitter **14** includes a coder **20** that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper **22** is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator **24**, which modulates the symbols for transmission via the channel.

A receiver in accordance with an embodiment of the invention is illustrated in FIG. **3**. The receiver **16** includes a demodulator **30** that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper **32** provides the likelihoods to a decoder **34** that decodes the encoded bit stream to provide a sequence of received bits to a sink.

Geometrically Shaped Constellations

Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

Selection of a Geometrically Shaped Constellation

Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

Upgrading Existing Communication Systems

A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

Joint Capacity and Parallel Decoding Capacity

A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. **4a**. The portions of the communication system that are considered part of the channel are indicated by the ghost line **40**. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD}=\sum_{i=0}^{l-1} I(X_i;Y)$$

where X_i is the ith bit of the l-bits transmitted symbol, and Y is the received symbol, and I(A;B) denotes the mutual information between random variables A and B.

Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. **4b**. The portions of the communication system that are considered part of the channel are indicated by the ghost line **42**. The joint capacity of the channel is given by:

$$C_{JOINT}=I(X;Y)$$

Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example

AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

Selecting a Constellation Having an Optimal Capacity

Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. **5**. The process **50** commences with the selection (**52**) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (**52**) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known a priori. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (**56**) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

The SNR at which the optimized M-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (**57**). A determination (**58**) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (**55**). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

The process illustrated in FIG. **5** can maximize PD capacity or joint capacity of an M-ary constellation for a given SNR. Although the process illustrated in FIG. **5** shows selecting an M-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an M-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q), or with multi-stage decoding. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

Optimizing the Capacity of an M-Ary Constellation at a Given SNR

Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process **56** shown in FIG. **5**, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged.

The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. **5**.

Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

Gains Achieved by Optimized Geometrically Spaced Constellations

The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}\log_2(1+SNR)$$

Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. **6***a* and **6***b*. FIG. **6***a* includes a chart **60** showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **62** exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. **6***b* includes a chart **64** showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **66** exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

In order to gain a better view of the differences between the curves shown in FIGS. **6***a* and **6***b* at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. **7**. It is interesting to note from the chart **70** in FIG. **7** that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

FIGS. **8***a* and **8***b* summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. **8***a* shows the SNR gap for each constellation optimized for PD capacity. FIG. **8***b* shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

Referring to FIG. **8***a*, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of ½. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. **9**. A chart **90** includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. **8***a*.

Capacity Optimized Pam Constellations

Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. **10***a*-**17***b*. The locus plots and design tables show PAM-4,8,16,32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

In FIG. **10***a*, a locus plot **100** shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot **105** showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. **10***b*. In FIG. **10***c*. the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. **10***d* the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. **11***a* and FIG. **11***b* where vertical slices of FIGS. **10***ab* and **10***cd* are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities={0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4)=2$ bps, these design points represent systems with information code rates R={¼, ⅜, ½, ⅝, ¾} respectively.

FIGS. **12***ab* and **12***cd* present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. **13***a* and **13***b* provide slices from these plots at SNRs corresponding to achievable capacities η={0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate R=η bps/$\log_2(8)$, resulting in R={⅙, ⅓, ½, ⅔, ⅚}. As an example of the relative performance of the constellations in these tables, consider FIG. **13***b* which shows a PD capacity optimized PAM-8 constellation optimized for SNR=9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. **8***a* and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=½). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

Similar information is presented in FIGS. **14***abcd*, and **15***ab* which provide loci plots and design tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. **16***abcd*, **17***ab* provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

Capacity Optimized PSK Constellations

Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart **180** comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. **18** where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4,8,16,32 and for PD capacity optimized PSK-4,8,16,32.

The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. **19**, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. **20**. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR=8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR=14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR=2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

We note now that the locus of points for PD optimized PSK-32 in FIG. **19** in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. **18** implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8,16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. **19**. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to ⅕ to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. **18**.

Adaptive Rate Design

In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation/code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper/decoder configuration in order to match the new transmit-side configuration.

Conversely, the receiver could implement a quality of performance based optimized constellation/code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation/code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper/decoder modules.

Geometrically Shaped QAM Constellations

Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

In FIG. **21** we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. **13***b*). The label-point pairs in this PAM-8 constellation are {(000, −1.72), (001, −0.81), (010, 1.72), (011, −0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, −0.02)}. Examination of FIG. **21** shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8×8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-Dimensional Constellation Optimization

Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

Capacity Optimized Constellations for Fading Channels

Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y=a(t)\cdot X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and a(t) is the fading distribution, which is a function of time.

In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

Many fading channels follow a Rayleigh distribution. FIGS. **22***a*-**24***b* are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

What is claimed is:

1. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate using an LDPC code;

a mapper configured to map encoded bits to symbols in a QAM symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper; and

wherein the QAM symbol constellation is a geometrically spaced symbol constellation optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a QAM signal constellation that maximizes d_{min}.

2. The communication system of claim **1**, wherein the geometrically spaced symbol constellation is capacity optimized subject to additional constraints.

3. The communication system of claim **1**, wherein the channel is an AWGN channel.

4. The communication system of claim **1**, wherein the channel is a fading channel.

5. The communication system of claim **1**, wherein the QAM constellation is a QAM-64 constellation.

6. The communication system of claim **5**, wherein the LDPC code is a rate 1/2 LDPC code.

7. The communication system of claim **6**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.72, −0.81, 1.72, −0.62, 0.62, 0.02, 0.81, −0.02}.

8. The communication system of claim **5**, wherein the LDPC code is a rate 2/3 LDPC code.

9. The communication system of claim **8**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.64, −0.97, 1.64, −0.58, 0.58, 0.15, 0.97, −0.15}.

10. The communication system of claim **5**, wherein the LDPC code is a rate 5/6 LDPC code.

11. The communication system of claim **10**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.60, −1.03, −0.19, −0.58, 1.60, 1.03, 0.19, 0.58}.

12. The communication system of claim **1**, wherein the QAM constellation is a QAM-256 constellation.

13. The communication system of claim **12**, wherein the LDPC code is a rate 1/2 LDPC code.

14. The communication system of claim **13**, wherein the symbol constellation is formed using a PAM-16 constellation comprising the set of constellation points {−1.98, −1.29, 1.94, −1.17, −0.38, −0.65, −0.38, −0.68, 1.09, 0.76, 1.26, 0.76, 0.06, 0.29, 0.06, 0.29}.

15. The communication system of claim **12**, wherein the LDPC code is a rate 3/4 LDPC code.

16. The communication system of claim **15**, wherein the symbol constellation is formed using a PAM-16 constellation comprising the set of constellation points {−1.84, −1.42, 1.84, −1.11, −0.40, −0.65, −0.29, −0.83, 1.11, 0.84, 1.42, 0.65, 0.05, 0.29, −0.05, 0.40}.

17. A digital communication system, comprising:

a receiver configured to receive signals transmitted via a communication channel using a QAM symbol constellation;

wherein the receiver, comprises:

a demodulator configured to demodulate the signal received via the communication channel;

a demapper configured to estimate likelihoods of symbols in a QAM symbol constellation from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper using an LDPC code; and

wherein the QAM symbol constellation is a geometrically spaced symbol constellation optimized for capacity using parallel decode capacity that provides a given capacity at a reduced signal-to-noise ratio compared to a QAM signal constellation that maximizes d_{min}.

18. The communication system of claim **17**, wherein the geometrically spaced symbol constellation is capacity optimized subject to additional constraints.

19. The communication system of claim **17**, wherein the channel is an AWGN channel.

20. The communication system of claim **17**, wherein the channel is a fading channel.

21. The communication system of claim **17**, wherein the QAM constellation is a QAM-64 constellation.

22. The communication system of claim **21**, wherein the LDPC code is a rate 1/2 LDPC code.

23. The communication system of claim **22**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.72, −0.81, 1.72, −0.62, 0.62, 0.02, 0.81, −0.02}.

24. The communication system of claim **21**, wherein the LDPC code is a rate 2/3 LDPC code.

25. The communication system of claim **24**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.64, −0.97, 1.64, −0.58, 0.58, 0.15, 0.97, −0.15}.

26. The communication system of claim **21**, wherein the LDPC code is a rate 5/6 LDPC code.

27. The communication system of claim **26**, wherein the symbol constellation is formed using a PAM-8 constellation comprising the set of constellation points {−1.60, −1.03, −0.19, −0.58, 1.60, 1.03, 0.19, 0.58}.

28. The communication system of claim **17**, wherein the QAM constellation is a QAM-256 constellation.

29. The communication system of claim **28**, wherein the LDPC code is a rate 1/2 LDPC code.

30. The communication system of claim **29**, wherein the symbol constellation is formed using a PAM-16 constellation comprising the set of constellation points {−1.98, −1.29, 1.94, −1.17, −0.38, −0.65, −0.38, −0.68, 1.09, 0.76, 1.26, 0.76, 0.06, 0.29, 0.06, 0.29}.

31. The communication system of claim **28**, wherein the LDPC code is a rate 3/4 LDPC code.

32. The communication system of claim **31**, wherein the symbol constellation is formed using a PAM-16 constellation comprising the set of constellation points {−1.84, −1.42, 1.84, −1.11, −0.40, −0.65, −0.29, −0.83, 1.11, 0.84, 1.42, 0.65, 0.05, 0.29, −0.05, 0.40}.

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U.S. PATENT: *10,693,700*
ISSUE DATE: *June 23, 2020*

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(54) **RECEIVERS INCORPORATING NON-UNIFORM MULTIDIMENSIONAL CONSTELLATIONS AND CODE RATE PAIRS**

(71) Applicant: **Constellation Designs, LLC**, Anaheim, CA (US)

(72) Inventors: **Maged F. Barsoum**, San Jose, CA (US); **Christopher R. Jones**, Pacific Palisades, CA (US)

(73) Assignee: **Constellation Designs, LLC**, Anaheim, CA (US)

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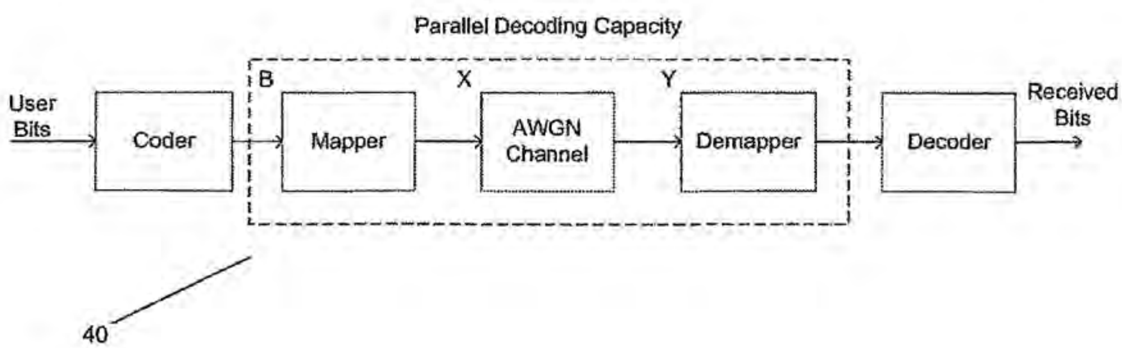
Primary Examiner — Tesfaldet Bocure
(74) *Attorney, Agent, or Firm* — KPPB LLP

(57) **ABSTRACT**

Communication systems are described that use unequally spaced constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. One embodiment is a digital communications system including a transmitter transmitting signals via a communication channel, the transmitter including a coder capable of receiving user bits and outputting encoded bits at a rate, a mapper capable of mapping encoded bits to symbols in a constellation, and a modulator capable of generating a modulated signal for transmission via the communication channel using symbols generated by the mapper, wherein the constellation is unequally spaced and characterizable by assignment of locations and labels of constellation points to maximize parallel decode capacity of the constellation at a given signal-to-noise ratio so that the constellation provides a given capacity at a reduced signal-to-noise ratio compared to a uniform constellation that maximizes the minimum distance between constellation points of the uniform constellation.

30 Claims, 43 Drawing Sheets



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continuation of application No. 15/682,475, filed on Aug. 21, 2017, now Pat. No. 10,149,179, which is a continuation of application No. 15/200,800, filed on Jul. 1, 2016, now Pat. No. 9,743,292, which is a continuation of application No. 14/491,731, filed on Sep. 19, 2014, now Pat. No. 9,385,832, which is a continuation of application No. 13/618,630, filed on Sep. 14, 2012, now Pat. No. 8,842,761, which is a continuation of application No. 13/118,921, filed on May 31, 2011, now Pat. No. 8,270,511, which is a continuation of application No. 12/156,989, filed on Jun. 5, 2008, now Pat. No. 7,978,777.

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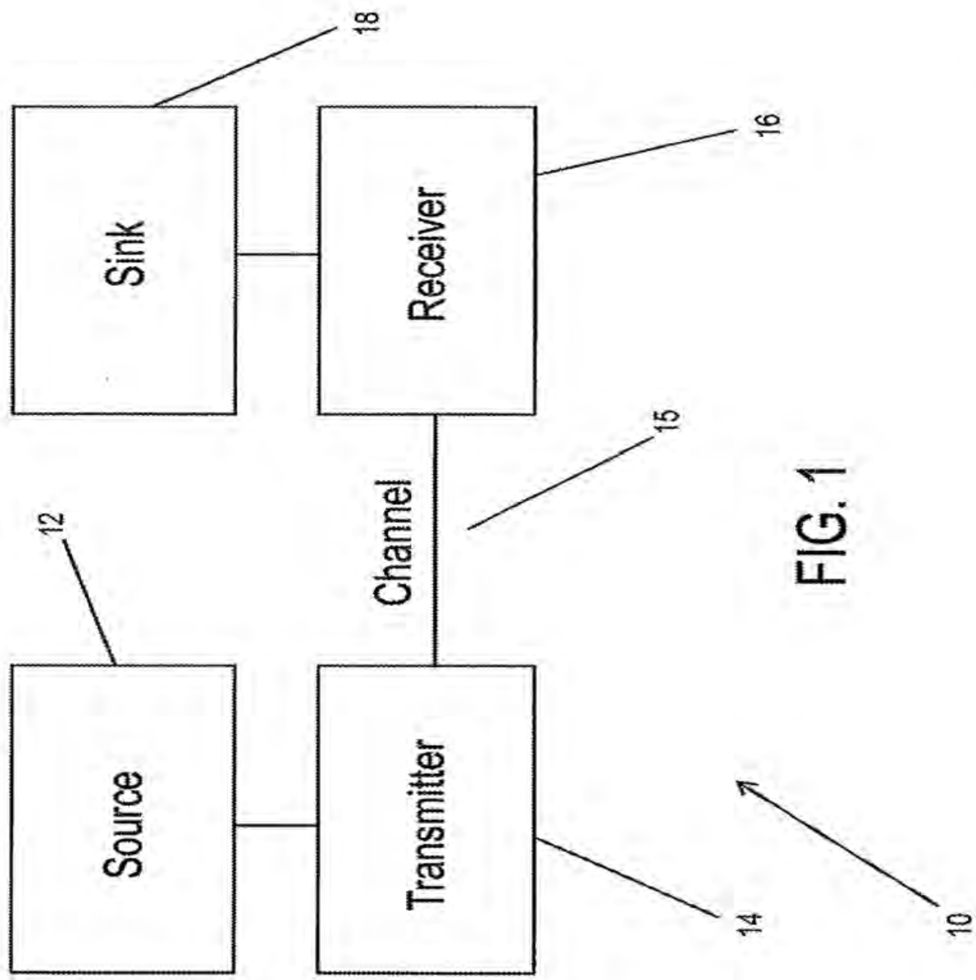


FIG. 1

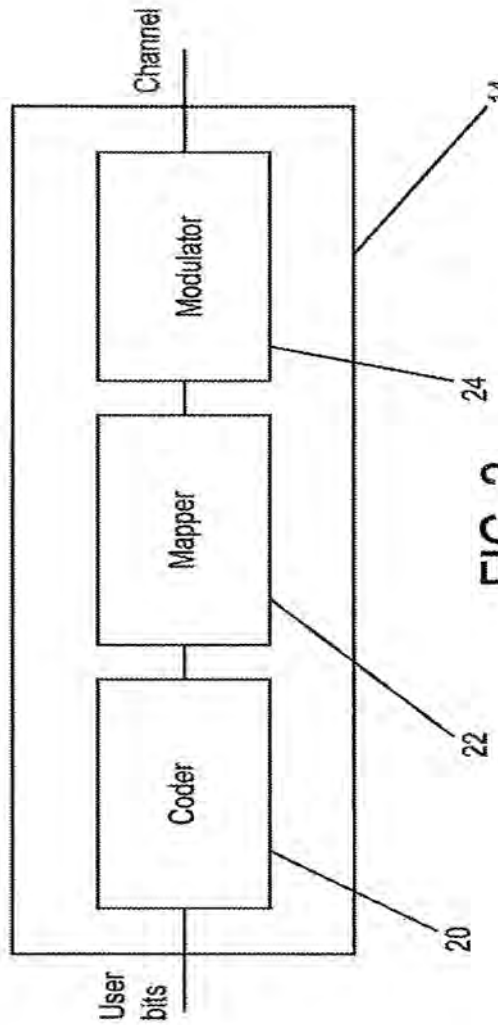

Channel
14
Modulator
24
Mapper
22
Coder
20
User bits

FIG. 2

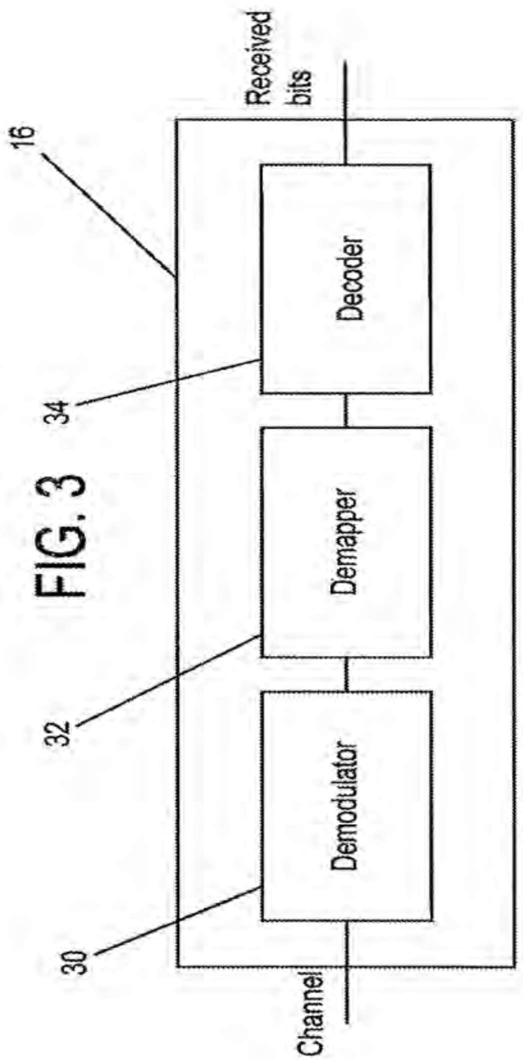

Received bits
16
Decoder
34
Demapper
32
Demodulator
30
Channel

FIG. 3

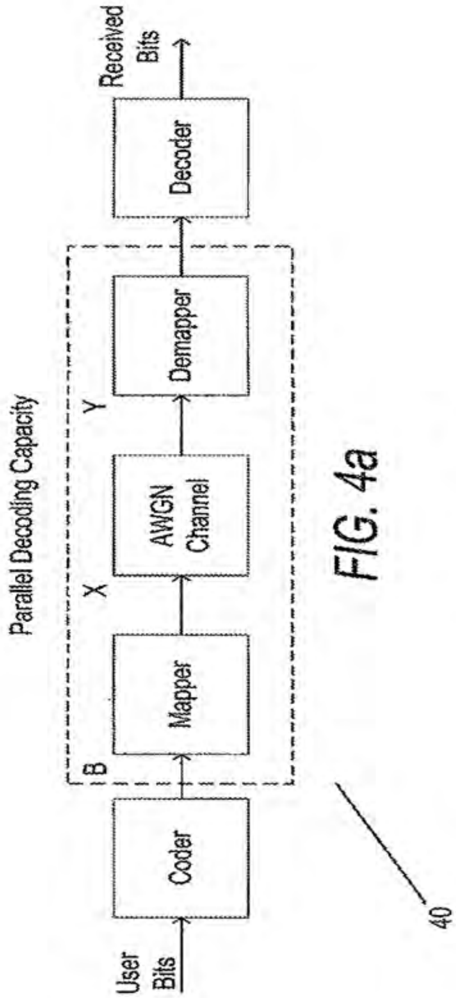

Parallel Decoding Capacity
User Bits
Coder
B
Mapper
X
AWGN Channel
Y
Demapper
Decoder
Received Bits
40

FIG. 4a

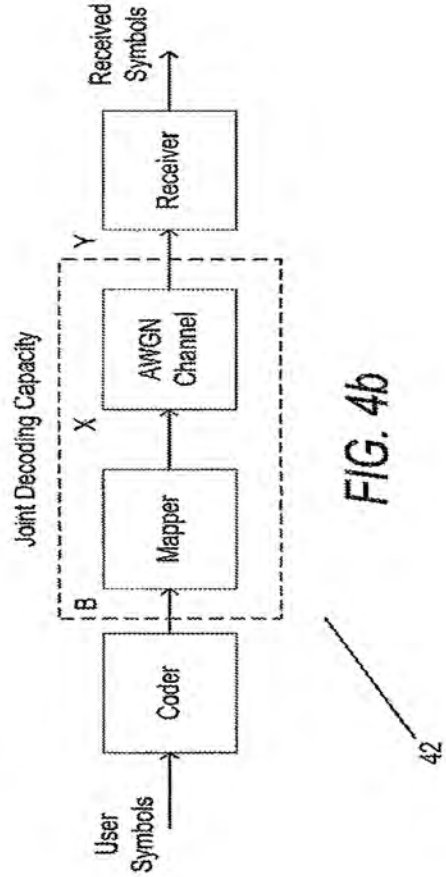

Joint Decoding Capacity
User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
42

FIG. 4b

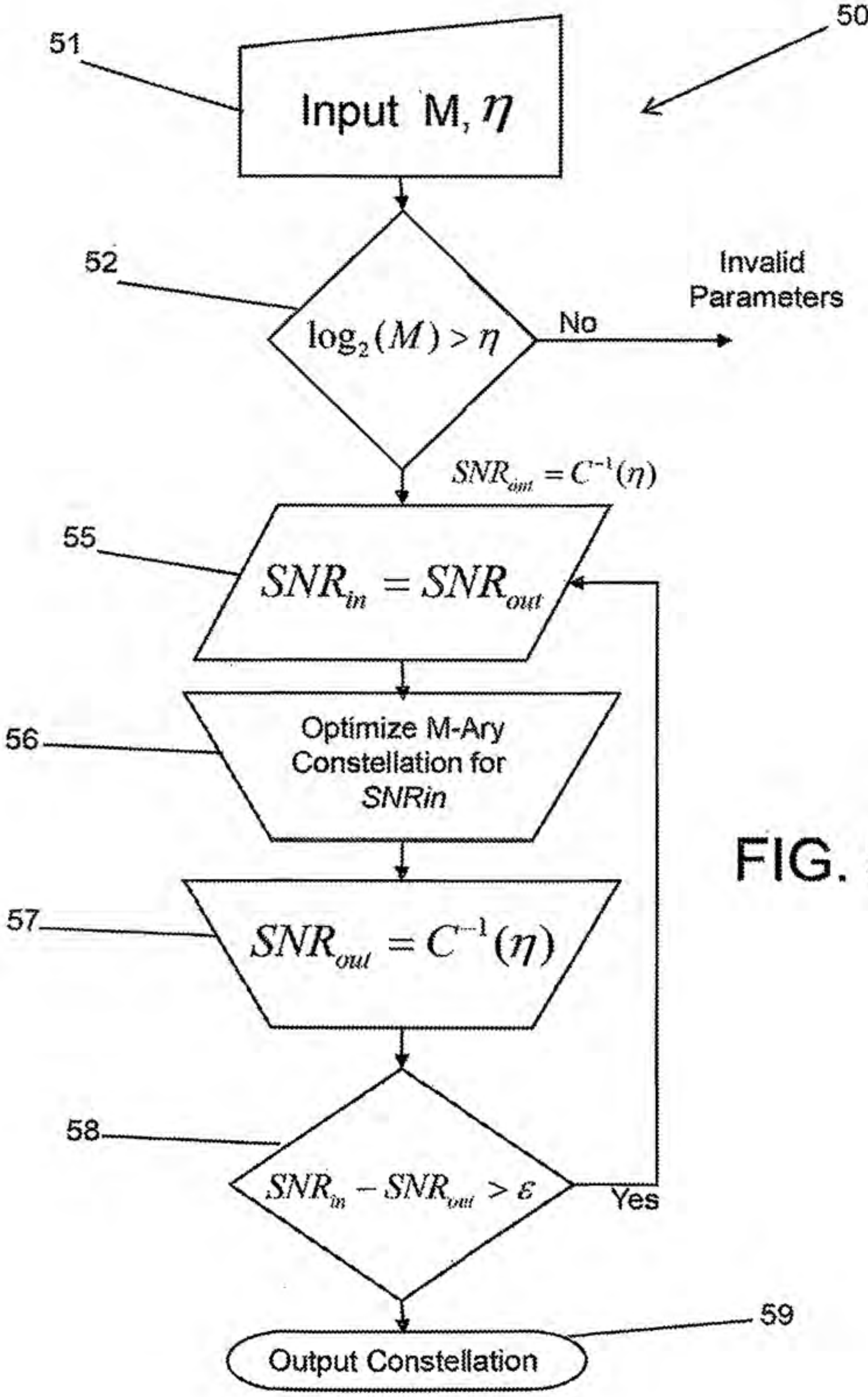

50
51
Input M, η
52
$\log_2(M) > \eta$
No
Invalid Parameters
$SNR_{out} = C^{-1}(\eta)$
55
$SNR_{in} = SNR_{out}$
56
Optimize M-Ary Constellation for SNRin
57
$SNR_{out} = C^{-1}(\eta)$
58
$SNR_{in} - SNR_{out} > \varepsilon$
Yes
59
Output Constellation

FIG. 5

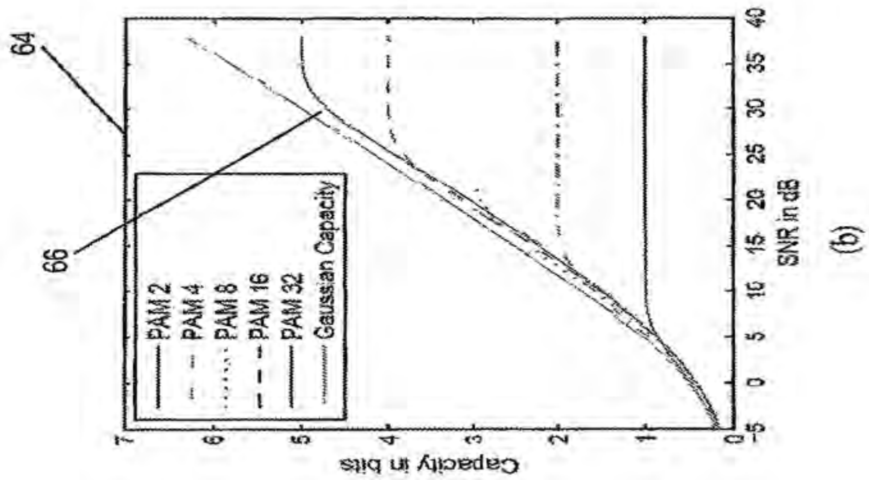

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

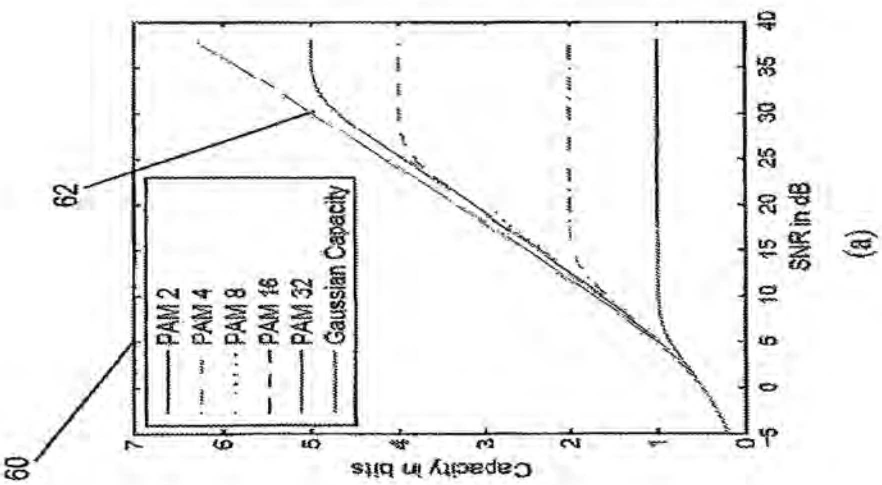
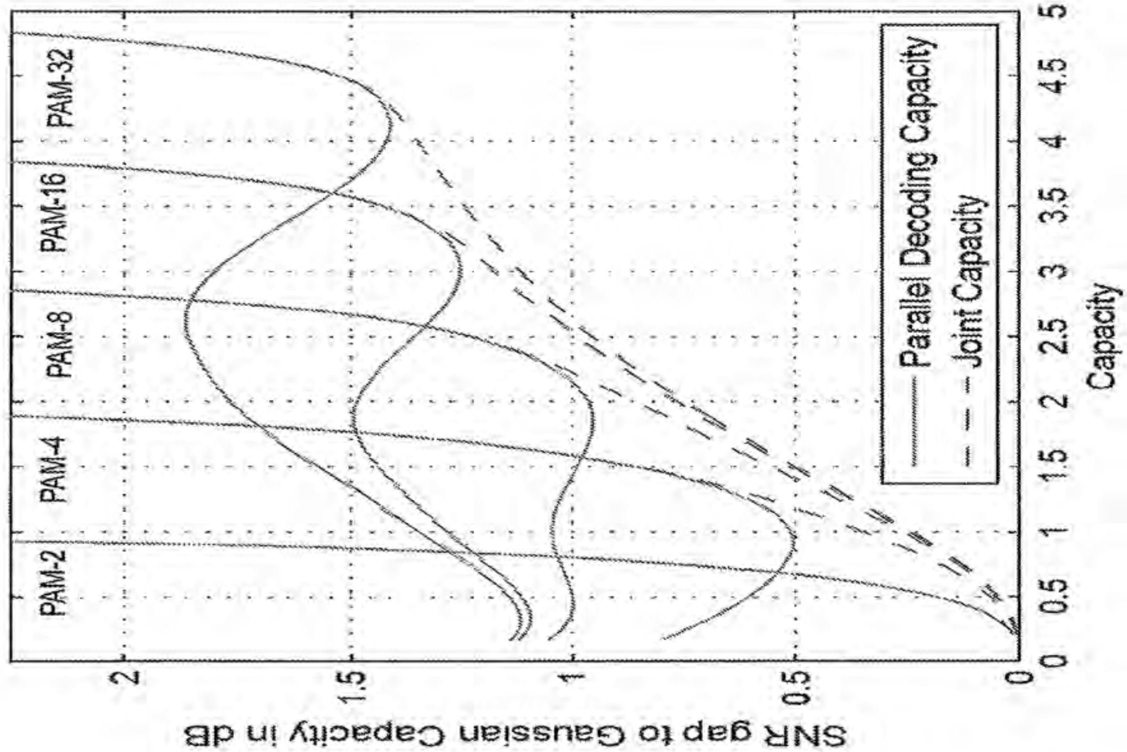

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

FIG. 7


PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
Parallel Decoding Capacity
Joint Capacity
Capacity
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

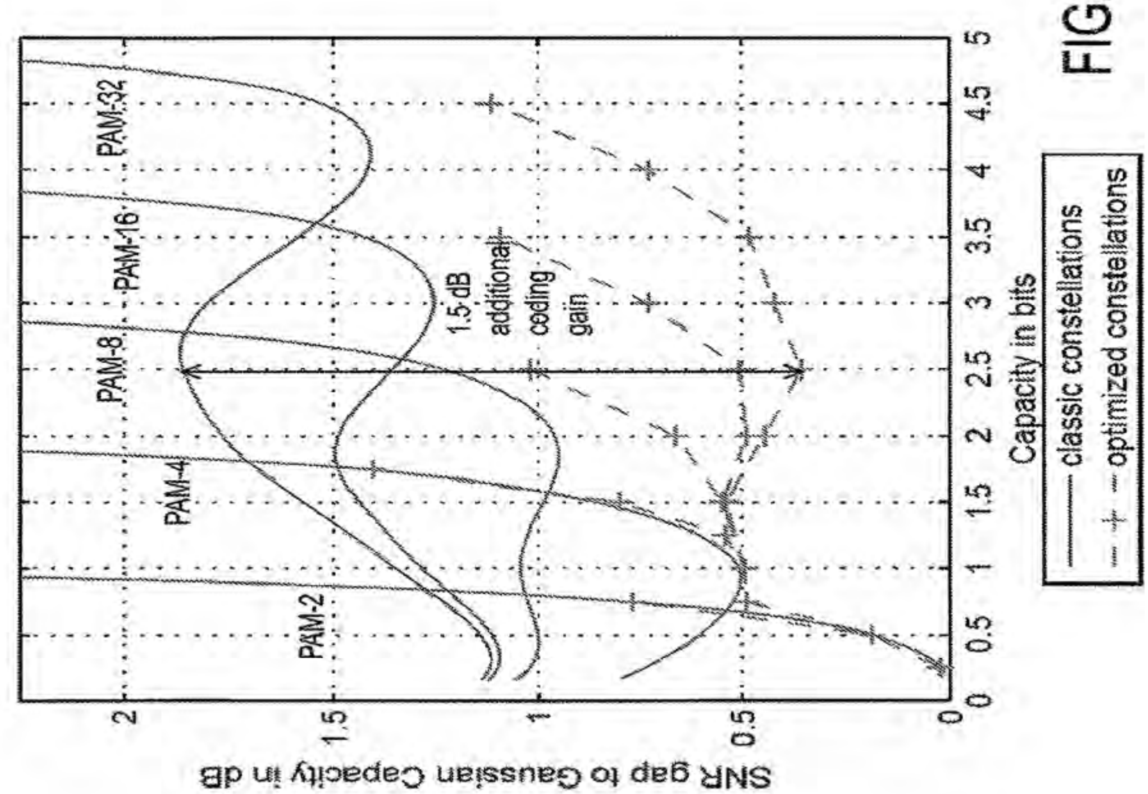

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5 dB
additional
coding
gain
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
classic constellations
optimized constellations

FIG. 8a

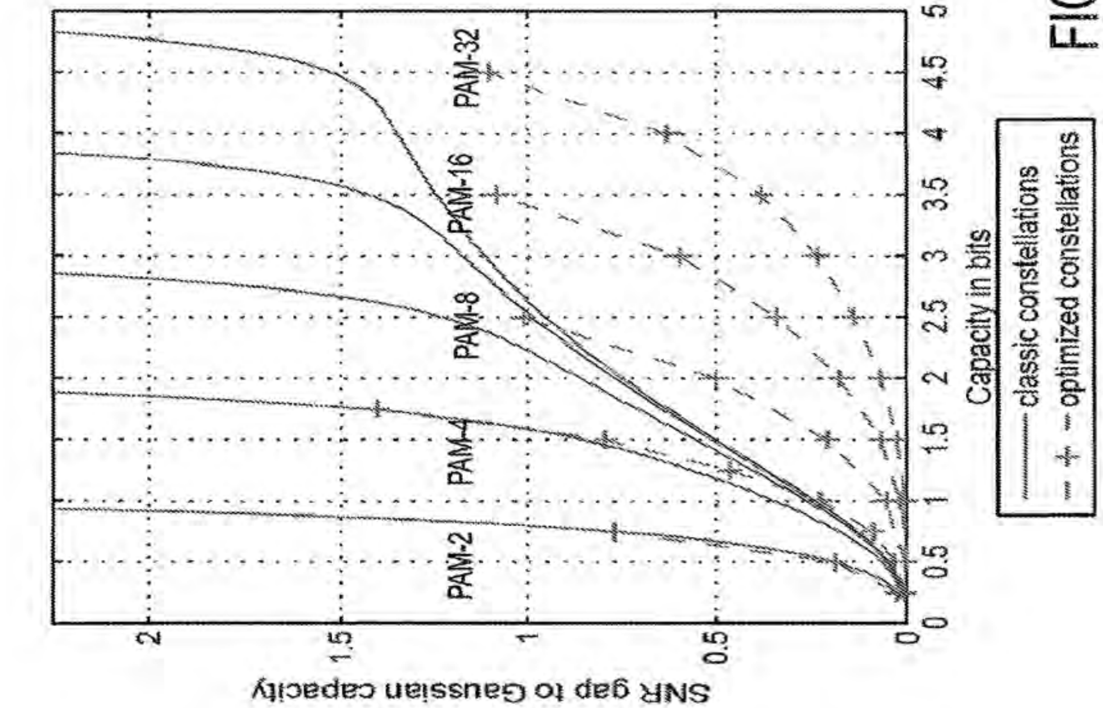

SNR gap to Gaussian capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
classic constellations
optimized constellations

FIG. 8b

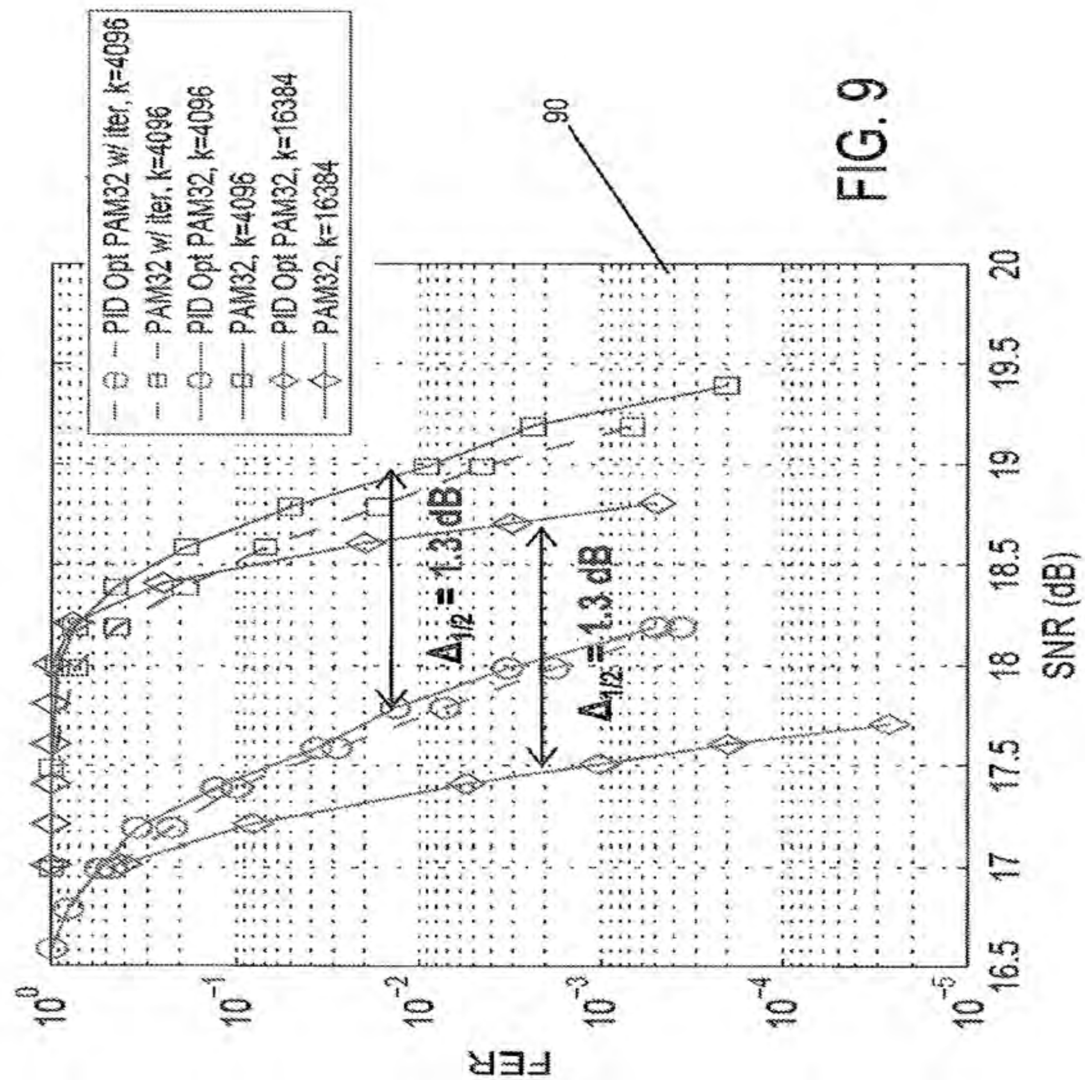

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
Δ1/2 = 1.3 dB
Δ1/2 = 1.3 dB
FER
SNR (dB)
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

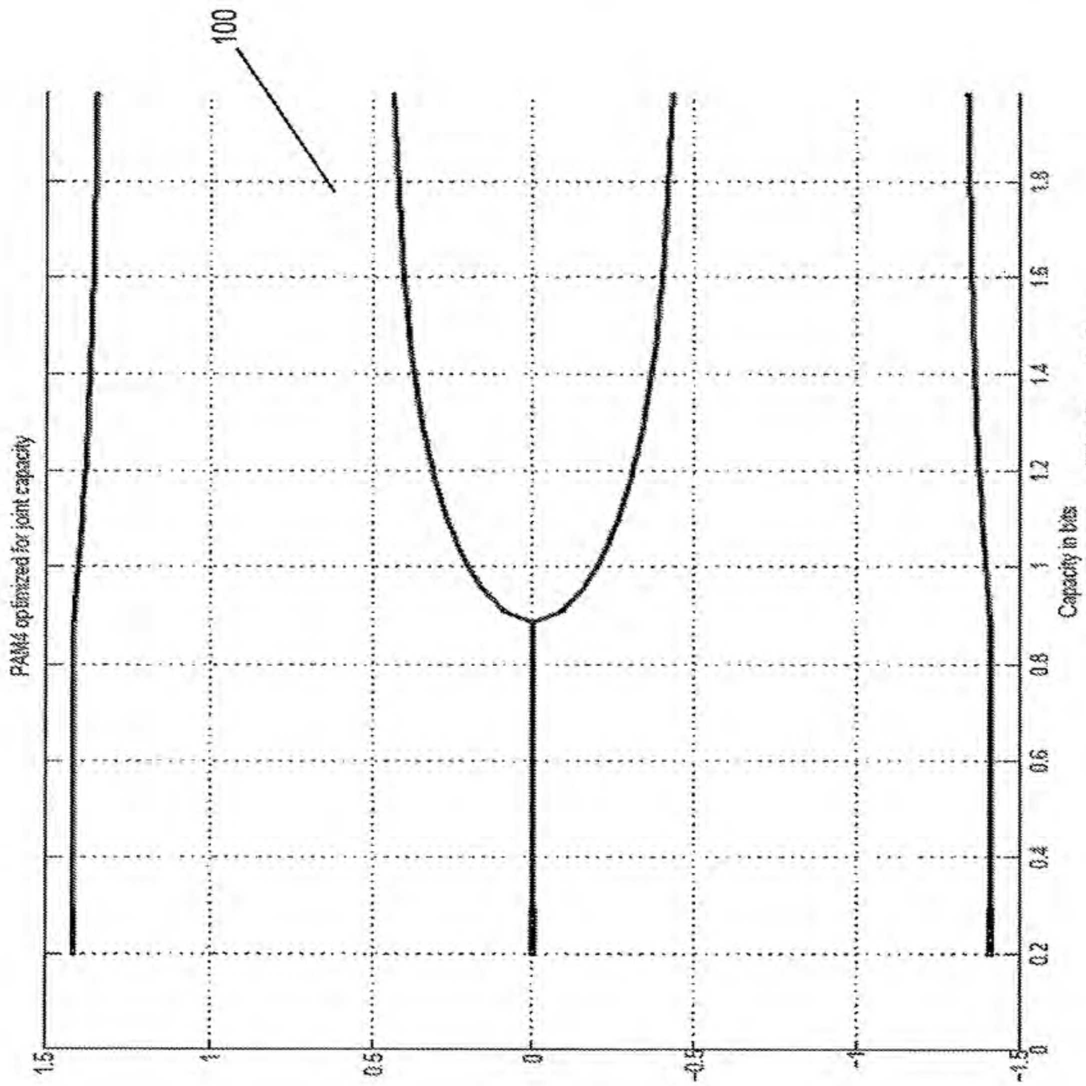

100
PAM4 optimized for joint capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10a

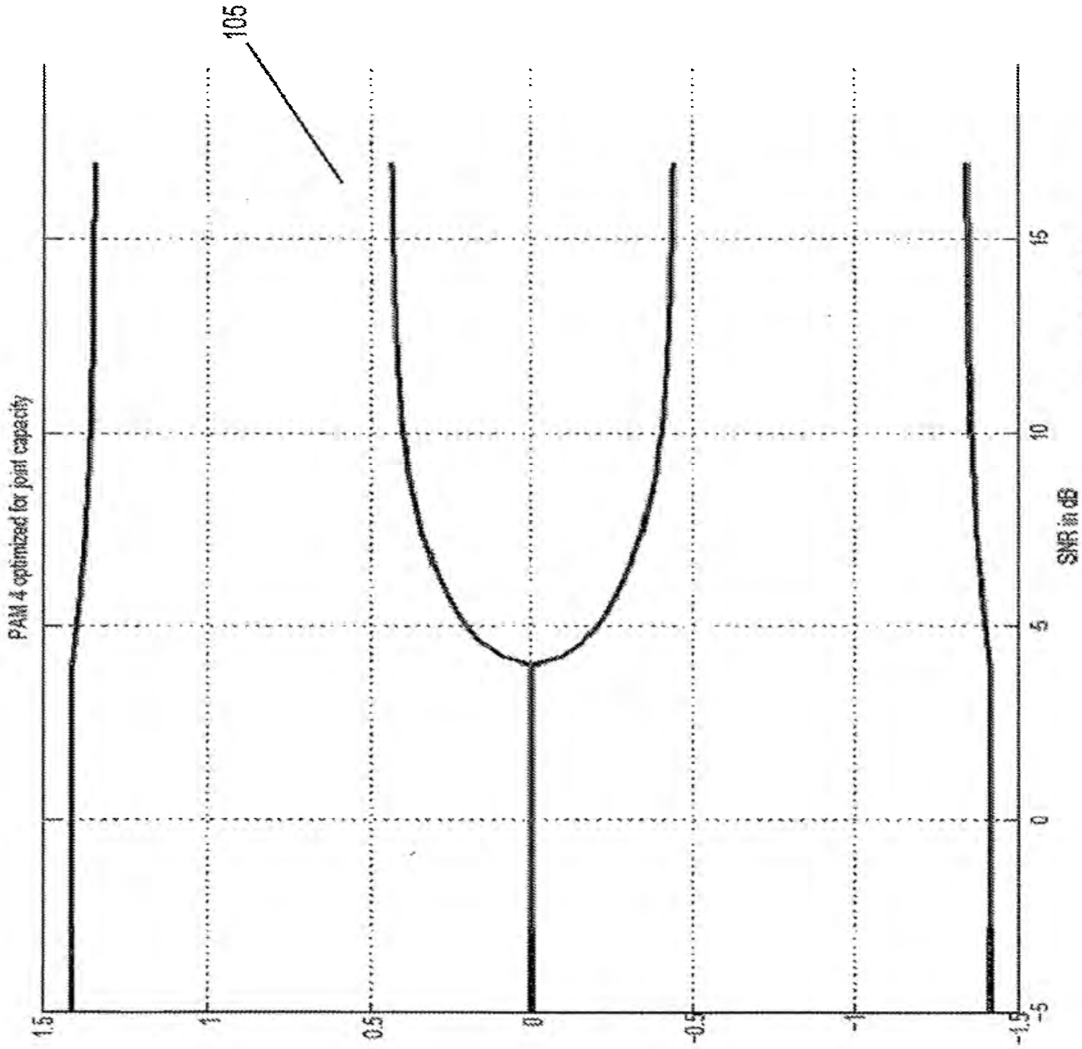

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10b

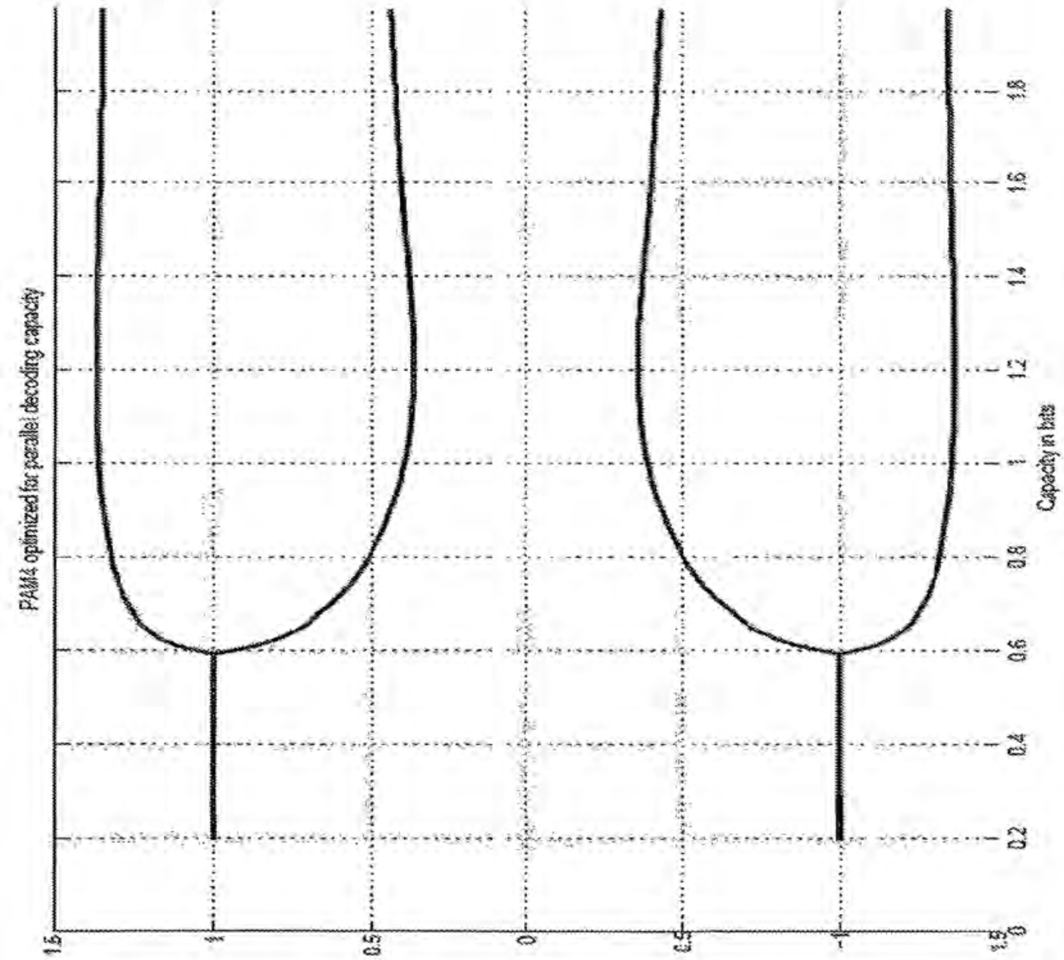

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

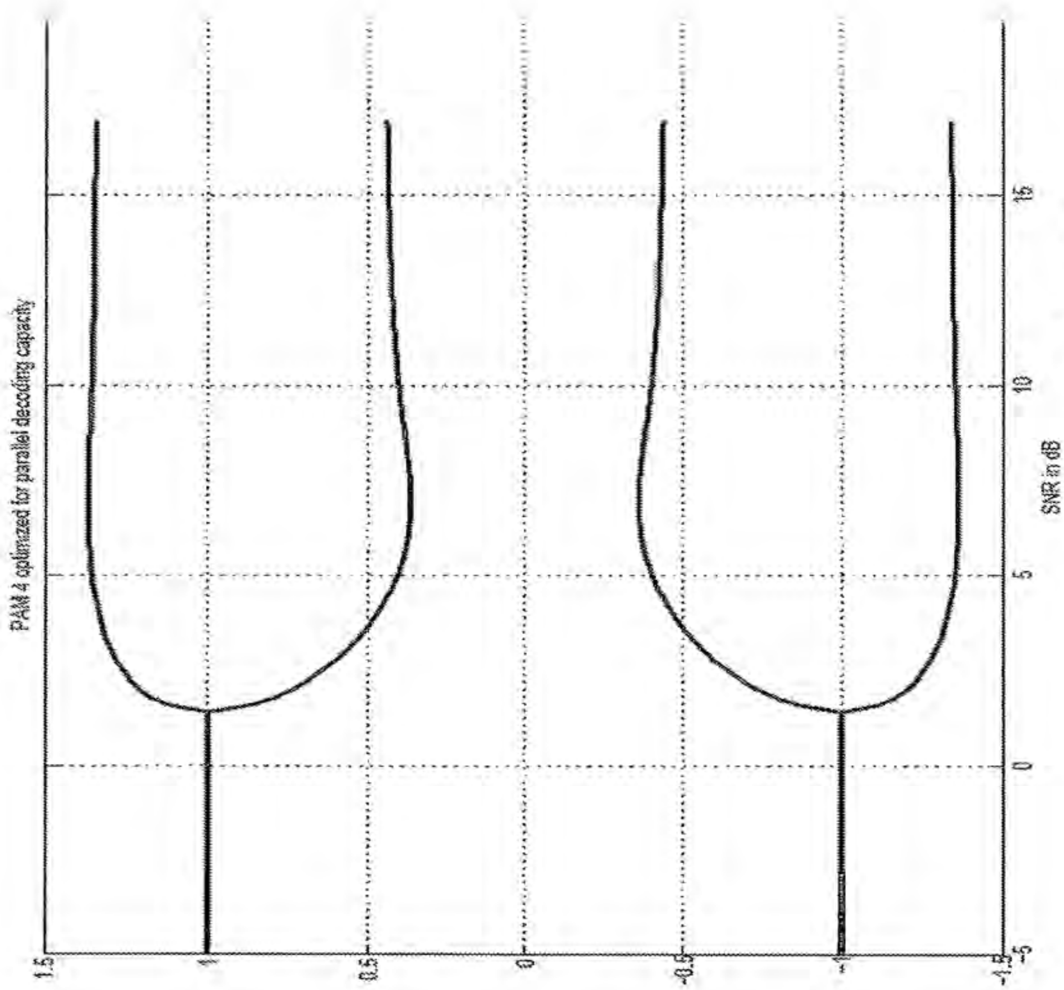

PAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

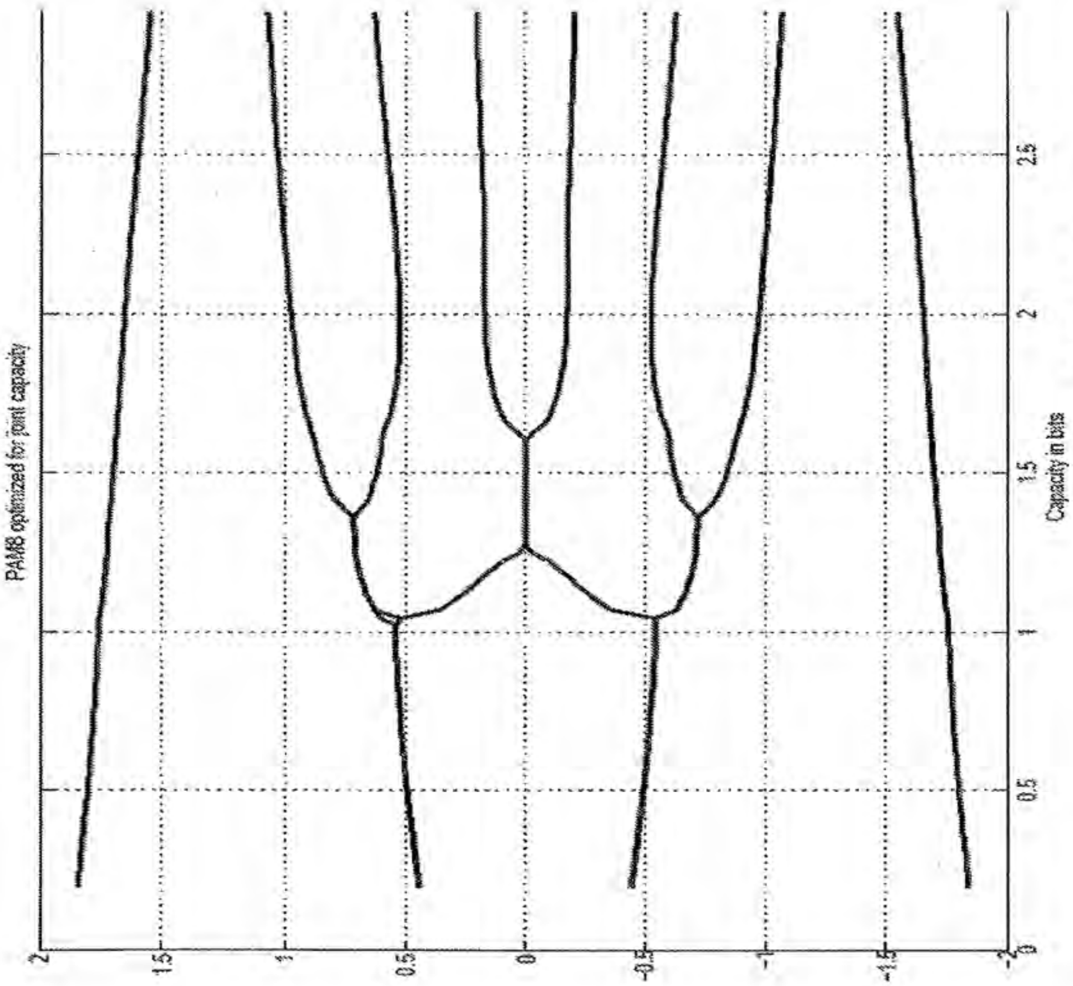

PAM8 optimized for joint capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12a

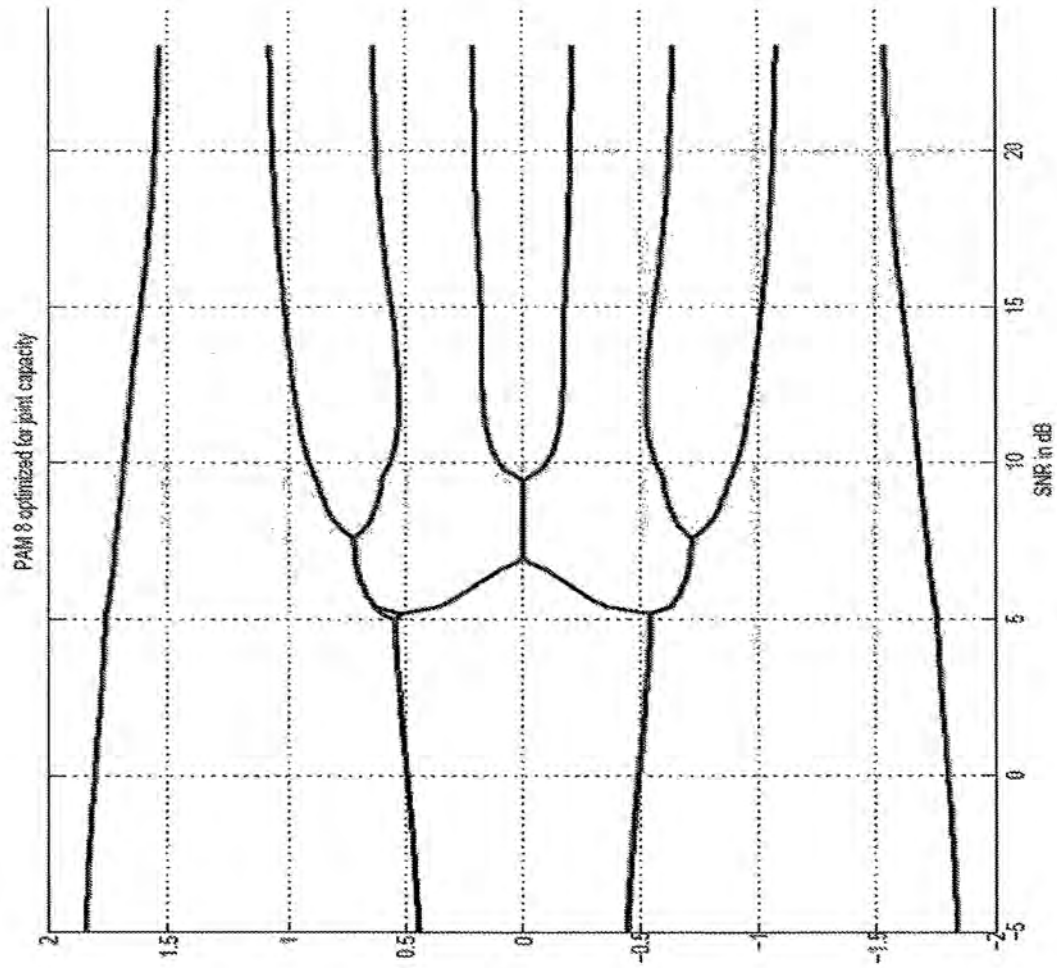
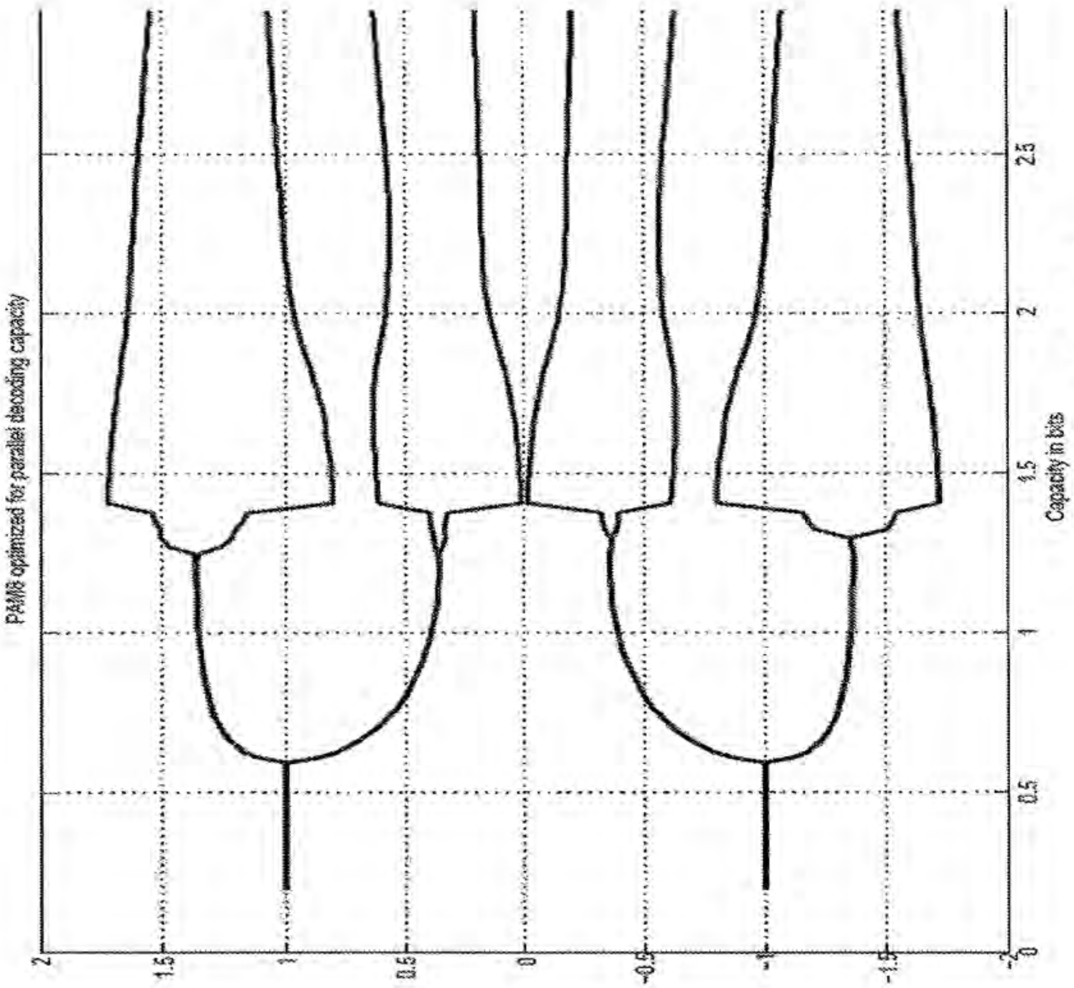

PAM 8 optimized for joint capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12b



PAM8 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12c

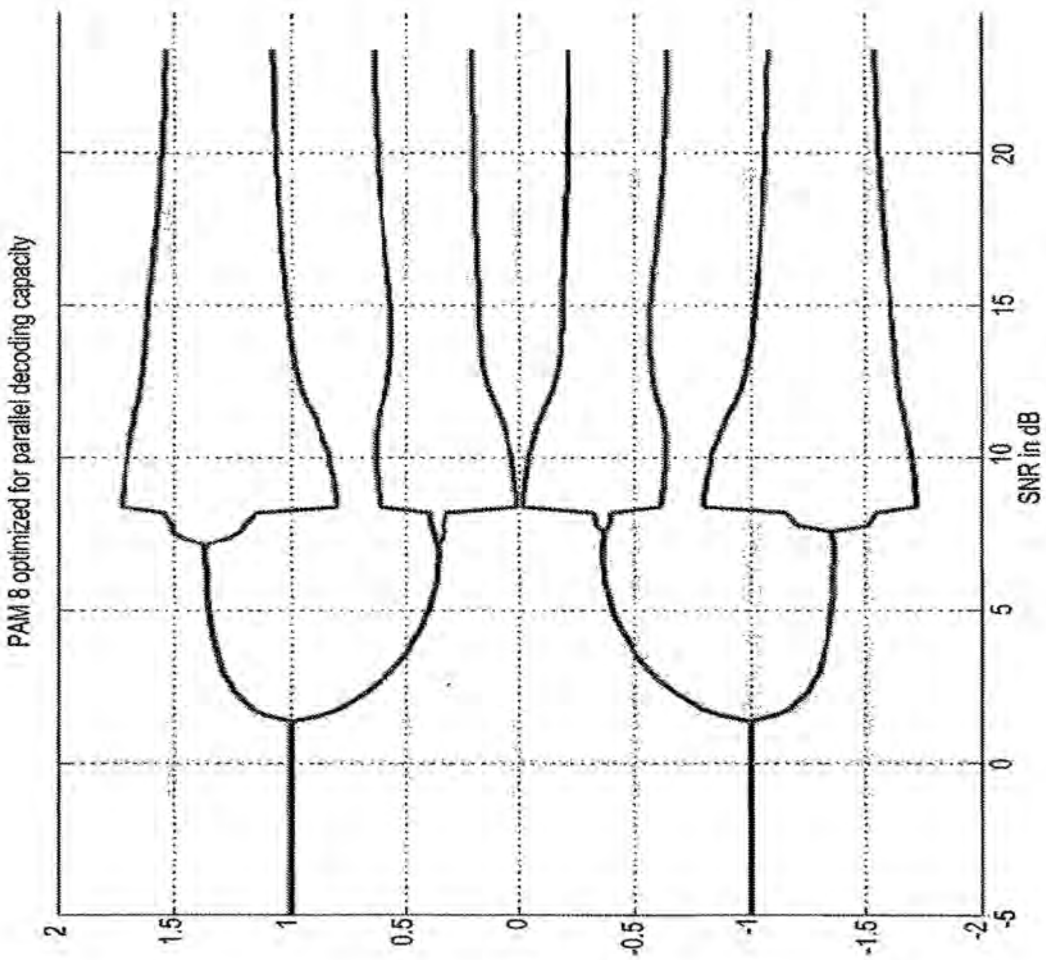

PAM 8 optimized for parallel decoding capacity
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20
SNR in dB

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5,27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

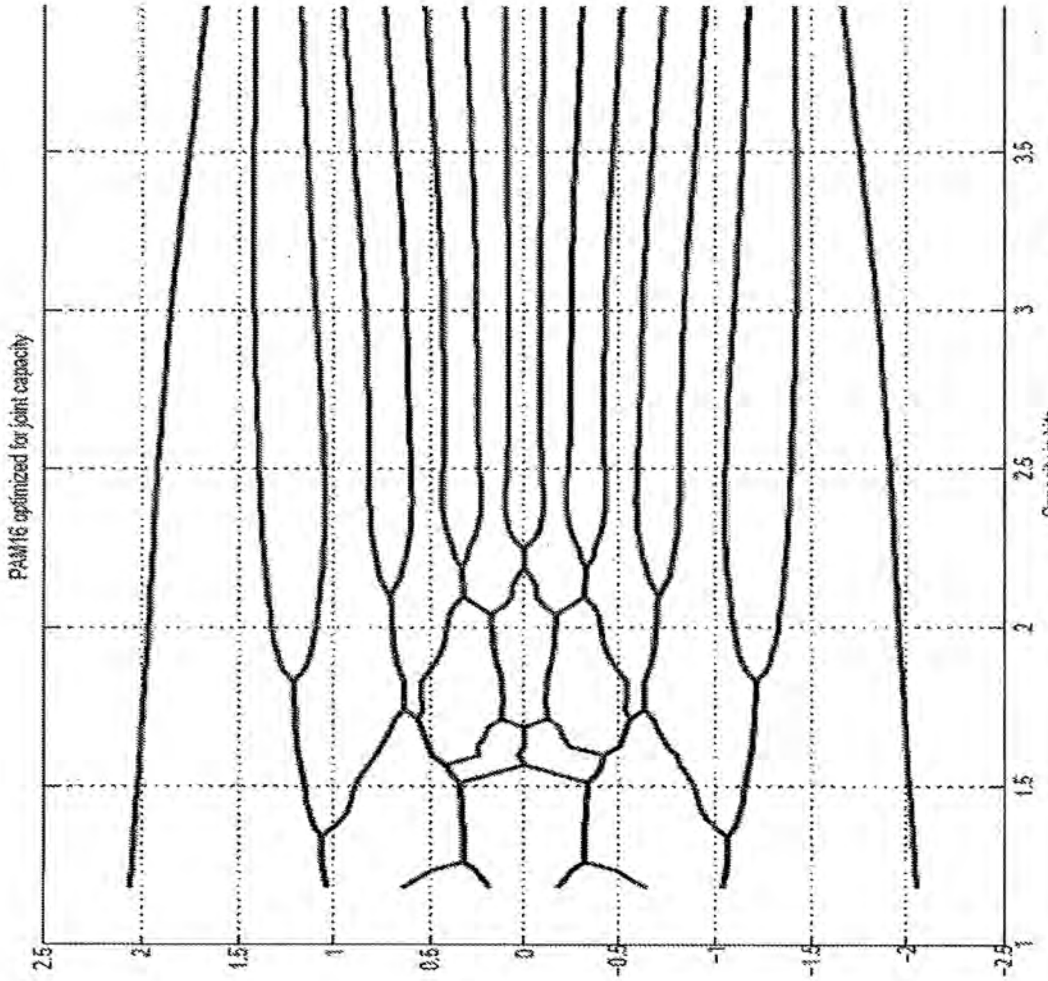

PAM16 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.5
2
2.5
3
3.5

FIG. 14a

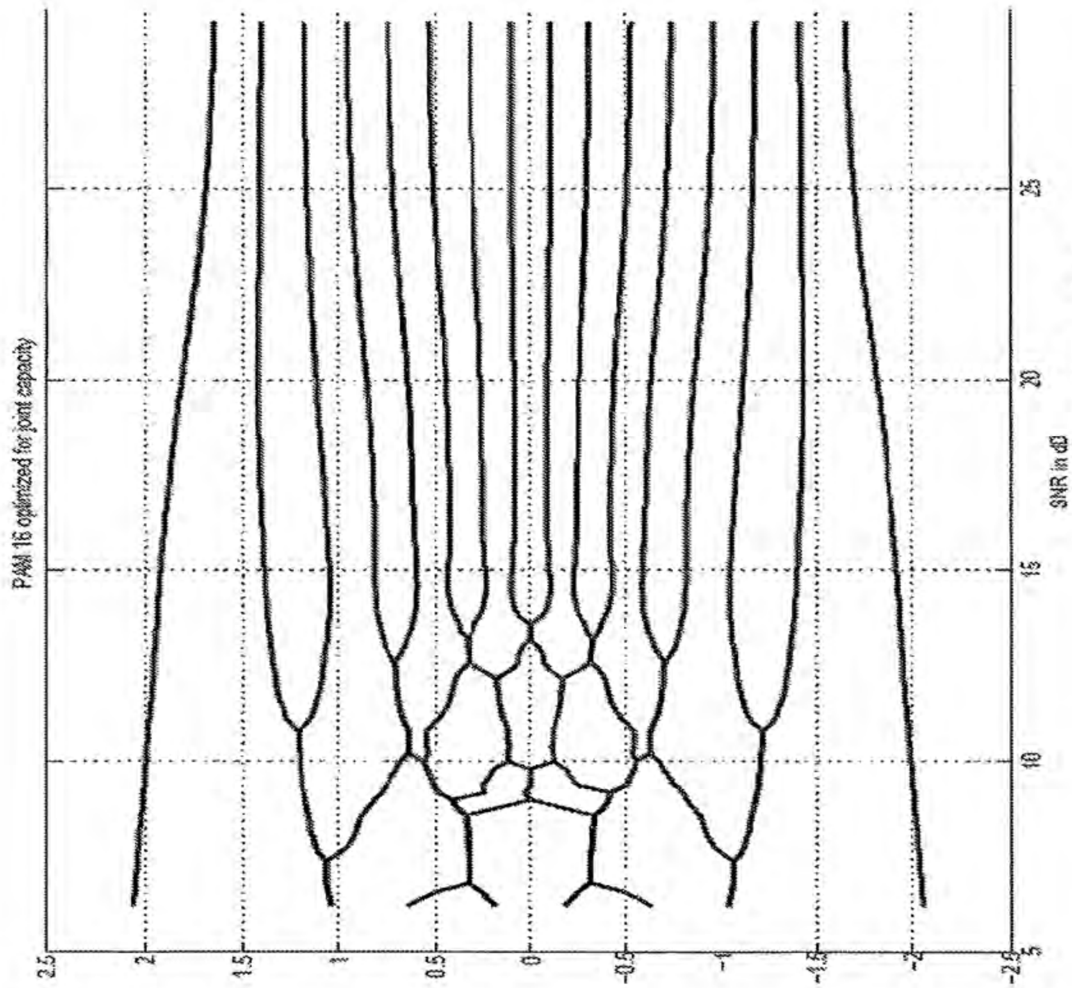

PAM 16 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14b

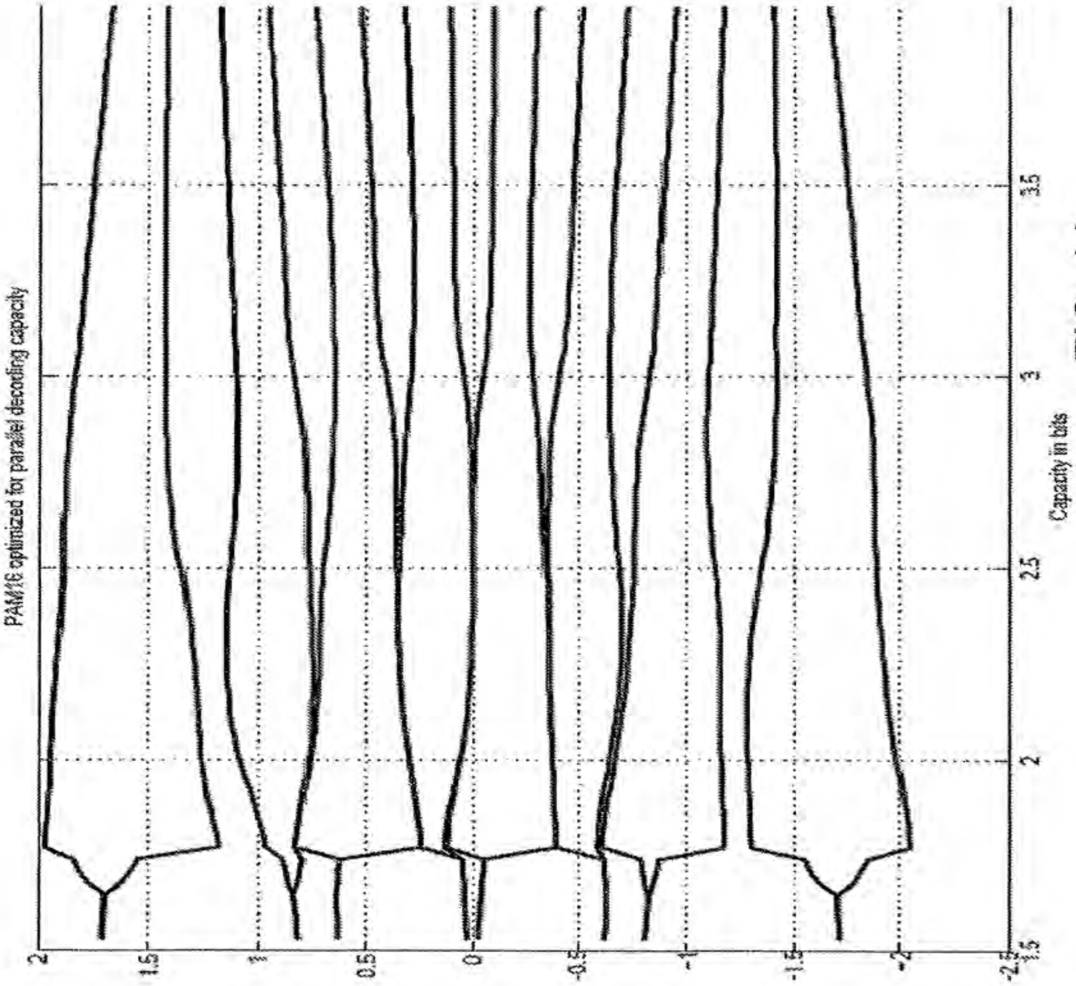
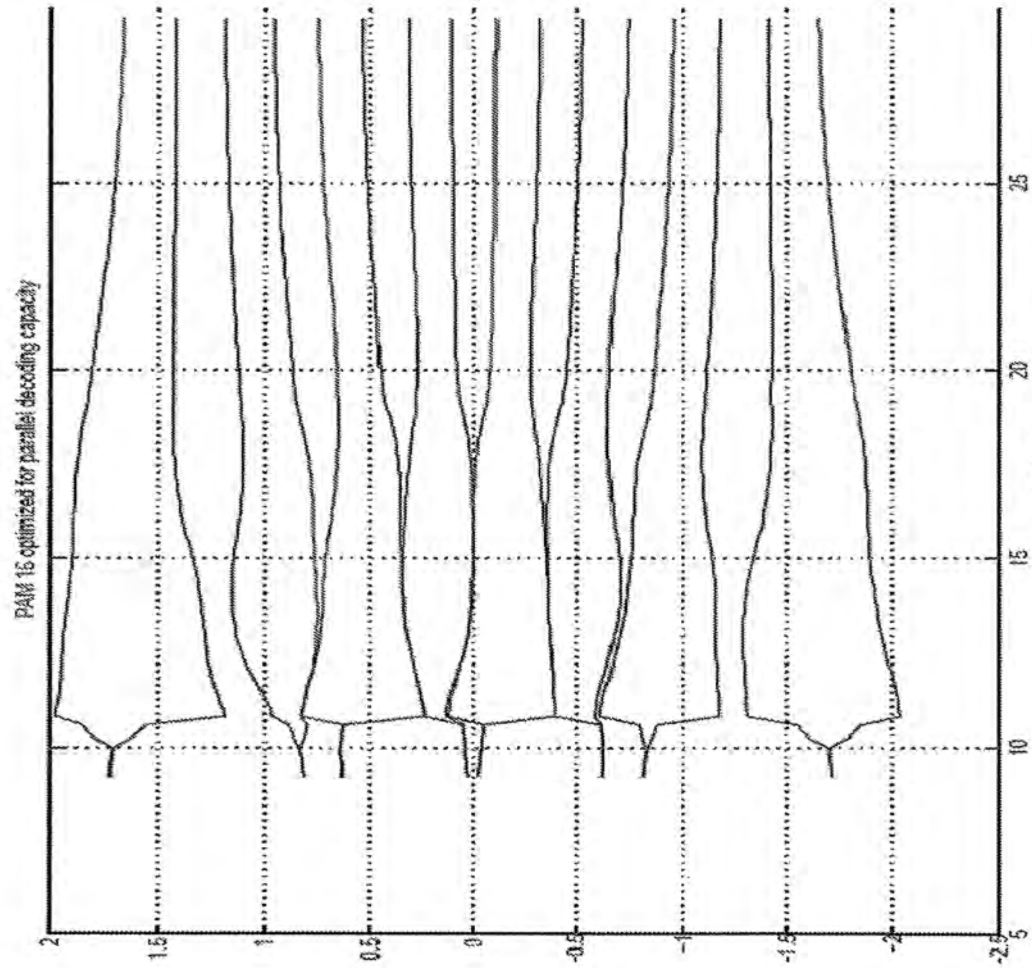

PAM16 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5

FIG. 14c



PAM 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different.

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

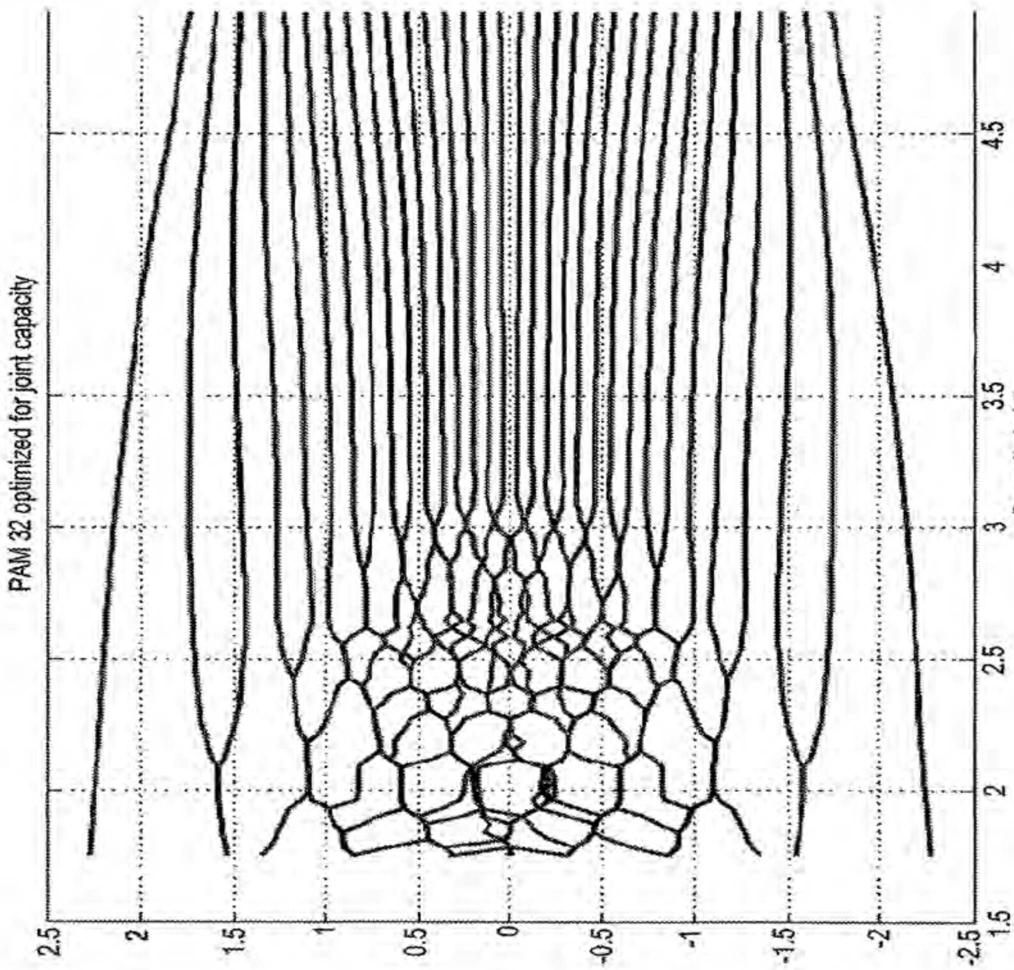

PAM 32 optimized for joint capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16a

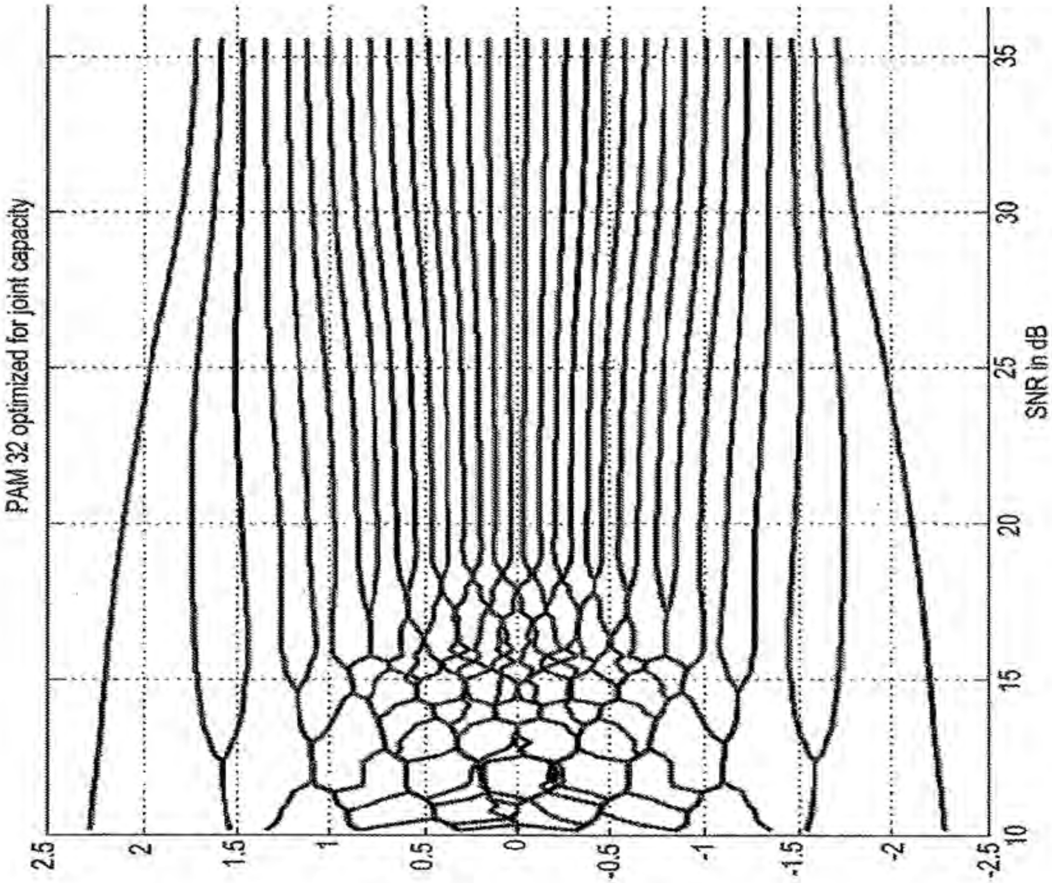

PAM 32 optimized for joint capacity
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35
SNR in dB

FIG. 16b

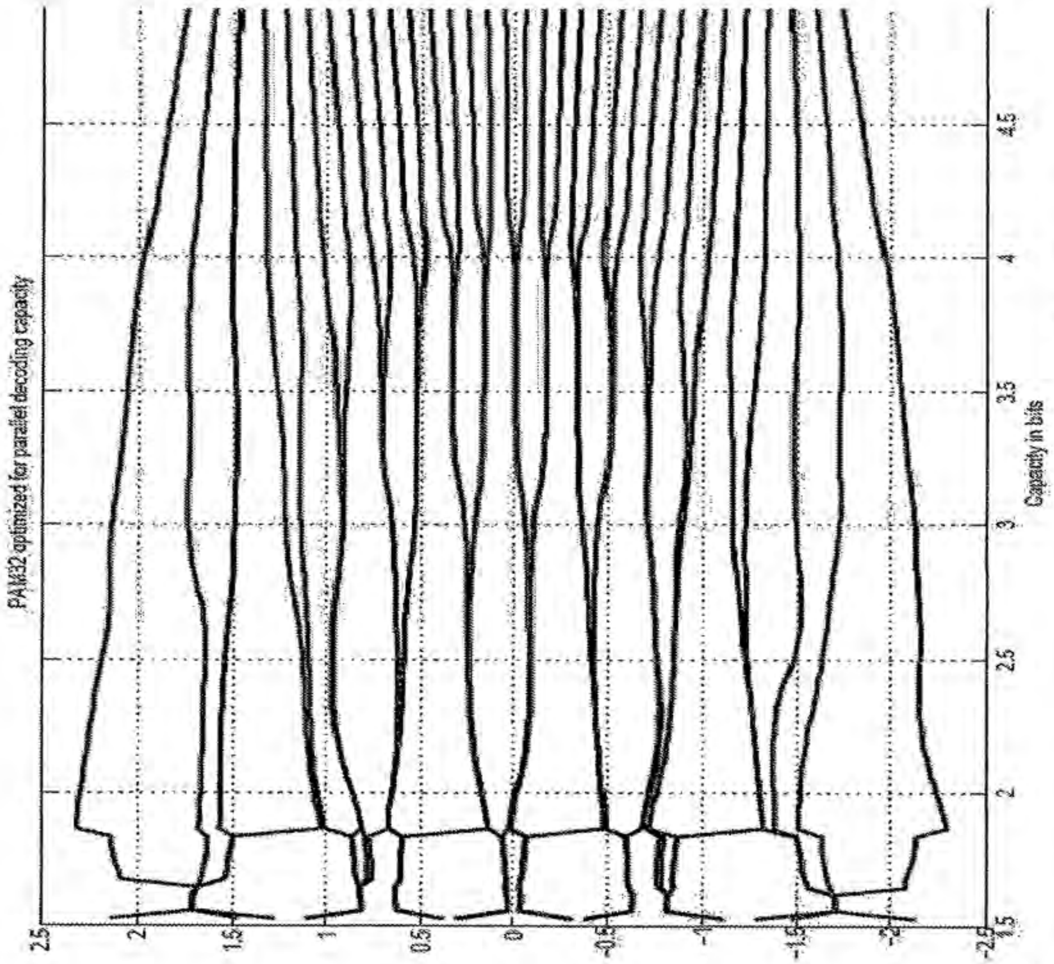

PAM32 optimized for parallel decoding capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16c

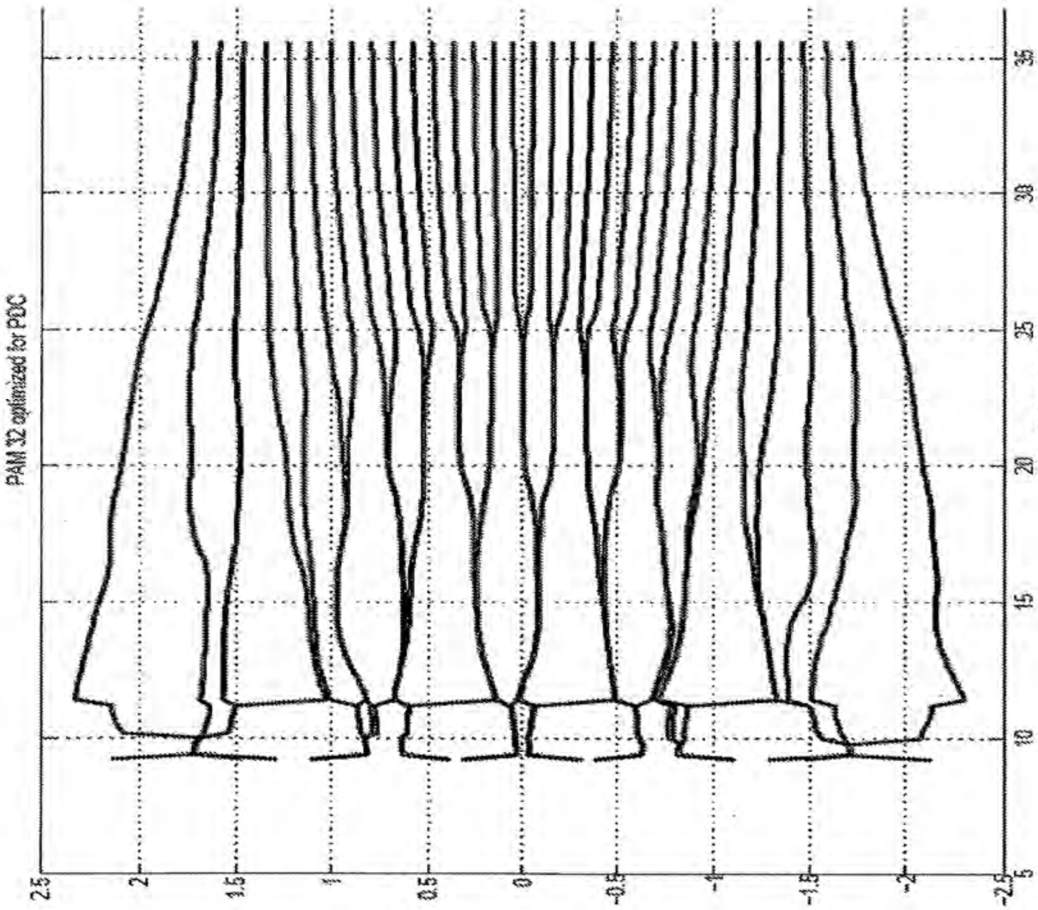


FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

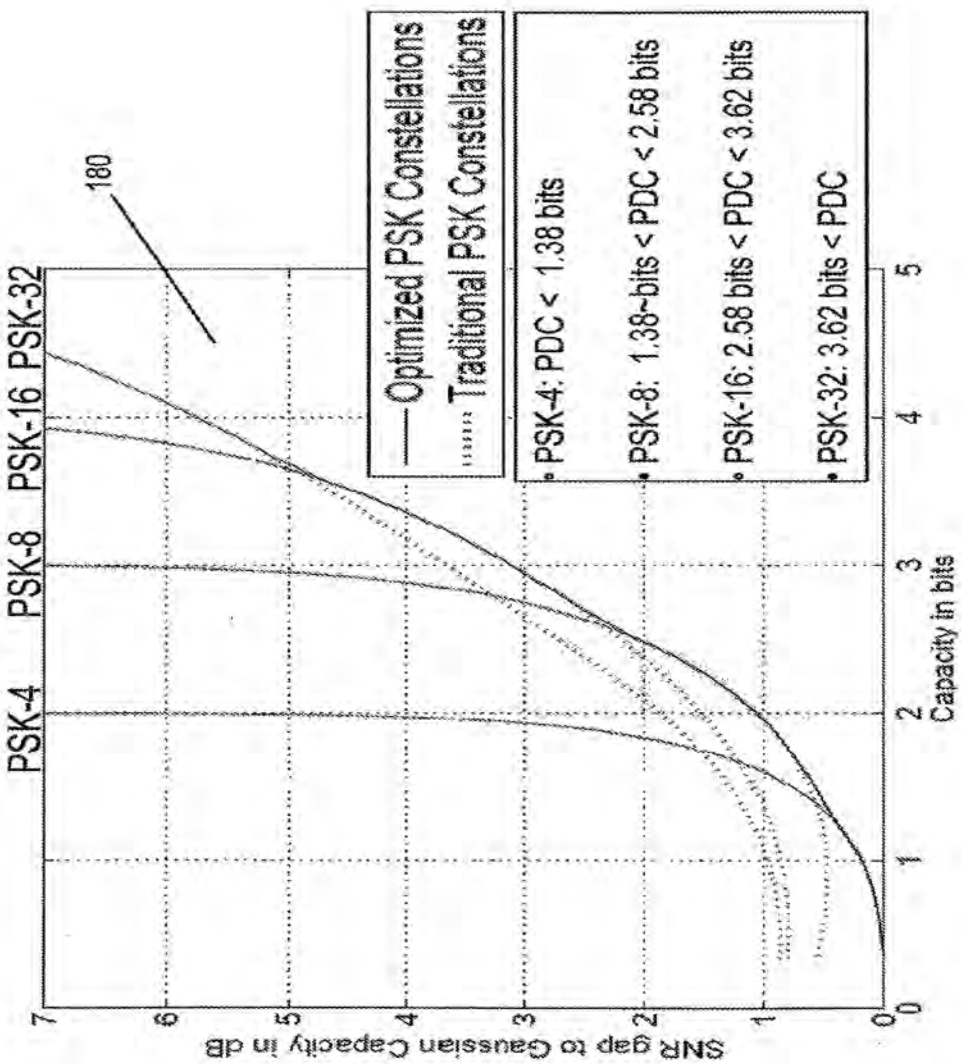

PSK-4
PSK-8
PSK-16
PSK-32
180
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38~bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

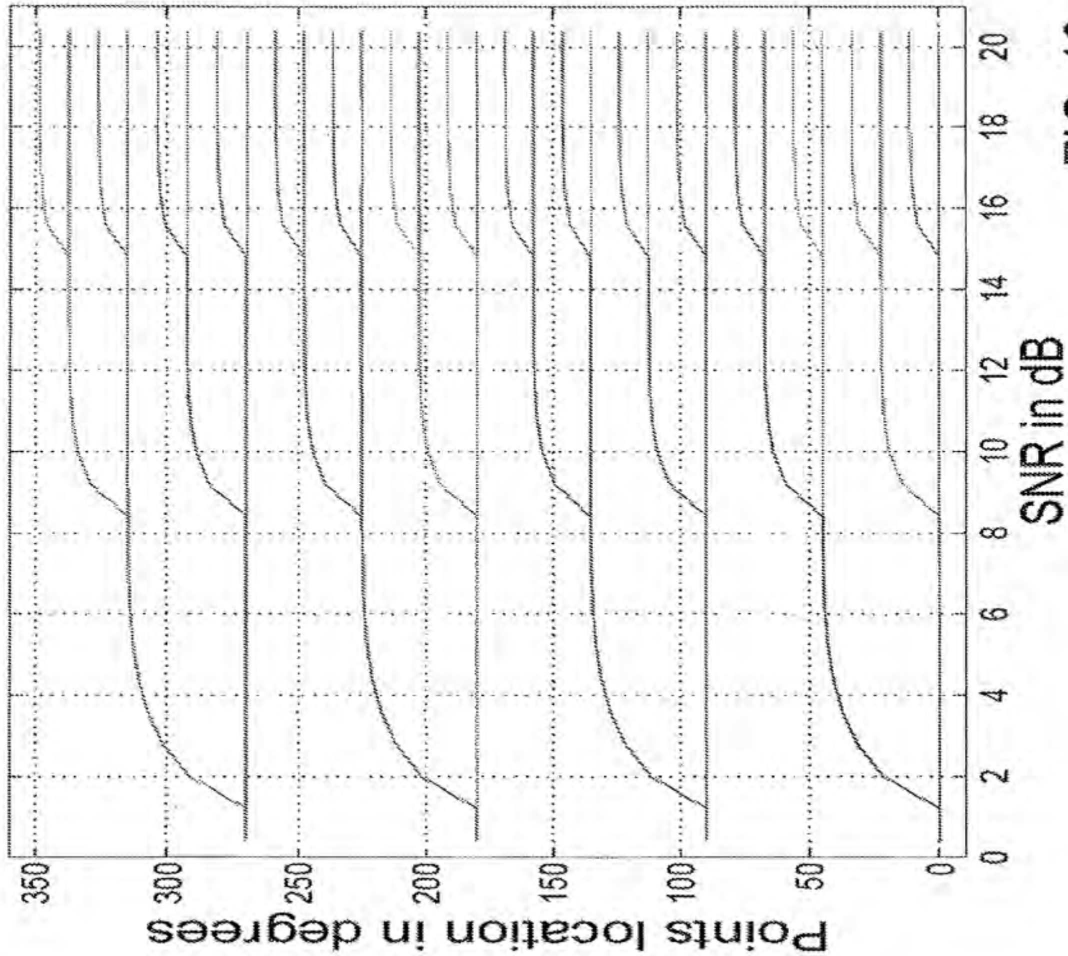

Points location in degrees
0
50
100
150
200
250
300
350
SNR in dB
0
2
4
6
8
10
12
14
16
18
20

FIG. 19

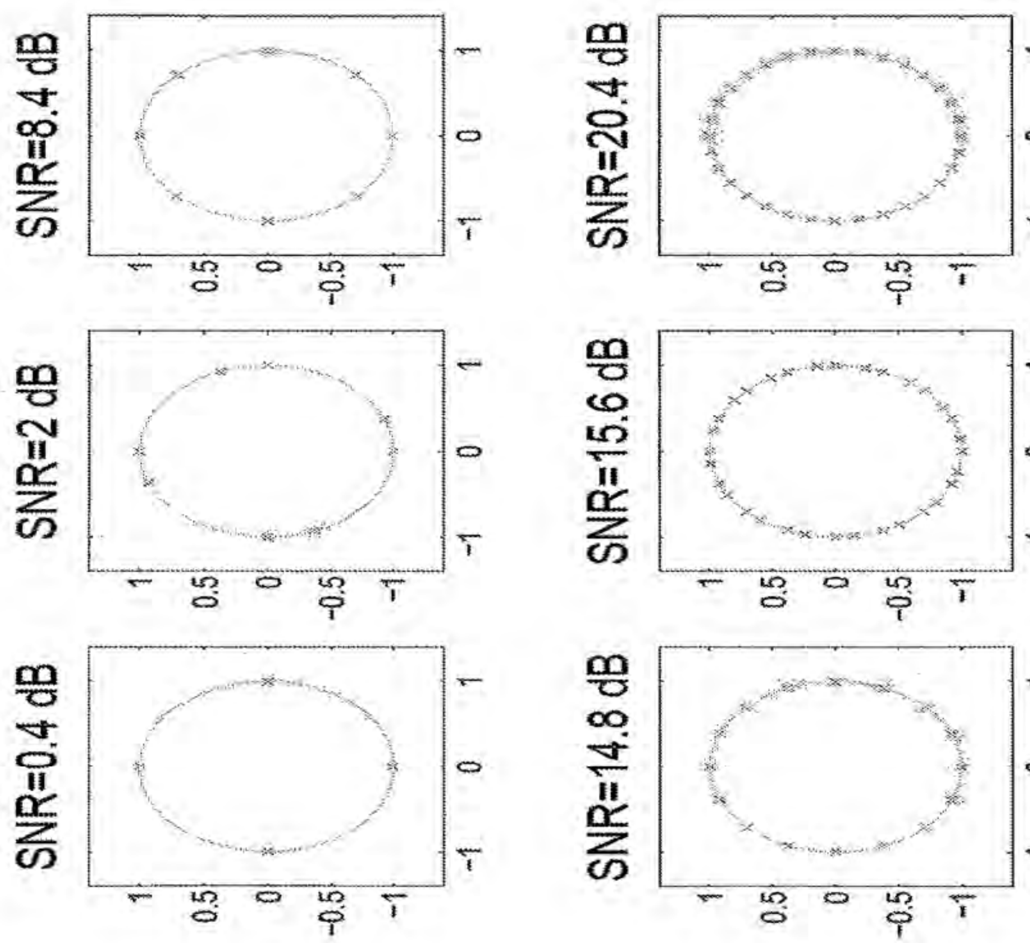
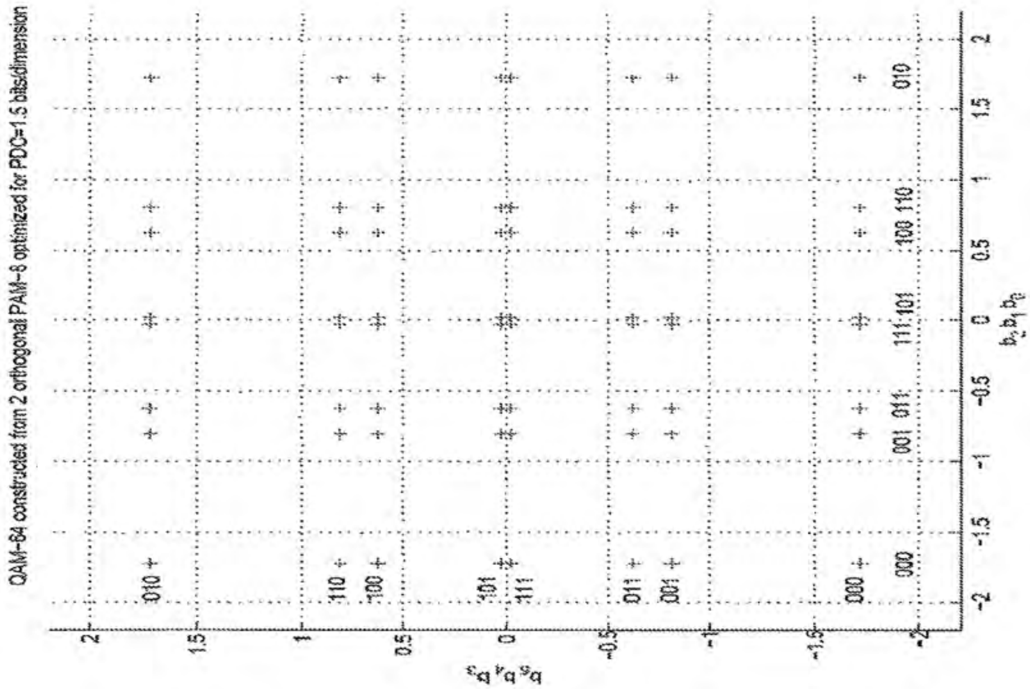


FIG. 20



QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
$b_2b_1b_0$
$b_5b_4b_3$
010
110
100
101
111
011
001
000

FIG. 21

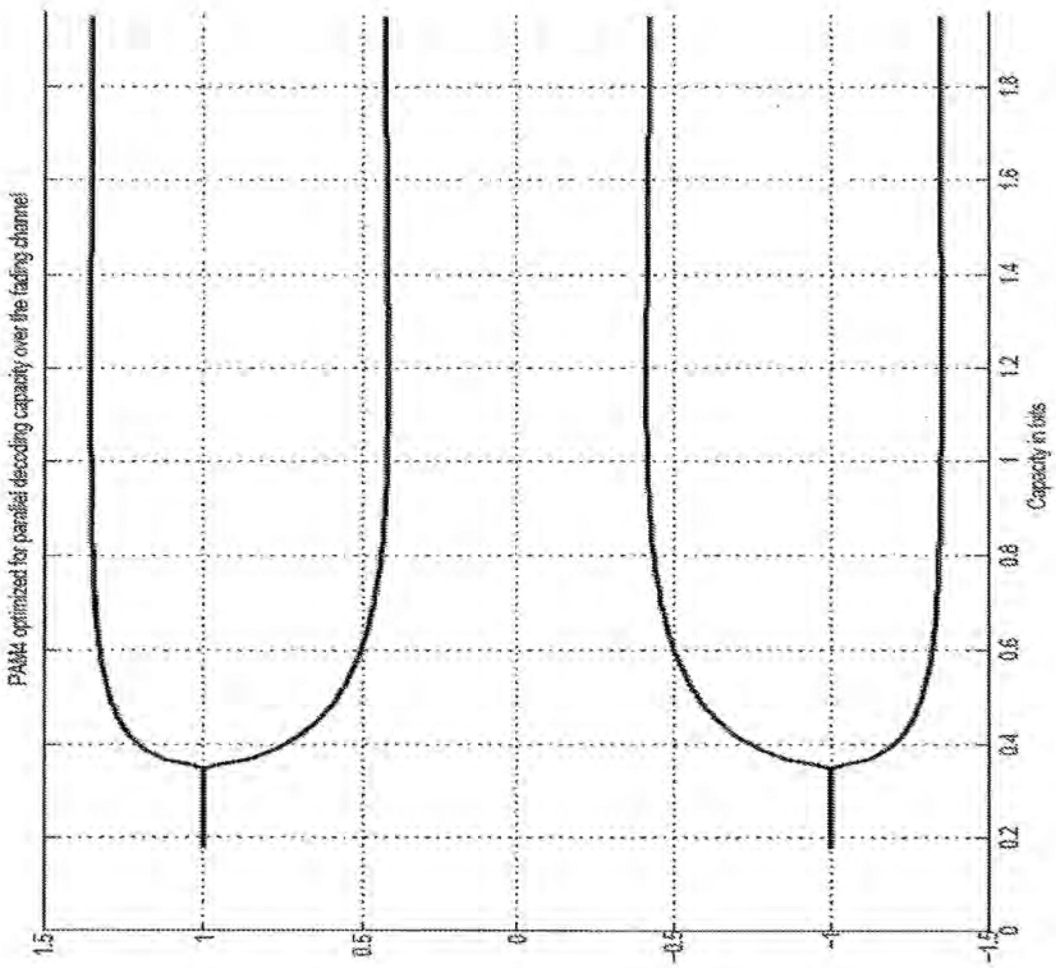

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 22a

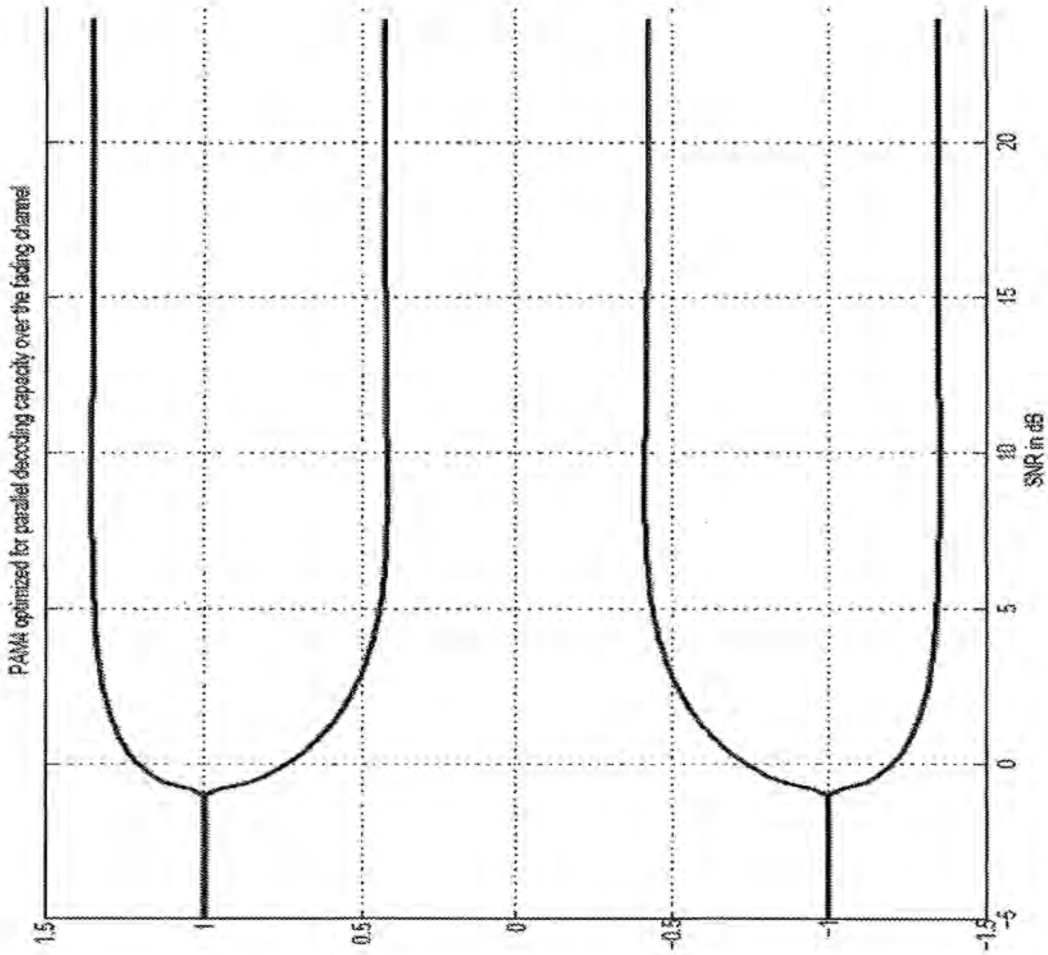

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15
20

FIG. 22b

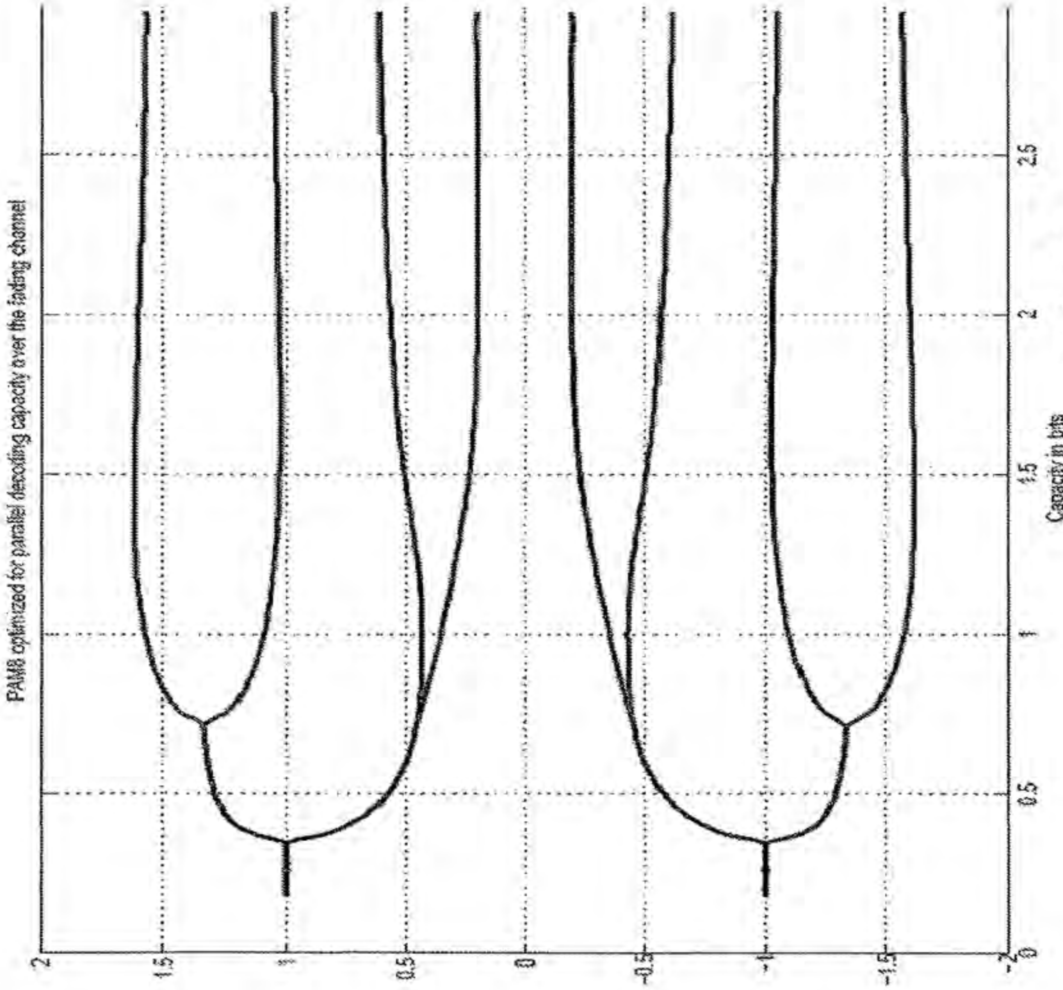

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23a

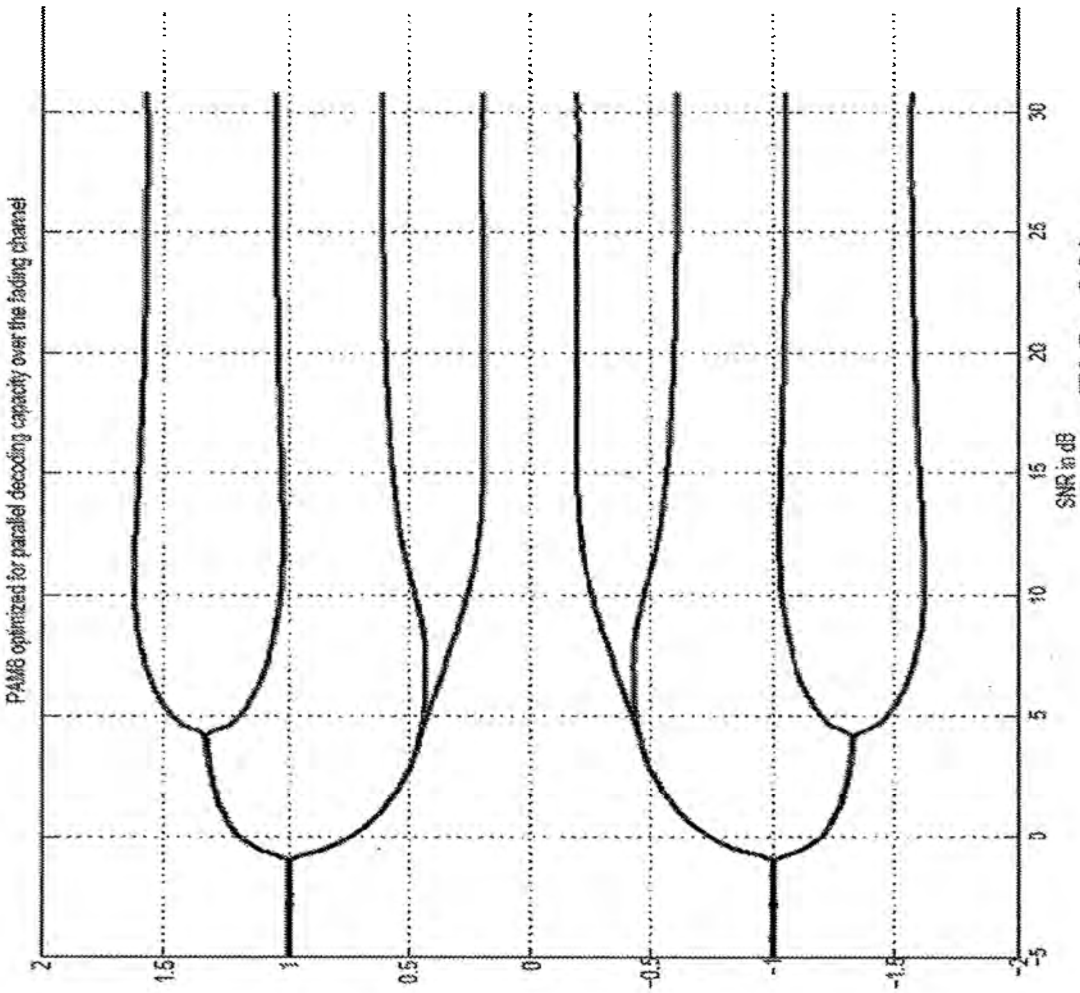

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB

FIG. 23b

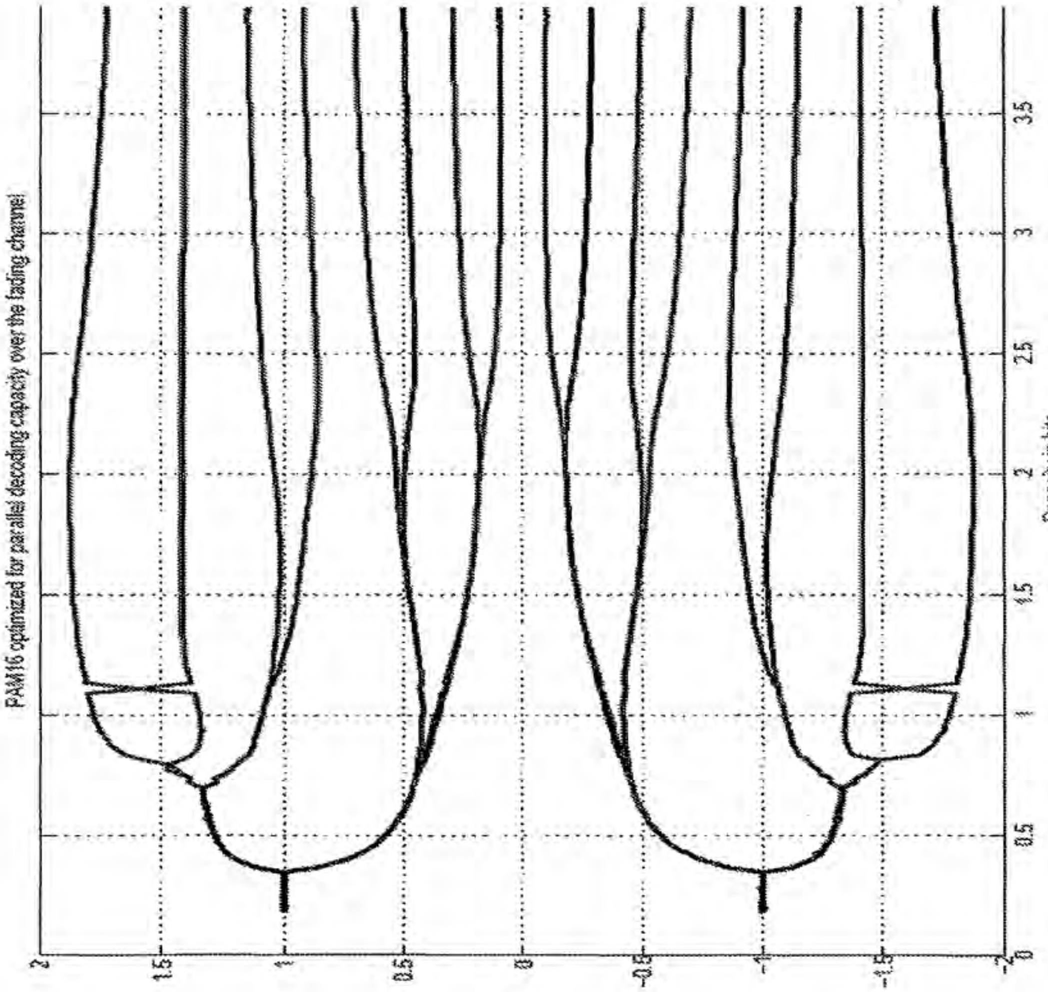

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24a

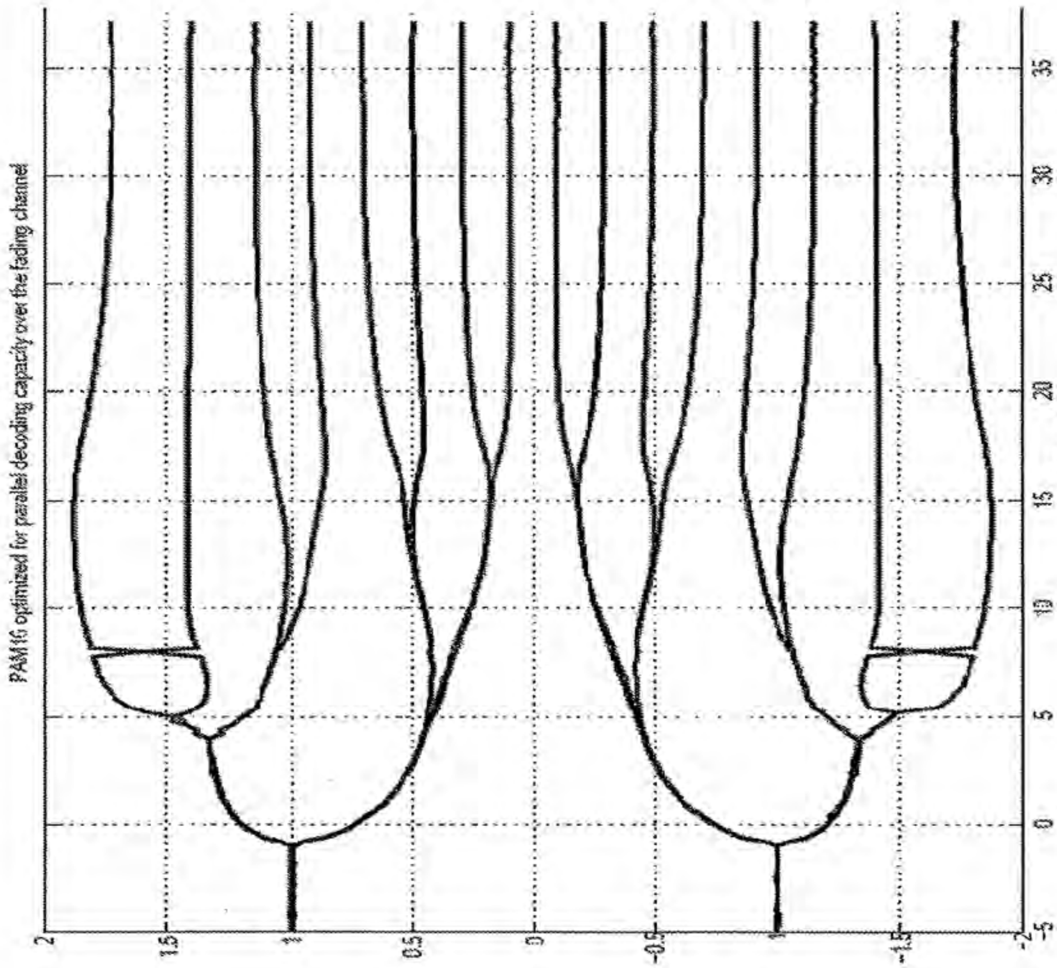

PAM 16 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
35
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24b

RECEIVERS INCORPORATING NON-UNIFORM MULTIDIMENSIONAL CONSTELLATIONS AND CODE RATE PAIRS

RELATED APPLICATIONS

This application is a continuation of application Ser. No. 16/206,991 filed Nov. 30, 2018 and issued as U.S. Pat. No. 10,567,980 on Feb. 18, 2020, which application is a continuation of application Ser. No. 15/682,475 filed Aug. 21, 2017 and issued on Dec. 4, 2018 as U.S. Pat. No. 10,149,179, which application is a continuation of application Ser. No. 15/200,800 filed Jul. 1, 2016 and issued on Aug. 22, 2017 as U.S. Pat. No. 9,743,292, which application is a continuation of application Ser. No. 14/491,731 filed Sep. 19, 2014 and issued on Jul. 5, 2016 as U.S. Pat. No. 9,385,832, which application is a continuation of application Ser. No. 13/618,630 filed Sep. 14, 2012 and issued on Sep. 23, 2014 as U.S. Pat. No. 8,842,761, which application is a continuation of application Ser. No. 13/118,921 filed May 31, 2011 and issued on Sep. 18, 2012 as U.S. Pat. No. 8,270,511, which application is a continuation of application Ser. No. 12/156,989 filed Jun. 5, 2008 and issued on Jul. 12, 2011 as U.S. Pat. No. 7,978,777, which application claimed priority to U.S. Provisional Application 60/933,319 filed Jun. 5, 2007, the disclosures of which are incorporated herein by reference.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

FIG. **1** is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

FIG. **2** is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

FIG. **3** is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

FIG. **4*a*** is a conceptual illustration of the joint capacity of a channel.

FIG. **4*b*** is a conceptual illustration of the parallel decoding capacity of a channel.

FIG. **5** is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

FIG. **6*a*** is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

FIG. **6*b*** is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8, 16,32.

FIG. **7** is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2,4,8,16,32 constellations.

FIG. **8*a*** is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **8*b*** is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **9** is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

FIGS. **10*a*-10*d*** are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **11*a*** and **11*b*** are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

FIGS. **12*a*-12*d*** are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **13*a*** and **13*b*** are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

FIGS. **14*a*-14*d*** are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **15*a*** and **15*b*** are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

FIGS. **16*a*-16*d*** are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **17*a*** and **17*b*** are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

FIG. **18** is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

FIG. **19** is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

FIG. **20** is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

FIG. **21** illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

FIGS. **22*a*** and **22*b*** are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **23*a*** and **23*b*** are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **24*a*** and **24*b*** are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

Communication Systems

A communication system in accordance with an embodiment of the invention is shown in FIG. **1**. The communication system **10** includes a source **12** that provides user bits to a transmitter **14**. The transmitter transmits symbols over a channel to a receiver **16** using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

A transmitter in accordance with an embodiment of the invention is shown in FIG. **2**. The transmitter **14** includes a coder **20** that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper **22** is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator **24**, which modulates the symbols for transmission via the channel.

A receiver in accordance with an embodiment of the invention is illustrated in FIG. **3**. The receiver **16** includes a demodulator **30** that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper **32** provides the likelihoods to a decoder **34** that decodes the encoded bit stream to provide a sequence of received bits to a sink.

Geometrically Shaped Constellations

Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

Selection of a Geometrically Shaped Constellation

Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

Upgrading Existing Communication Systems

A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

Joint Capacity and Parallel Decoding Capacity

A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. **4***a*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **40**. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{l-1} I(X_i; Y)$$

where X is the ith bit of the l-bits transmitted symbol, and Y is the received symbol, and I(A;B) denotes the mutual information between random variables A and B.

Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4*b*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **42**. The joint capacity of the channel is given by:

$$C_{JOINT}=I(X;Y)$$

Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

Selecting a Constellation Having an Optimal Capacity

Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. **5**. The process **50** commences with the selection (**52**) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (**52**) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known a priori. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (**56**) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

The SNR at which the optimized M-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (**57**). A determination (**58**) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (**55**). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

The process illustrated in FIG. **5** can maximize PD capacity or joint capacity of an M-ary constellation for a given SNR. Although the process illustrated in FIG. **5** shows selecting an M-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an M-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q), or with multi-stage decoding. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

Optimizing the Capacity of an M-Ary Constellation at a Given SNR

Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process **56** shown in FIG. **5**, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged.

The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. **5**.

Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

Gains Achieved by Optimized Geometrically Spaced Constellations

The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}\log_2(1+SNR)$$

Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. **6***a* and **6***b*. FIG. **6***a* includes a chart **60** showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **62** exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. **6***b* includes a chart **64** showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **66** exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

In order to gain a better view of the differences between the curves shown in FIGS. **6***a* and **6***b* at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. **7**. It is interesting to note from the chart **70** in FIG. **7** that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

FIGS. **8***a* and **8***b* summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. **8***a* shows the SNR gap for each constellation optimized for PD capacity. FIG. **8***b* shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

Referring to FIG. **8***a*, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. **9**. A chart **90** includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. **8***a*.

Capacity Optimized Pam Constellations

Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. **10***a*-**17***b*. The locus plots and design tables show PAM-4,8,16,32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

In FIG. **10***a*, a locus plot **100** shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot **105** showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. **10***b*. In FIG. **10***c*. the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. **10***d* the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. **11***a* and FIG. **11***b* where vertical slices of FIGS. **10***ab* and **10***cd* are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities={0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4)=2$ bps, these design points represent systems with information code rates R={1/4, 3/8, 1/2, 5/8, 3/4} respectively.

FIGS. **12***ab* and **12***cd* present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. **13***a* and **13***b* provide slices from these plots at SNRs corresponding to achievable capacities η={0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate $R=\eta$ bps/$\log_2(8)$, resulting in R={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. **13***b* which shows a PD capacity optimized PAM-8 constellation optimized for SNR=9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. **8***a* and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

Similar information is presented in FIGS. **14***abcd*, and **15***ab* which provide loci plots and design tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. **16***abcd*, **17***ab* provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

Capacity Optimized PSK Constellations

Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart **180** comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. **18** where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4,8,16,32 and for PD capacaity optimized PSK-4,8,16,32.

The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. **19**, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. **20**. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR=8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR=14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR=2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

We note now that the locus of points for PD optimized PSK-32 in FIG. **19** in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. **18** implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8,16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. **19**. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. **18**.

Adaptive Rate Design

In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation/code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper/decoder configuration in order to match the new transmit-side configuration.

Conversely, the receiver could implement a quality of performance based optimized constellation/code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation/code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper/decoder modules.

Geometrically Shaped QAM Constellations

Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

In FIG. **21** we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. **13***b*). The label-point pairs in this PAM-8 constellation are {(000, −1.72), (001, −0.81), (010, 1.72), (011, −0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, −0.02)}. Examination of FIG. **21** shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8×8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-Dimensional Constellation Optimization

Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

Capacity Optimized Constellations for Fading Channels

Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y=a(t)\cdot X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and a(t) is the fading distribution, which is a function of time.

In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

Many fading channels follow a Rayleigh distribution. FIGS. **22*a*-24*b*** are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

What is claimed is:

1. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a multidimensional symbol constellation selected from a plurality of multidimensional symbol constellations; and

a decoder, coupled to the demapper, capable of using the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code;

wherein the plurality of multidimensional symbol constellations comprises a plurality of different non-uniform multidimensional symbol constellations having the same number of constellation points, where the constellation points are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

wherein the receiver is capable of selecting an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs, where each of the plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs.

2. The communication system of claim **1**, wherein:

the demodulator is capable of using a demodulation scheme that is based on quadrature amplitude modulation (QAM) and the degrees of freedom available to the multidimensional symbol constellations are in phase and quadrature components;

each of the plurality of different non-uniform multidimensional symbol constellations is a sixteen point quadrature amplitude modulated (QAM) symbol constellation; and

the LDPC code rate and multidimensional symbol constellation pairs that include one of the plurality of different non-uniform multidimensional symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8.

3. The communication system of claim **2**, wherein:

the plurality of multidimensional symbol constellations further comprises an additional plurality of different non-uniform multidimensional symbol constellations that are quadrature amplitude modulated (QAM) symbol constellations and comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

the constellation points of the additional plurality of different non-uniform multidimensional symbol constellations are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellation;

each of the additional plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different sixty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 2/6 and less than or equal to 5/6;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different two-hundred-fifty-six-point symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 7/8;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different one-thousand-twenty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 4/10 and less than or equal to 9/10.

4. The communication system of claim **1**, wherein each of the plurality of different non-uniform multidimensional symbol constellations is capable of providing a greater parallel decoding capacity at a specific SNR than a similar multidimensional symbol constellation at the same SNR, where the similar multidimensional symbol constellation differs only in that the constellation points in the similar multidimensional symbol constellation are uniformly spaced in each degree of freedom.

5. The communication system of claim **1**, wherein each of the plurality of different non-uniform multidimensional symbol constellations is capable of providing a greater parallel decoding capacity at a specific SNR than the other symbol constellations in the plurality of multidimensional symbol constellations at the same SNR.

6. The communication system of claim 1, wherein each of the plurality of different non-uniform multidimensional symbol constellations are characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a specific SNR subject to at least one constraint.

7. The communication system of claim 1, wherein:

the receiver is capable of measuring the quality of the communication channel;

the receiver is capable of selecting an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs based at least in part on a quality measurement;

the receiver is capable of sending a request to a remote transmitter to use a selected LDPC code rate and multidimensional symbol constellation pair.

8. The communication system of claim 1, wherein the demodulator is capable of using a demodulation scheme that is based on phase shift keying and the degrees of freedom available to the multidimensional symbol constellations are amplitude and phase.

9. The communication system of claim 1, wherein the receiver is capable of substituting the plurality of multidimensional symbol constellations by an upgrade to at least one of the receiver software and firmware.

10. The communication system of claim 1, further comprising a transmitter capable of transmitting signals via the communication channel, where the transmitter comprises:

a coder capable of receiving bits and outputting encoded bits using the Low Density Parity Check (LDPC) code;

a mapper, coupled to the coder, capable of mapping the encoded bits to symbols in the selected multidimensional symbol constellation; and

a modulator, coupled to the mapper, capable of generating a signal for transmission via the communication channel based upon symbols selected by the mapper.

11. A communication system, comprising:

a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator that demodulates a received signal into a demodulated signal;

a demapper that determines likelihoods using the demodulated signal and a multidimensional symbol constellation selected from a plurality of multidimensional symbol constellations; and

a decoder that uses the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code;

wherein the demapper is interposed between the demodulator and the decoder and the demapper receives information from the demodulator and provides information to the decoder;

wherein the plurality of multidimensional symbol constellations comprises a plurality of different non-uniform multidimensional symbol constellations having the same number of constellation points, where the constellation points are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

wherein the receiver selects an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs, where each of the plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs.

12. The communication system of claim 11, wherein:

the demodulator uses a demodulation scheme that is based on quadrature amplitude modulation (QAM) and the degrees of freedom available to the multidimensional symbol constellations are in phase and quadrature components;

each of the plurality of different non-uniform multidimensional symbol constellations is a sixteen point quadrature amplitude modulated (QAM) symbol constellation; and

the LDPC code rate and multidimensional symbol constellation pairs that include one of the plurality of different non-uniform multidimensional symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8.

13. The communication system of claim 12, wherein:

the plurality of multidimensional symbol constellations further comprises an additional plurality of different non-uniform multidimensional symbol constellations that are quadrature amplitude modulated (QAM) symbol constellations and comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

the constellation points of the additional plurality of different non-uniform multidimensional symbol constellations are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

each of the additional plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different sixty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 2/6 and less than or equal to 5/6;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different two-hundred-fifty-six-point symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 7/8;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different one-thousand-twenty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 4/10 and less than or equal to 9/10.

14. The communication system of claim 11, wherein each of the plurality of different non-uniform multidimensional symbol constellations has a greater parallel decoding capacity at a specific SNR than a similar multidimensional symbol constellation at the same SNR, where the similar multidimensional symbol constellation differs only in that the constellation points in the similar multidimensional symbol constellation are uniformly spaced in each degree of freedom.

15. The communication system of claim 11, wherein each of the plurality of different non-uniform multidimensional symbol constellations has a greater parallel decoding capac-

ity at a specific SNR than the other symbol constellations in the plurality of multidimensional symbol constellations at the same SNR.

16. The communication system of claim **11**, wherein each of the plurality of different non-uniform multidimensional symbol constellations are characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a specific SNR subject to at least one constraint.

17. The communication system of claim **11**, wherein:

the receiver measures the quality of the communication channel;

the receiver selects an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs based at least in part on a quality measurement;

the receiver sends a request to a remote transmitter to use a selected LDPC code rate and multidimensional symbol constellation pair.

18. The communication system of claim **11**, wherein the demodulator uses a demodulation scheme that is based on phase shift keying and the degrees of freedom available to the multidimensional symbol constellations are amplitude and phase.

19. The communication system of claim **11**, wherein the receiver can substitute the plurality of multidimensional symbol constellations by an upgrade to at least one of the receiver software and firmware.

20. The communication system of claim **11**, further comprising a transmitter that transmits signals via the communication channel, where the transmitter comprises:

a coder that receives bits and outputs encoded bits using the Low Density Parity Check (LDPC) code;

a mapper that maps the encoded bits to symbols in the selected multidimensional symbol constellation; and

a modulator that generates a signal for transmission via the communication channel based upon symbols selected by the mapper;

wherein the mapper is interposed between the modulator and the coder and the mapper receives information from the coder and provides information to the modulator.

21. A communication system, comprising:

a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver uses a multidimensional symbol constellation to transform the received signals into received bits and the multidimensional symbol constellation is selected from a plurality of multidimensional symbol constellations;

wherein the plurality of multidimensional symbol constellations comprises a plurality of different non-uniform multidimensional symbol constellations having the same number of constellation points, where the constellation points are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

wherein the receiver selects an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs, where each of the plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs.

22. The communication system of claim **21**, wherein:

the receiver uses a demodulation scheme that is based on quadrature amplitude modulation (QAM) and the degrees of freedom available to the multidimensional symbol constellations are in phase and quadrature components;

each of the plurality of different non-uniform multidimensional symbol constellations is a sixteen point quadrature amplitude modulated (QAM) symbol constellation; and

the LDPC code rate and multidimensional symbol constellation pairs that include one of the plurality of different non-uniform multidimensional symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8.

23. The communication system of claim **22**, wherein:

the plurality of multidimensional symbol constellations further comprises an additional plurality of different non-uniform multidimensional symbol constellations that are quadrature amplitude modulated (QAM) symbol constellations and comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

the constellation points of the additional plurality of different non-uniform multidimensional symbol constellations are non-uniformly spaced in each degree of freedom available to the multidimensional symbol constellations;

each of the additional plurality of different non-uniform multidimensional symbol constellations is only included in one of the plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different sixty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 2/6 and less than or equal to 5/6;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different two-hundred-fifty-six-point symbol constellations have an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 7/8;

the LDPC code rate and multidimensional symbol constellation pairs that include one of the multiple different one-thousand-twenty-four-point symbol constellations have an LDPC code rate that is equal to or greater than 4/10 and less than or equal to 9/10.

24. The communication system of claim **21**, wherein each of the plurality of different non-uniform multidimensional symbol constellations has a greater parallel decoding capacity at a specific SNR than a similar multidimensional symbol constellation at the same SNR, where the similar multidimensional symbol constellation differs only in that the constellation points in the similar multidimensional symbol constellation are uniformly spaced in each degree of freedom.

25. The communication system of claim **21**, wherein each of the plurality of different non-uniform multidimensional symbol constellations has a greater parallel decoding capacity at a specific SNR than the other symbol constellations in the plurality of multidimensional symbol constellations at the same SNR.

26. The communication system of claim **21**, wherein each of the plurality of different non-uniform multidimensional symbol constellations are characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a specific SNR subject to at least one constraint.

27. The communication system of claim **21**, wherein:

the receiver measures the quality of the communication channel;

the receiver selects an LDPC code rate and multidimensional symbol constellation pair from a plurality of predetermined LDPC code rate and multidimensional symbol constellation pairs based at least in part on a quality measurement; and

the receiver sends a request to a remote transmitter to use a selected LDPC code rate and multidimensional symbol constellation pair.

28. The communication system of claim **21**, wherein the receiver uses a demodulation scheme that is based on phase shift keying and the degrees of freedom available to the multidimensional symbol constellations are amplitude and phase.

29. The communication system of claim **21**, wherein the receiver can substitute the plurality of multidimensional symbol constellations by an upgrade to at least one of the receiver software and firmware.

30. The communication system of claim **21**, further comprising a transmitter that transmits signals via the communication channel, where the transmitter uses the selected multidimensional symbol constellation to transform encoded bits into the transmitted signals.

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(12) **United States Patent**
Barsoum et al.

(10) **Patent No.: US 11,019,509 B2**
(45) **Date of Patent: May 25, 2021**

(54) **RECEIVERS INCORPORATING NON-UNIFORM CONSTELLATIONS WITH OVERLAPPING CONSTELLATION POINT LOCATIONS**

(71) Applicant: **Constellation Designs, LLC**, Anaheim, CA (US)

(72) Inventors: **Maged F. Barsoum**, San Jose, CA (US); **Christopher R. Jones**, Pacific Palisades, CA (US)

(73) Assignee: **Constellation Designs, LLC**, Anaheim, CA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

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(65) **Prior Publication Data**

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(Continued)

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H04L 27/06 (2006.01)
H04W 24/02 (2009.01)
(Continued)

(52) **U.S. Cl.**
CPC ***H04W 24/02*** (2013.01); ***H03M 13/255*** (2013.01); ***H03M 13/6325*** (2013.01);
(Continued)

(58) **Field of Classification Search**
CPC H04W 24/02; H04B 15/00; H04L 1/0003; H04L 1/0009; H04L 27/3405; H04L 27/3483; H04L 1/0002
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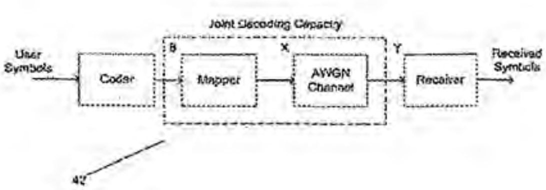
Primary Examiner — Tesfaldet Bocure
(74) *Attorney, Agent, or Firm* — KPPB LLP

(57) **ABSTRACT**

Communication systems are described that use unequally spaced constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. One embodiment is a digital communications system including a transmitter transmitting signals via a communication channel, the transmitter including a coder capable of receiving user bits and outputting encoded bits at a rate, a mapper capable of mapping encoded bits to symbols in a constellation, and a modulator capable of generating a modulated signal for transmission via the communication channel using symbols generated by the mapper, wherein the constellation is unequally spaced and characterizable by assignment of locations and labels of constellation points to maximize parallel decode capacity of the constellation at a given signal-to-noise ratio so that the constellation provides a given capacity at a reduced signal-to-noise ratio compared to a uniform constellation that maximizes the minimum distance between constellation points of the uniform constellation.

30 Claims, 43 Drawing Sheets



PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5.27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

Related U.S. Application Data

continuation of application No. 15/682,475, filed on Aug. 21, 2017, now Pat. No. 10,149,179, which is a continuation of application No. 15/200,800, filed on Jul. 1, 2016, now Pat. No. 9,743,292, which is a continuation of application No. 14/491,731, filed on Sep. 19, 2014, now Pat. No. 9,385,832, which is a continuation of application No. 13/618,630, filed on Sep. 14, 2012, now Pat. No. 8,842,761, which is a continuation of application No. 13/118,921, filed on May 31, 2011, now Pat. No. 8,270,511, which is a continuation of application No. 12/156,989, filed on Jun. 5, 2008, now Pat. No. 7,978,777.

(60) Provisional application No. 60/933,319, filed on Jun. 5, 2007.

(51) **Int. Cl.**

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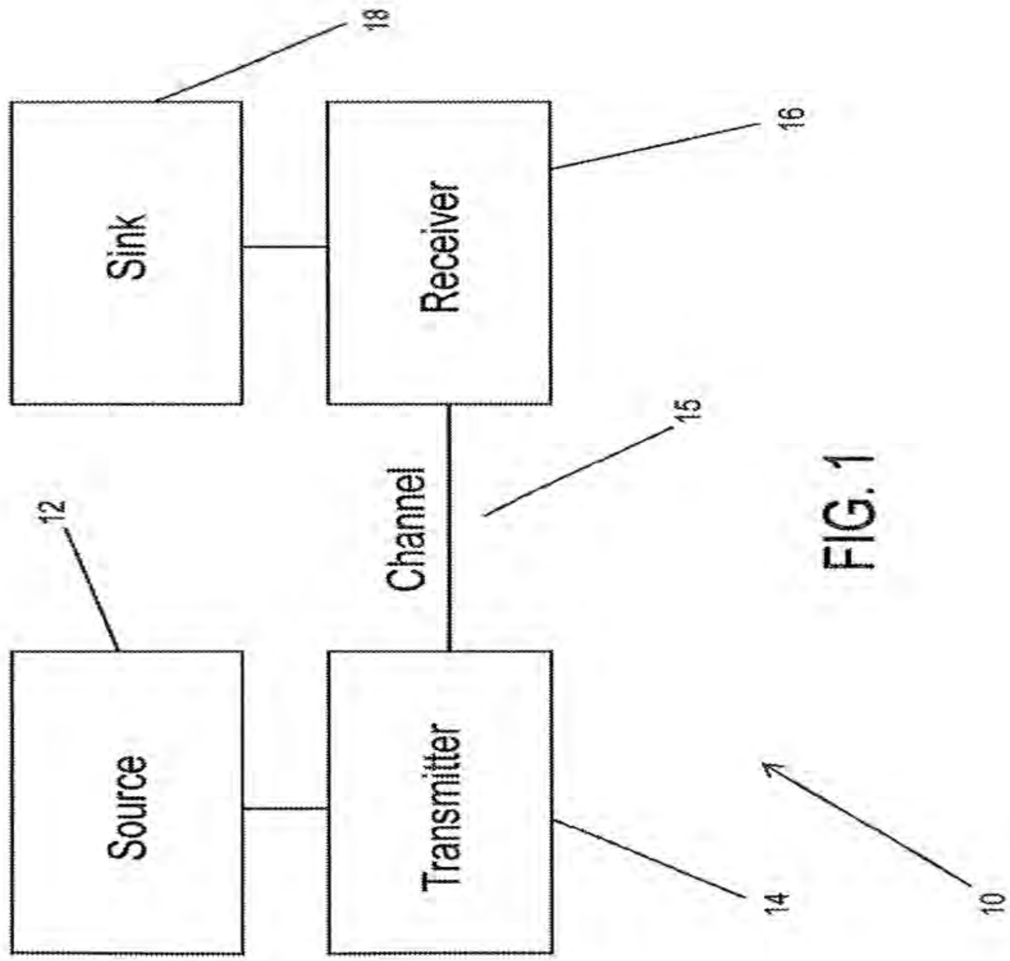


FIG. 1

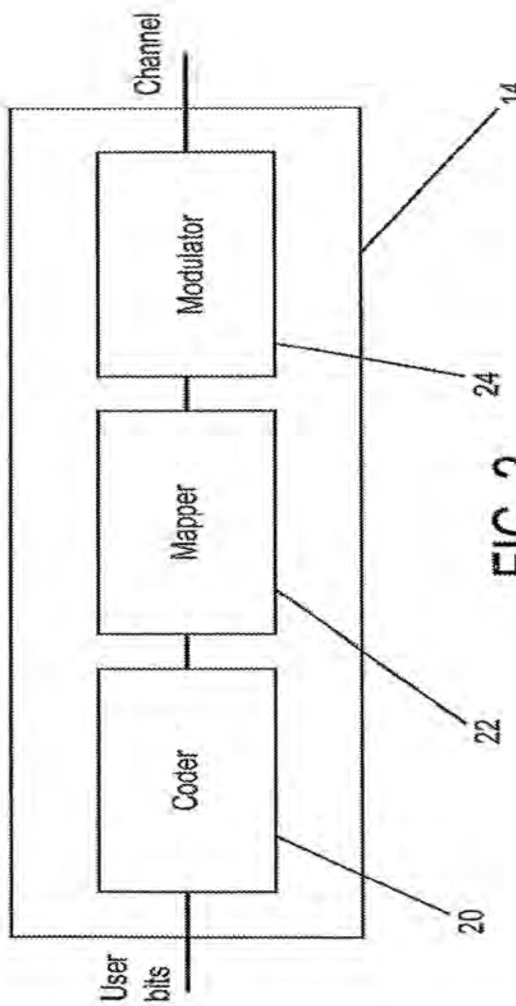

Channel
Modulator
Mapper
Coder
User bits
14
24
22
20

FIG. 2

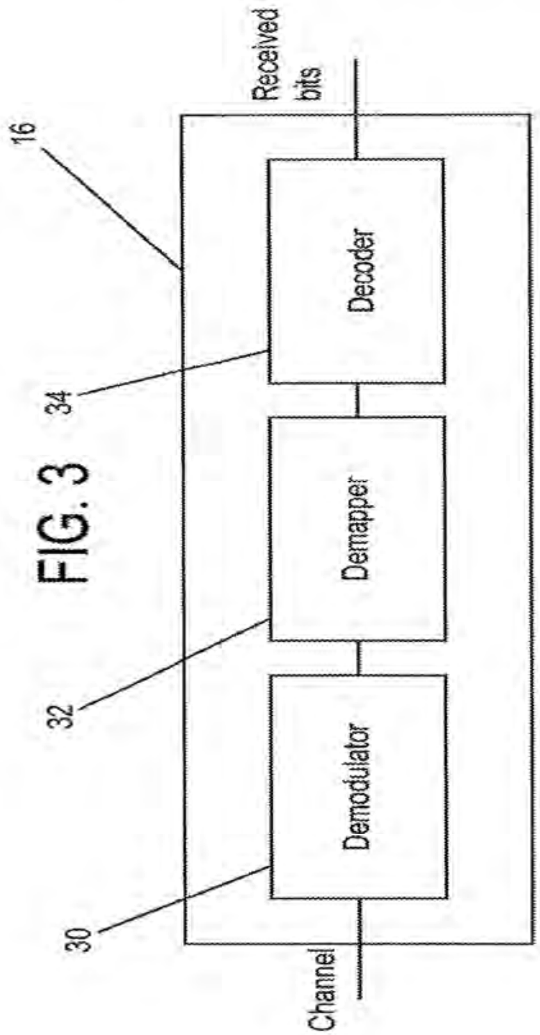

Received bits
Decoder
Demapper
Demodulator
Channel
16
34
32
30

FIG. 3

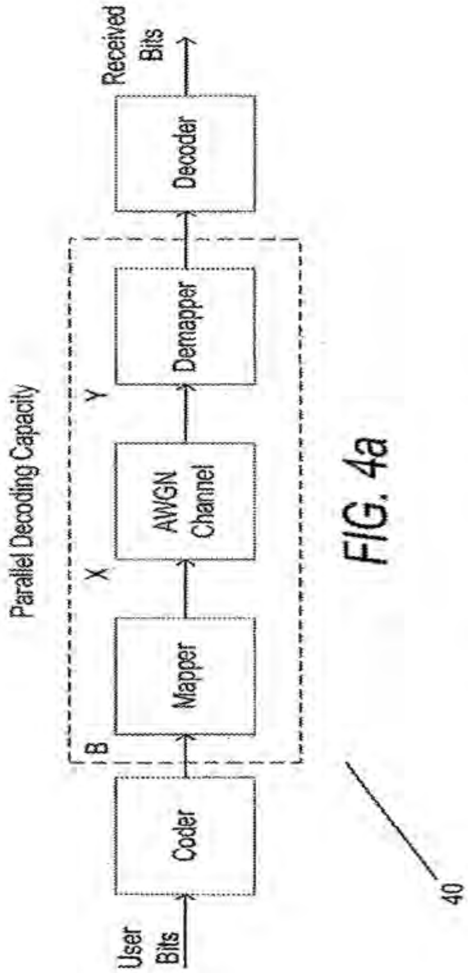

Parallel Decoding Capacity
User Bits
Coder
B
Mapper
X
AWGN Channel
Y
Demapper
Decoder
Received Bits
40

FIG. 4a

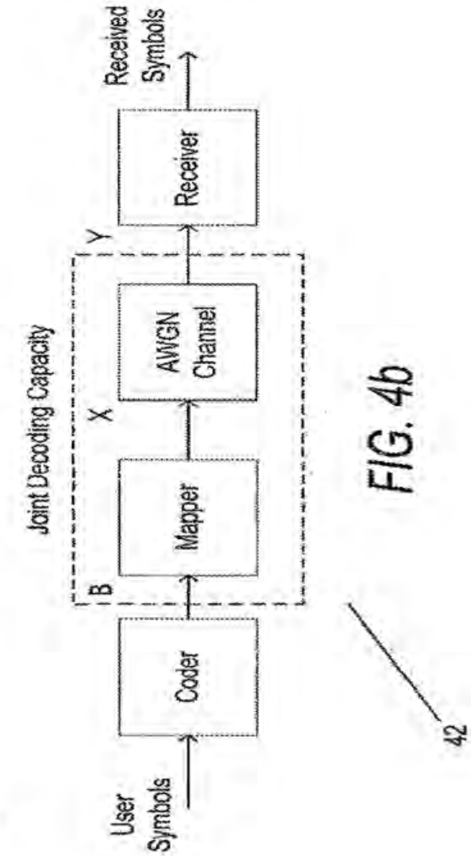

Joint Decoding Capacity
User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
42

FIG. 4b

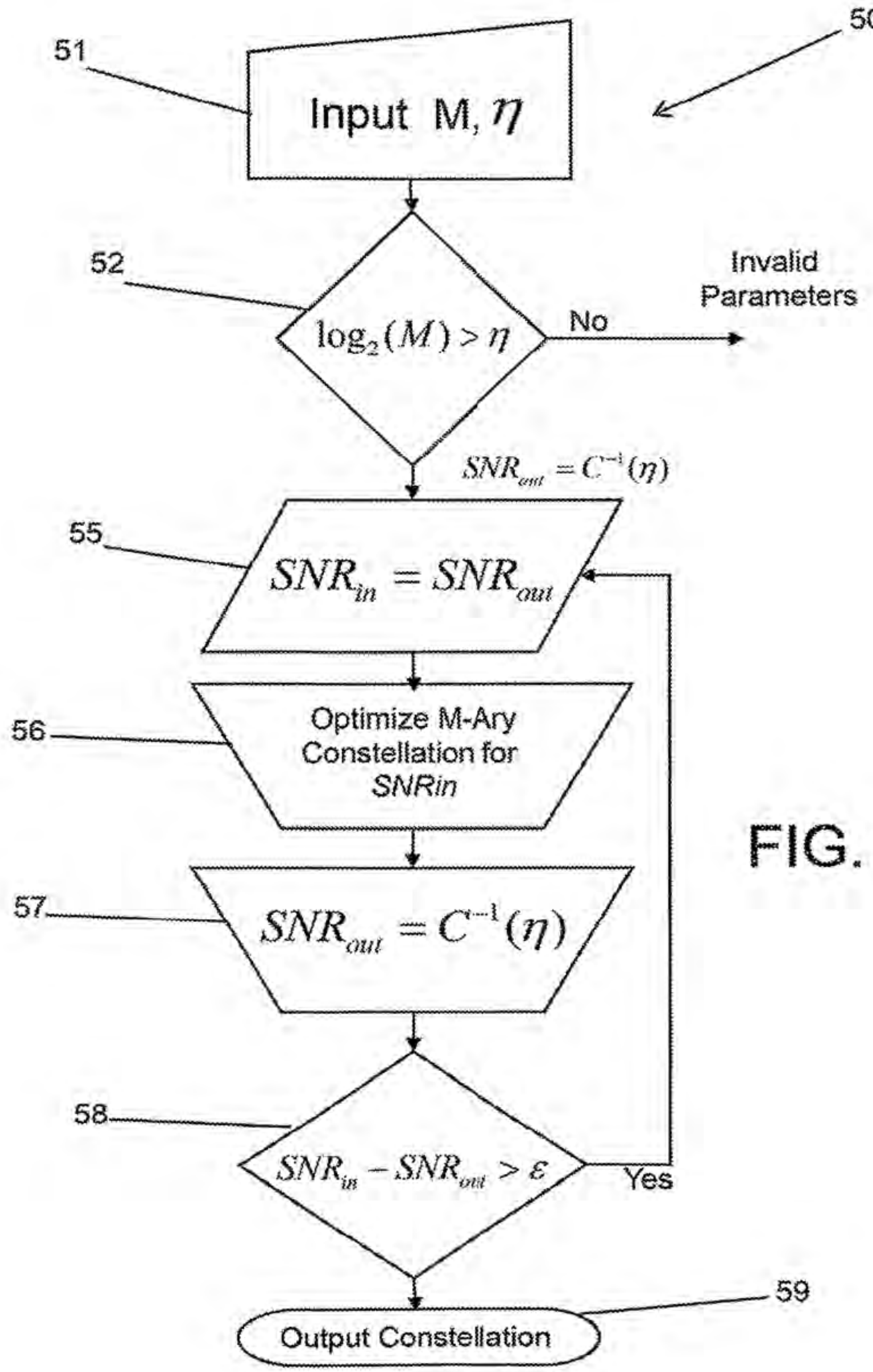

50
51
Input M, η
52
$\log_2(M) > \eta$
No
Invalid Parameters
$SNR_{out} = C^{-1}(\eta)$
55
$SNR_{in} = SNR_{out}$
56
Optimize M-Ary Constellation for SNRin
57
$SNR_{out} = C^{-1}(\eta)$
58
$SNR_{in} - SNR_{out} > \varepsilon$
Yes
59
Output Constellation

FIG. 5

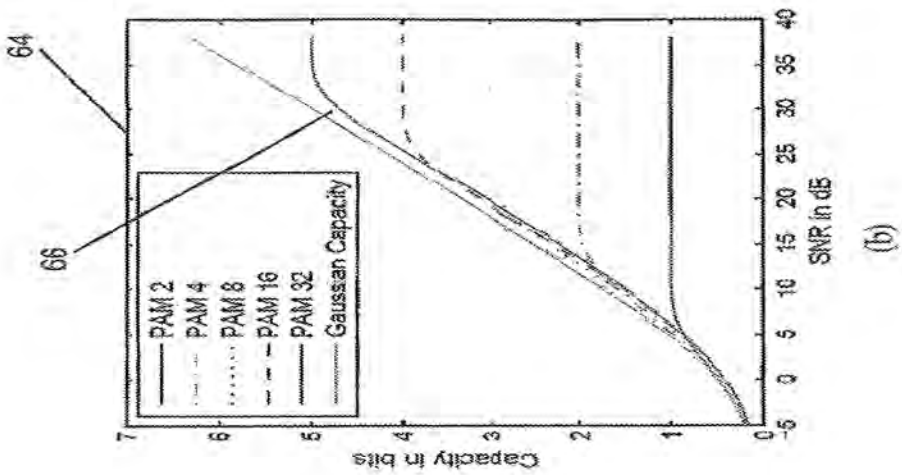

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

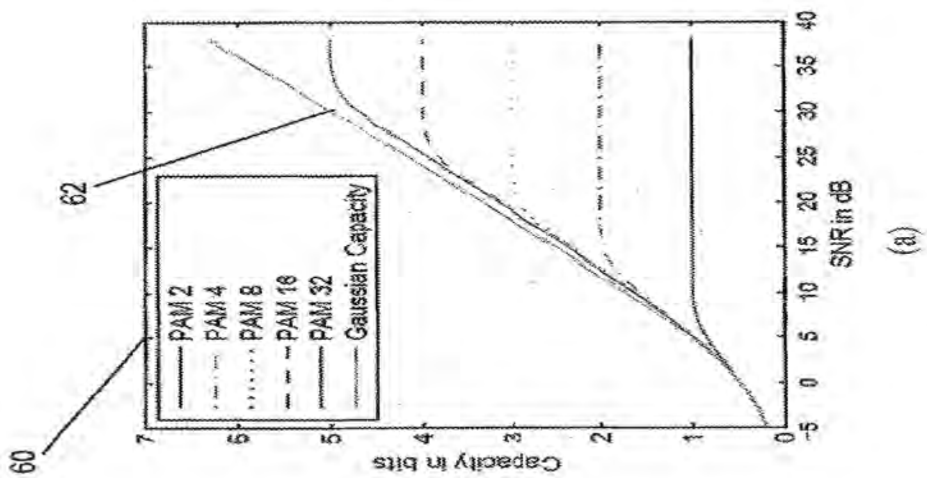

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

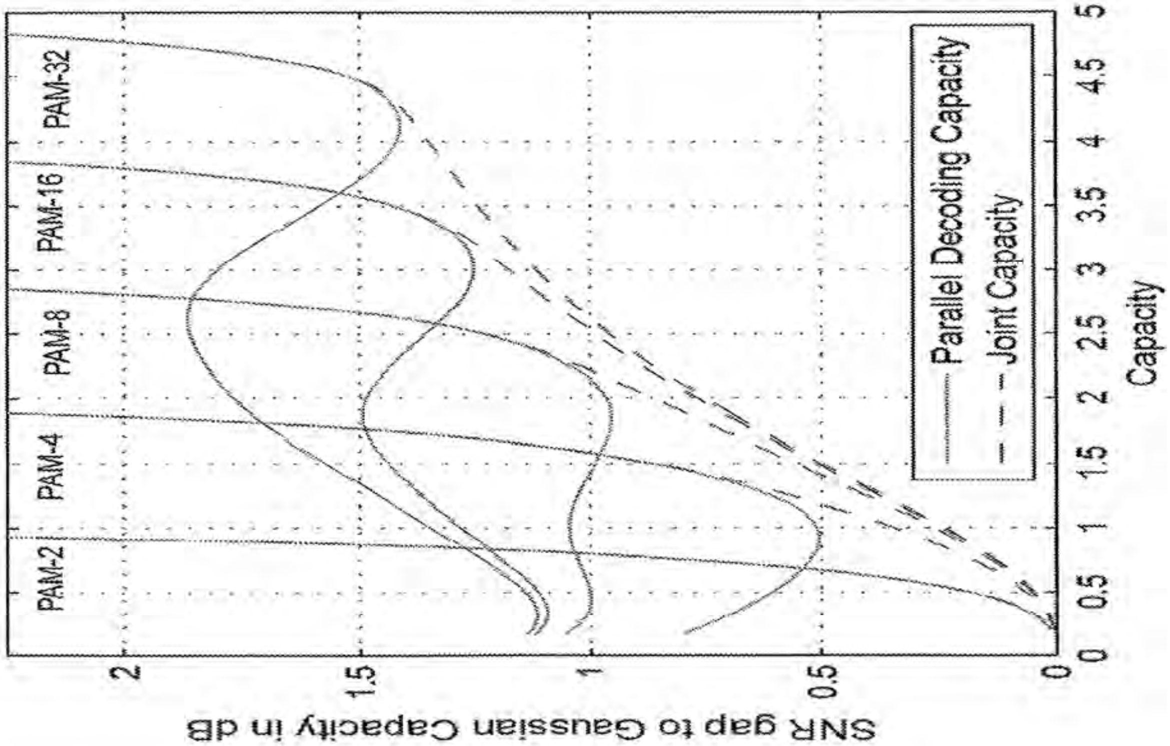

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
Parallel Decoding Capacity
Joint Capacity
Capacity
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

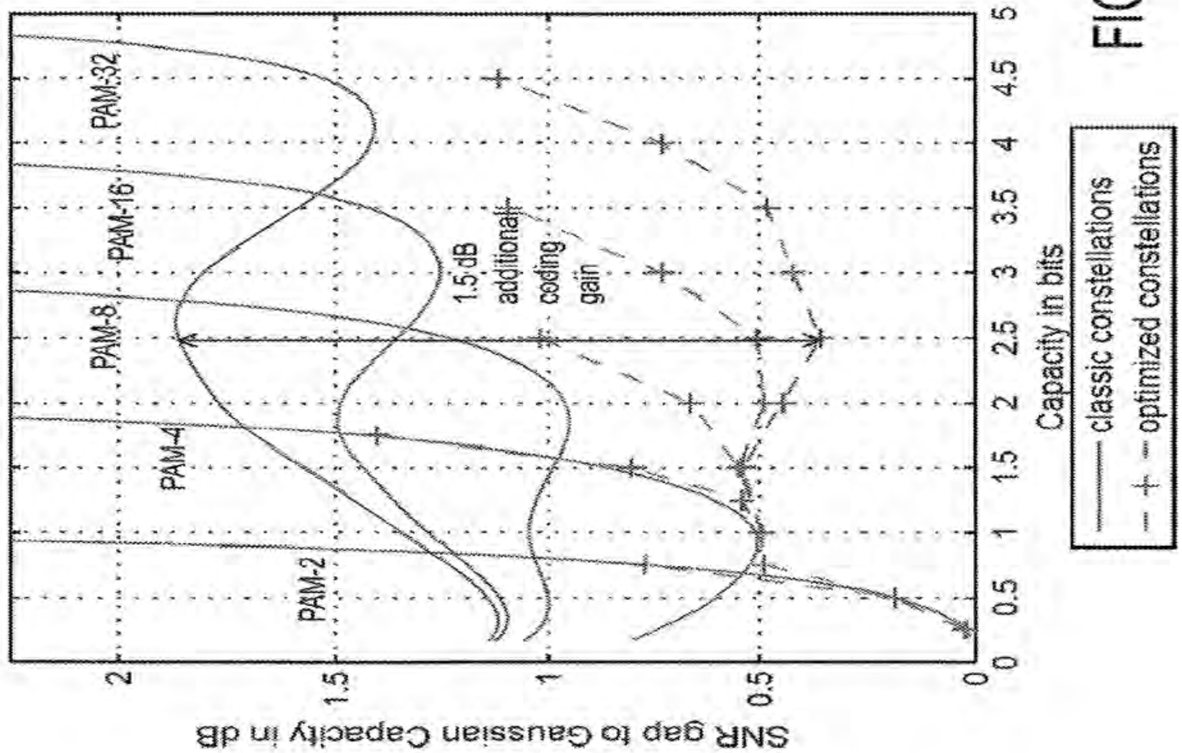
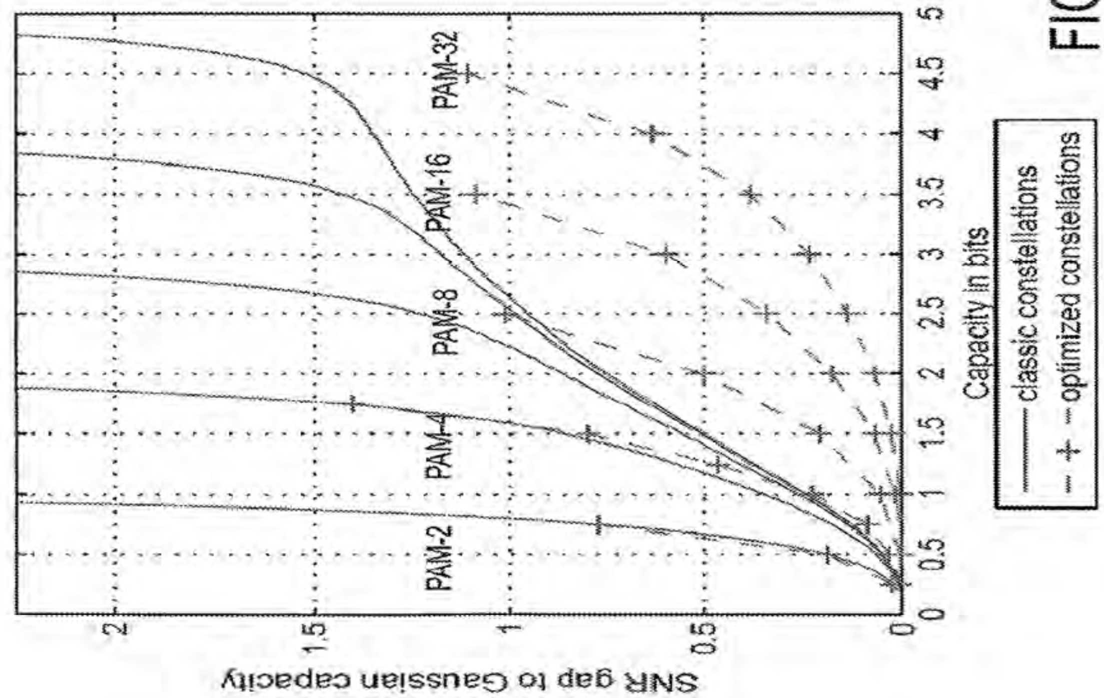

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5 dB additional coding gain
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Capacity in bits
SNR gap to Gaussian Capacity in dB
classic constellations
optimized constellations

FIG. 8a



PAM-32
PAM-16
PAM-8
PAM-4
PAM-2
Capacity in bits
SNR gap to Gaussian capacity
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
classic constellations
optimized constellations

FIG. 8b

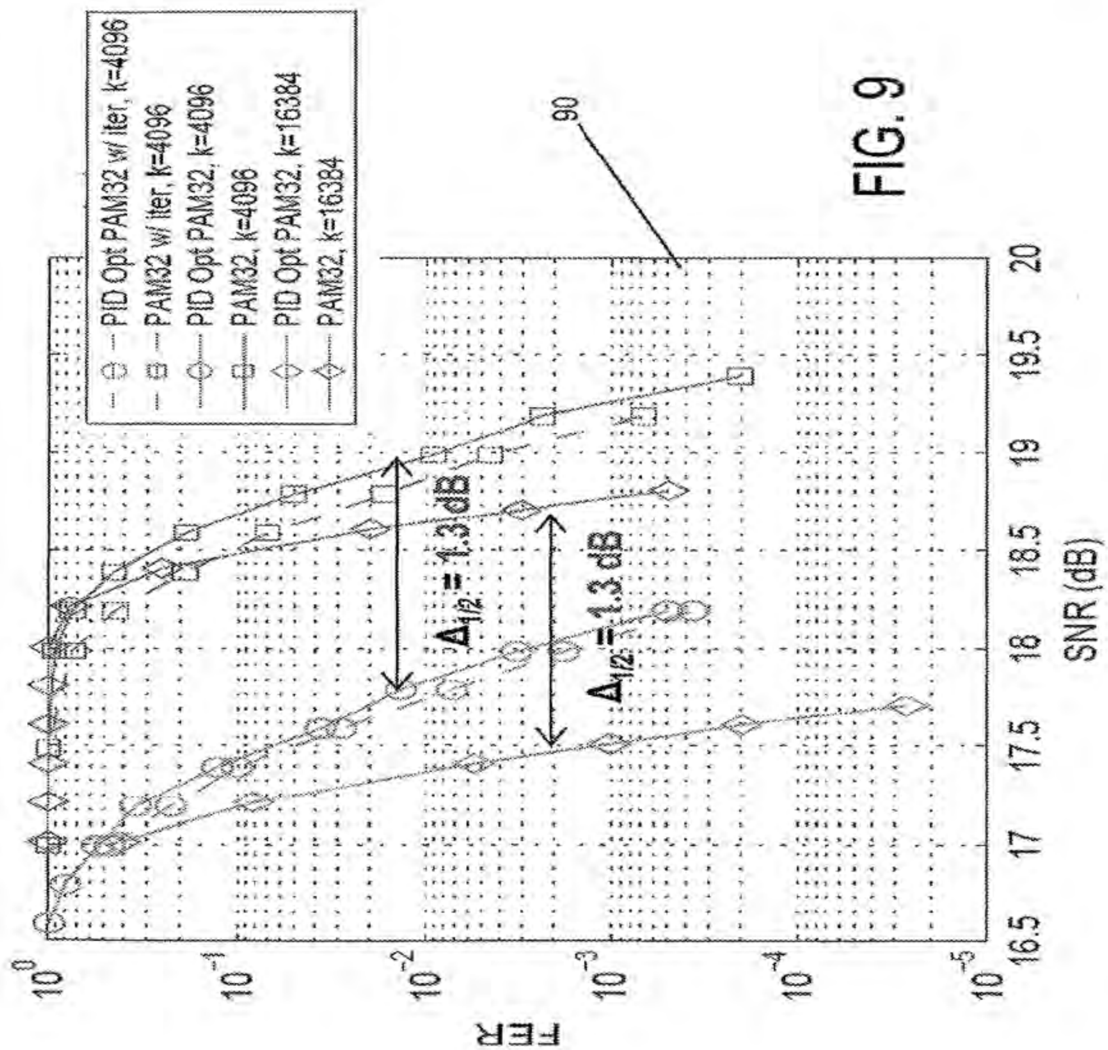

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

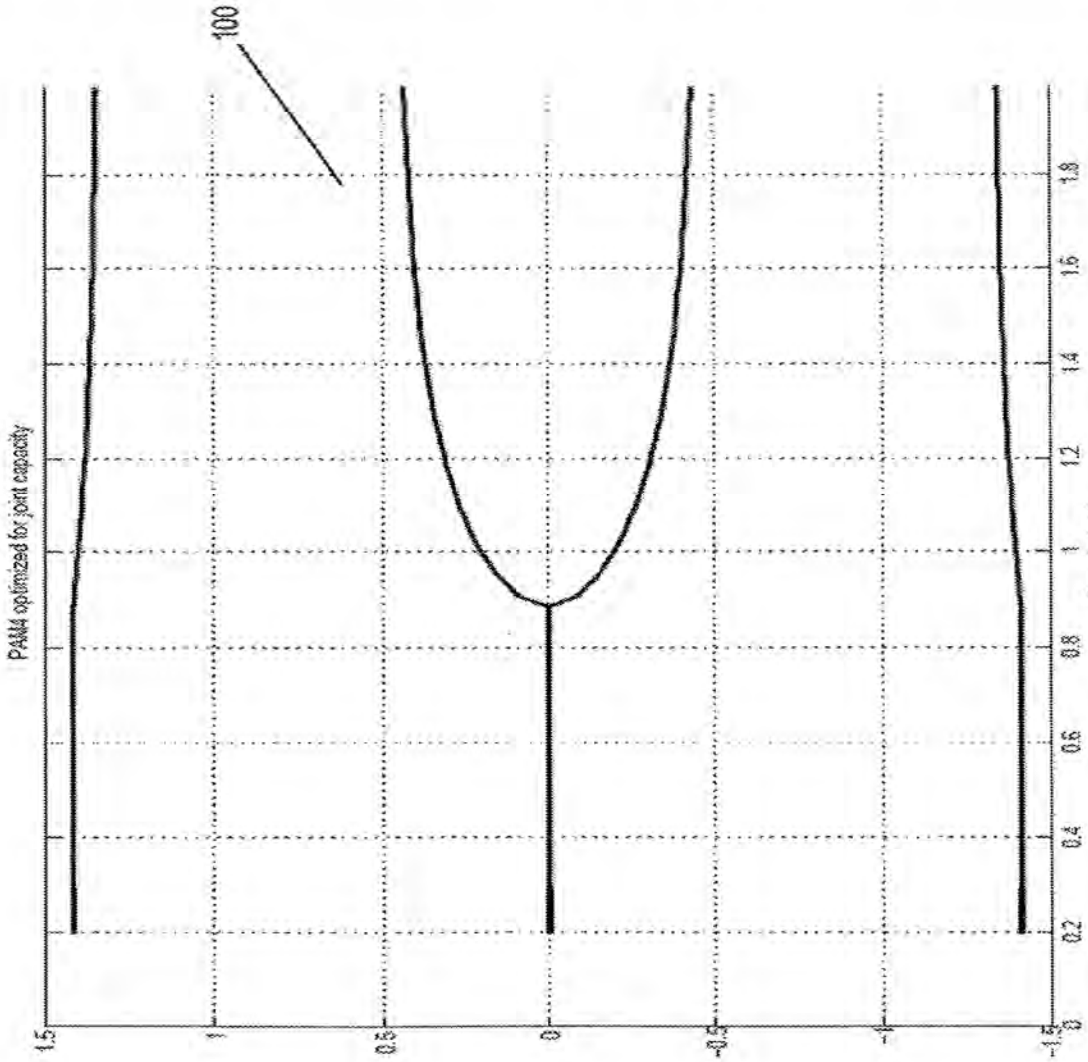

100
PAM4 optimized for joint capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10a

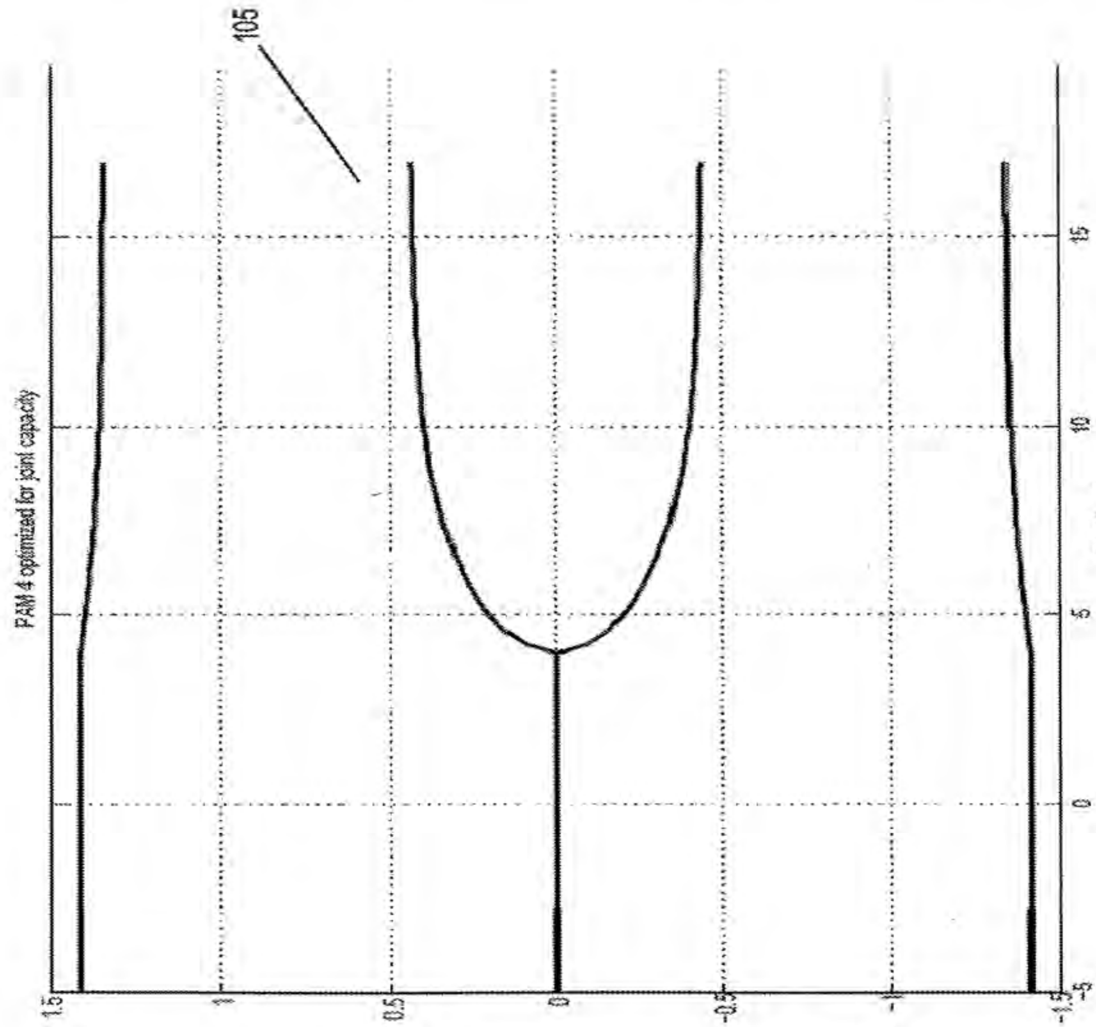

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10b

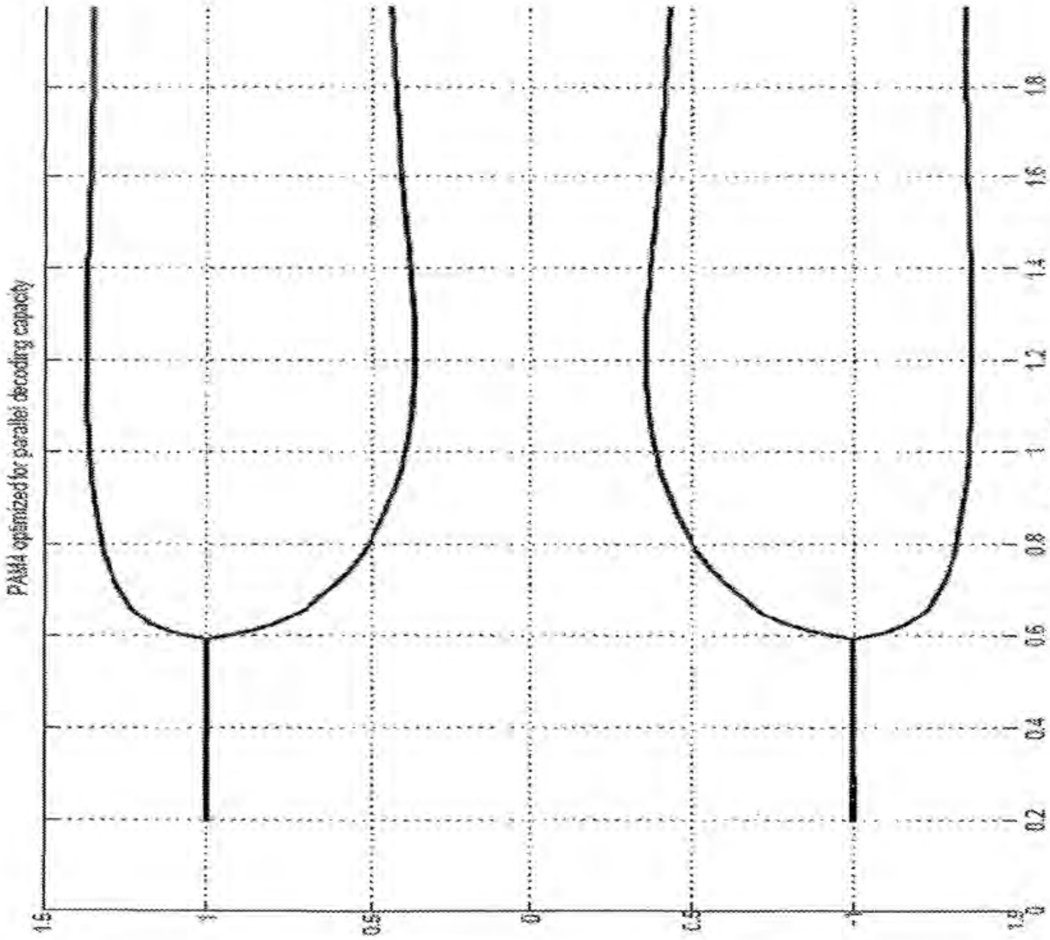

PAM4 optimized for parallel decoding capacity
Capacity in bits
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 10c

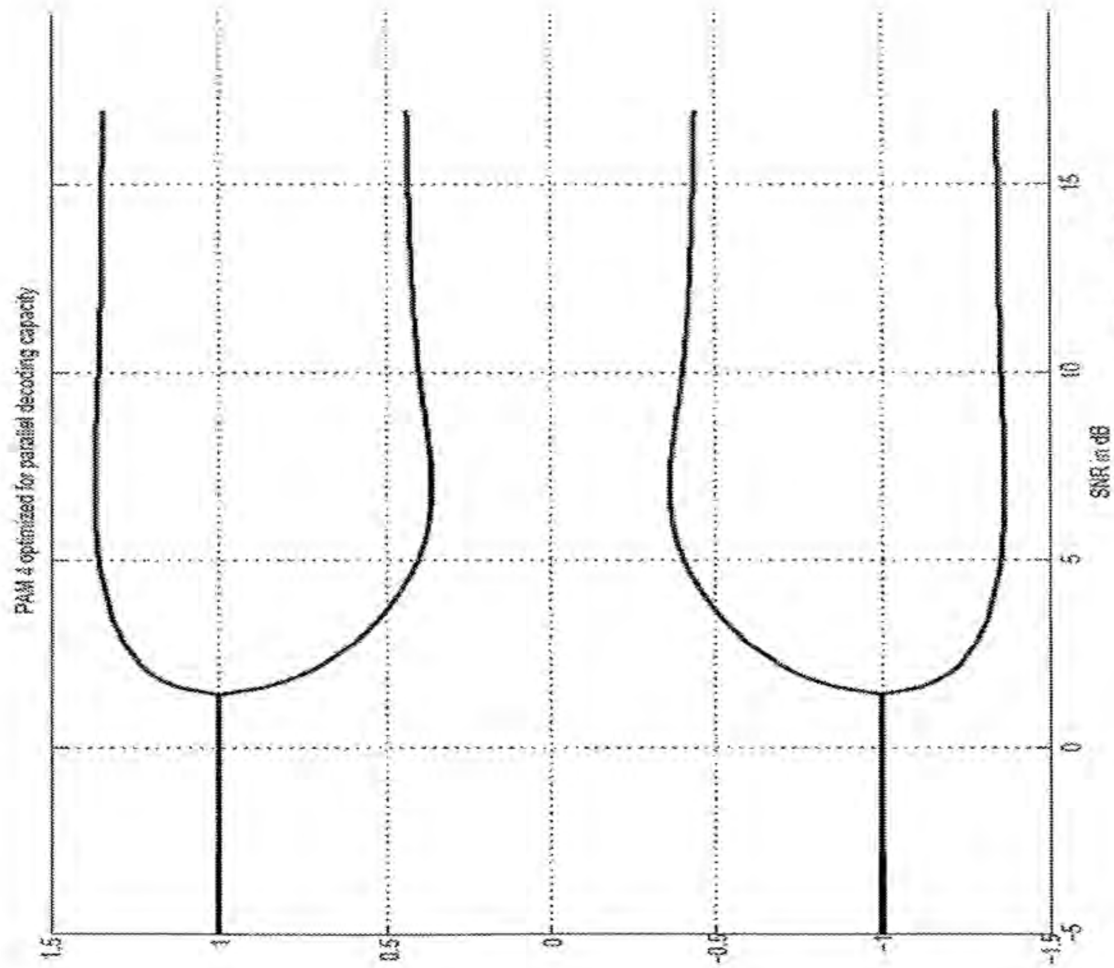

PAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

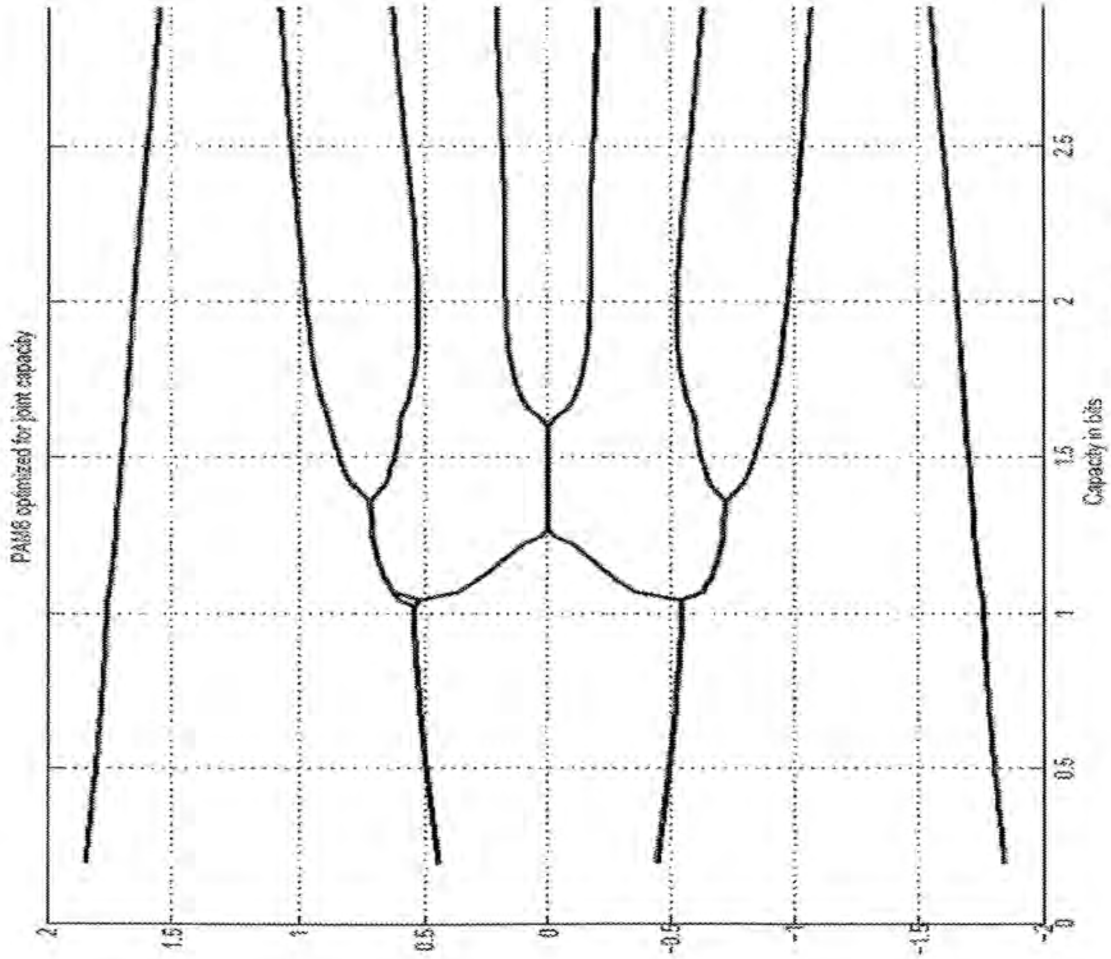

PAM6 optimized for joint capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12a

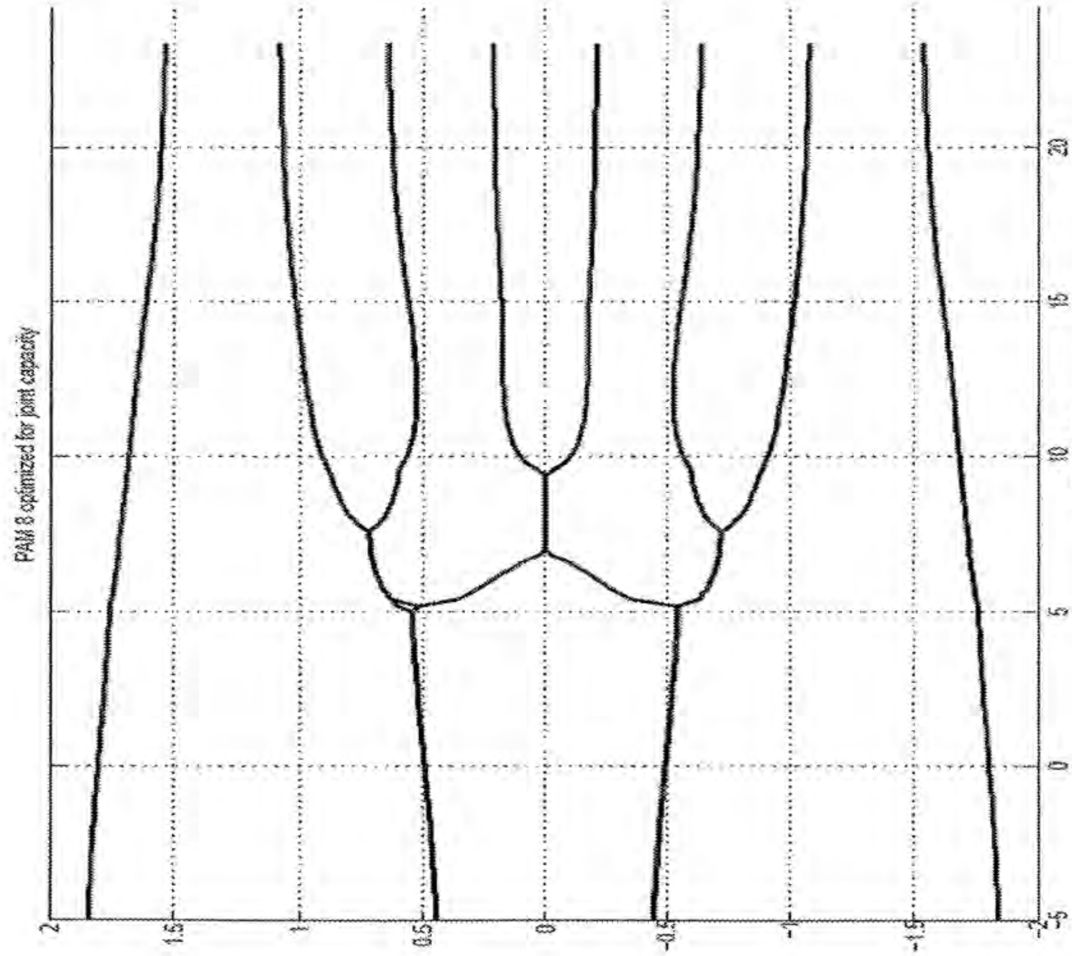

PAM 8 optimized for joint capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12b

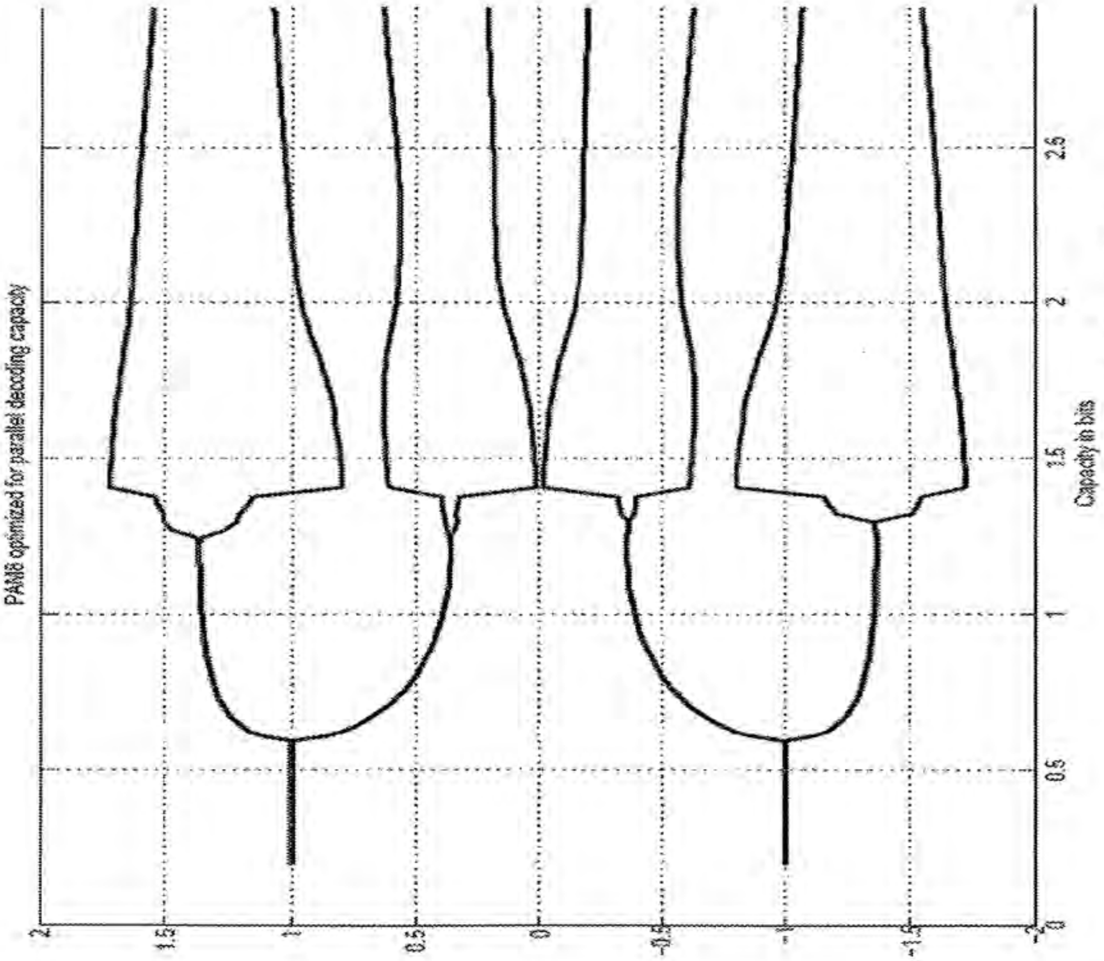

PAM8 optimized for parallel decoding capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12c

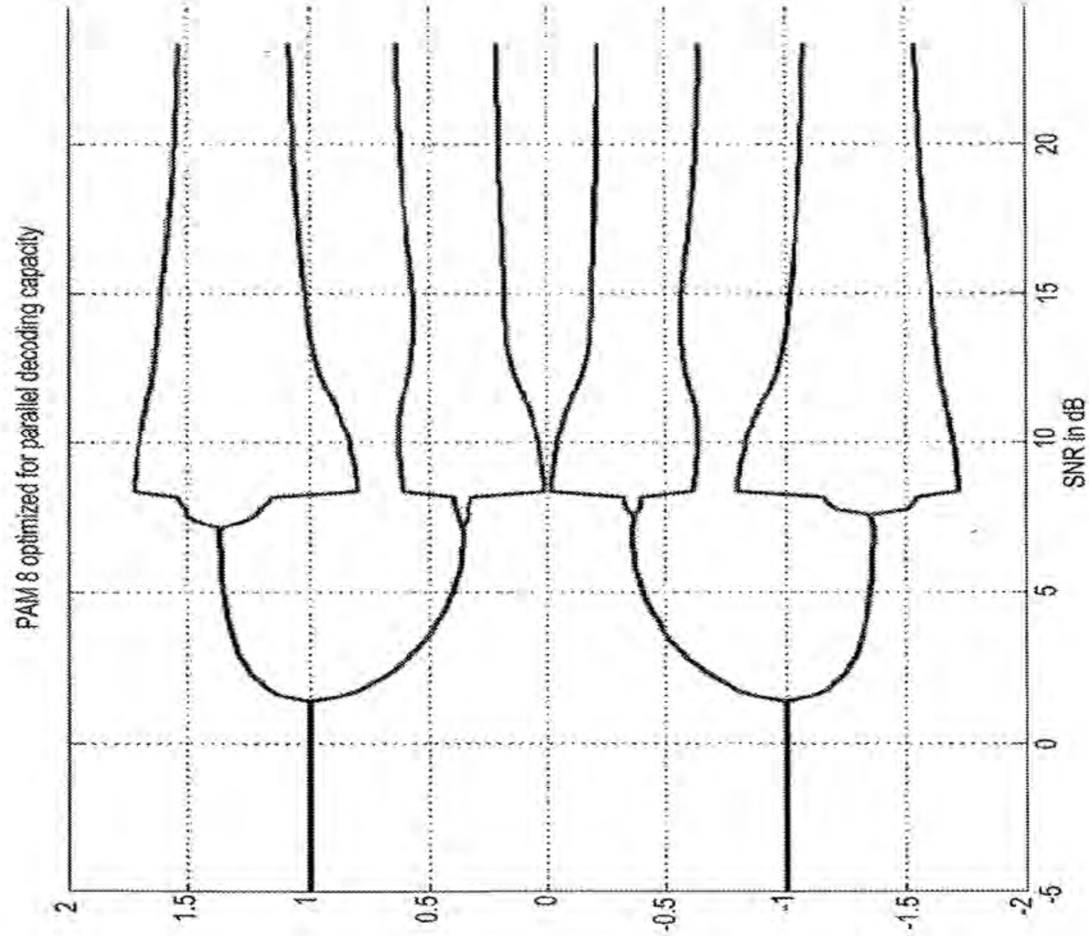

PAM 8 optimized for parallel decoding capacity
SNR in dB
-5
0
5
10
15
20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5,27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

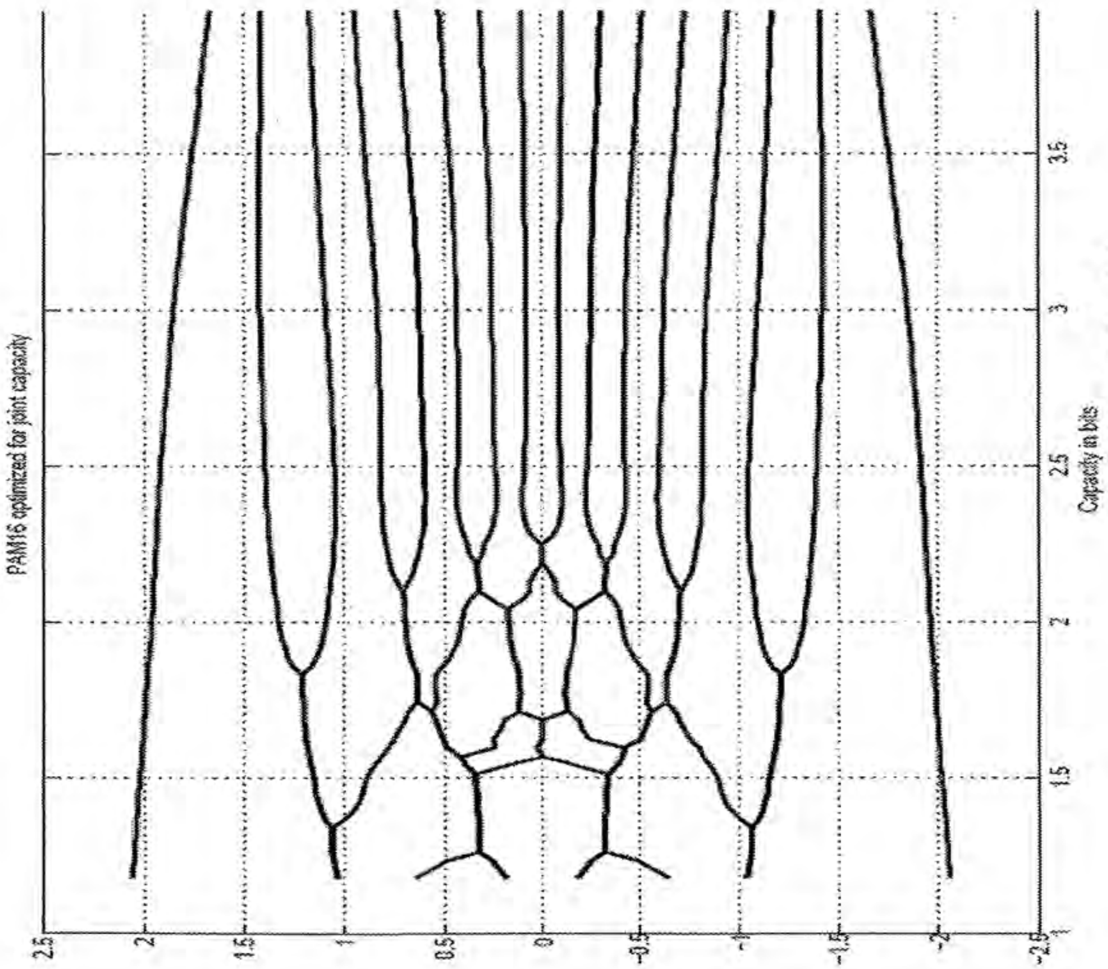
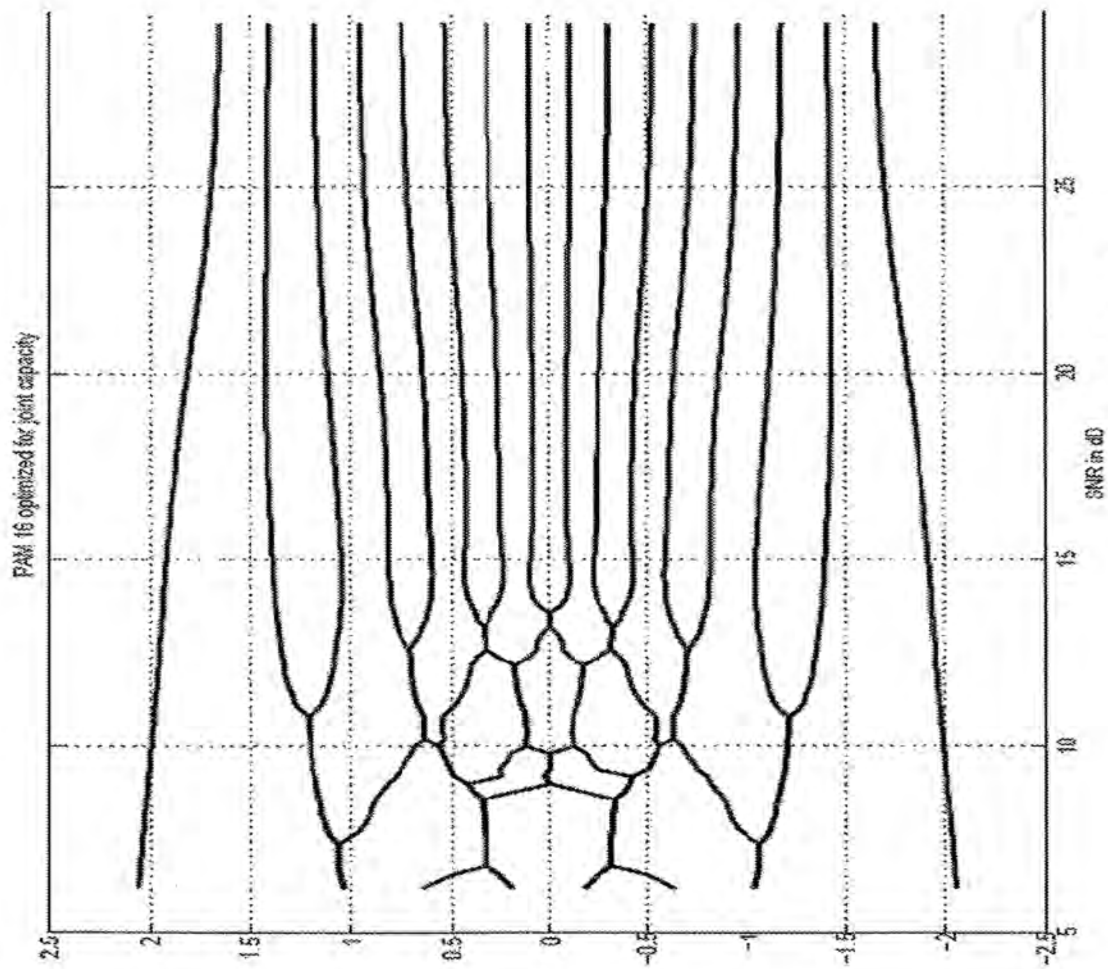

PAM16 optimized for joint capacity
Capacity in bits
1
1.5
2
2.5
3
3.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14a



PAM 16 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
5
10
15
20
25

FIG. 14b

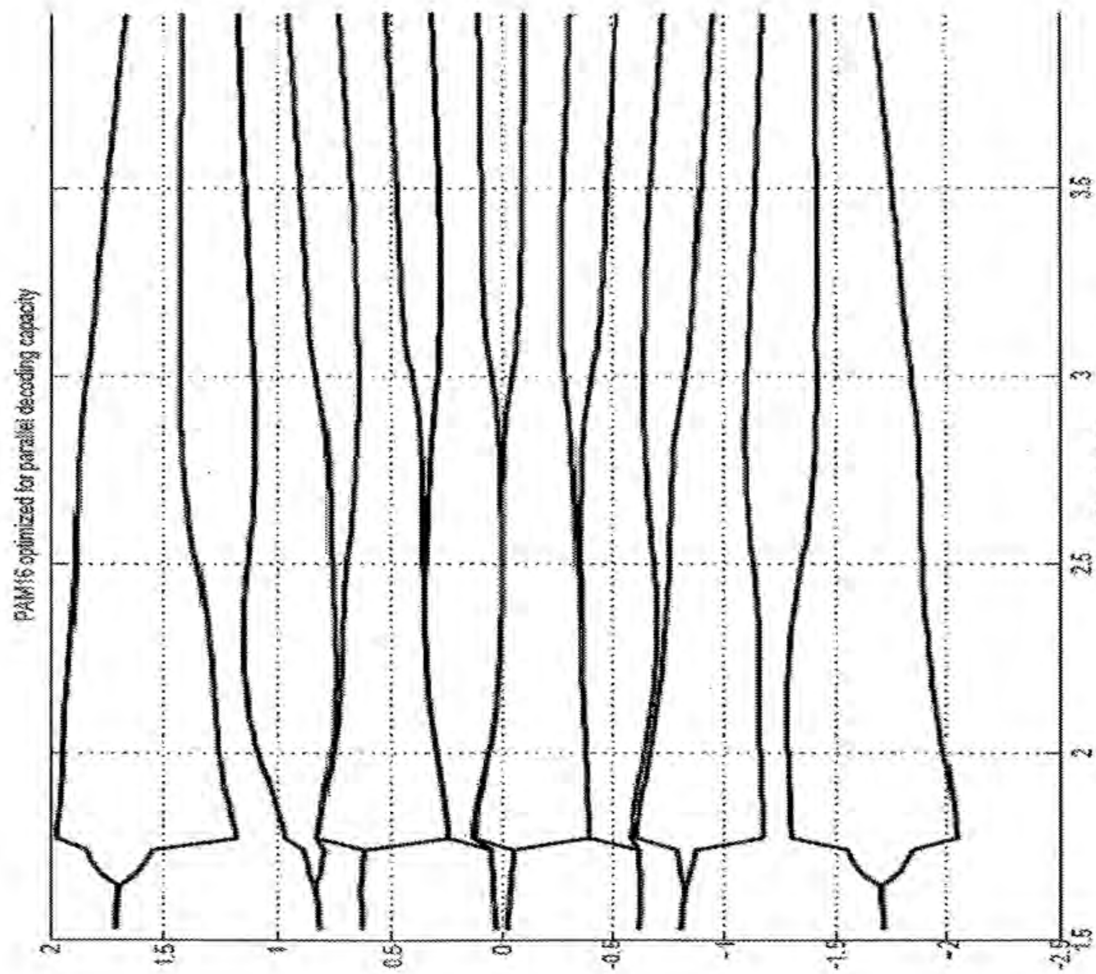

PAM16 optimized for parallel decoding capacity
Capacity in bits

FIG. 14c

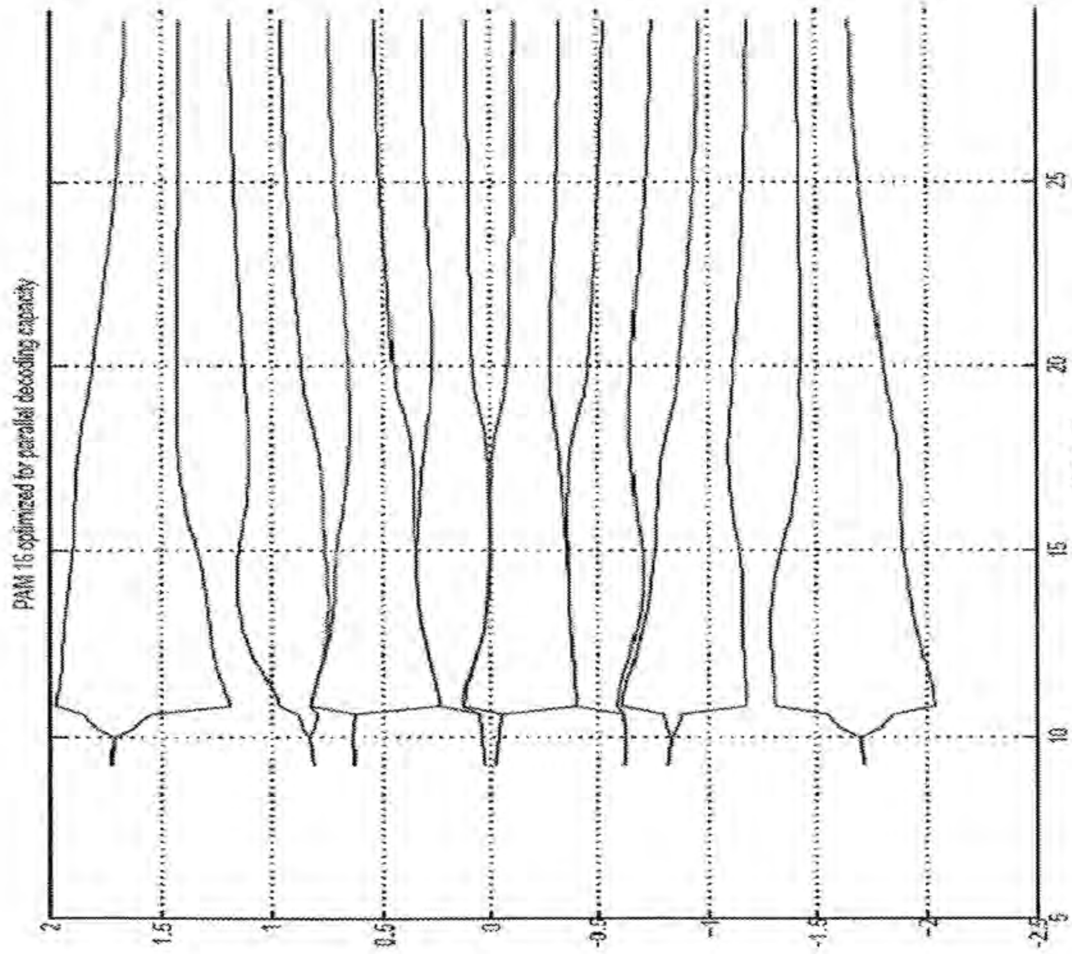

PAM 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

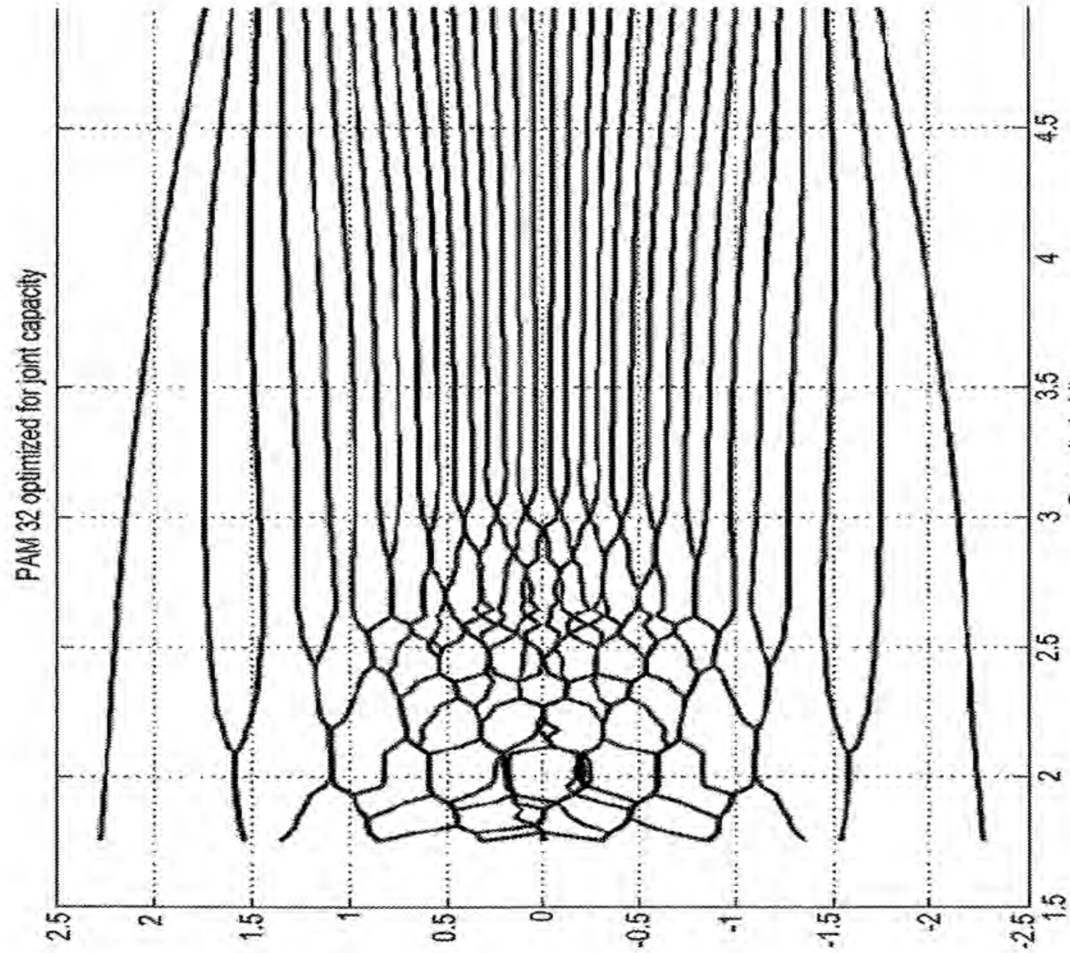
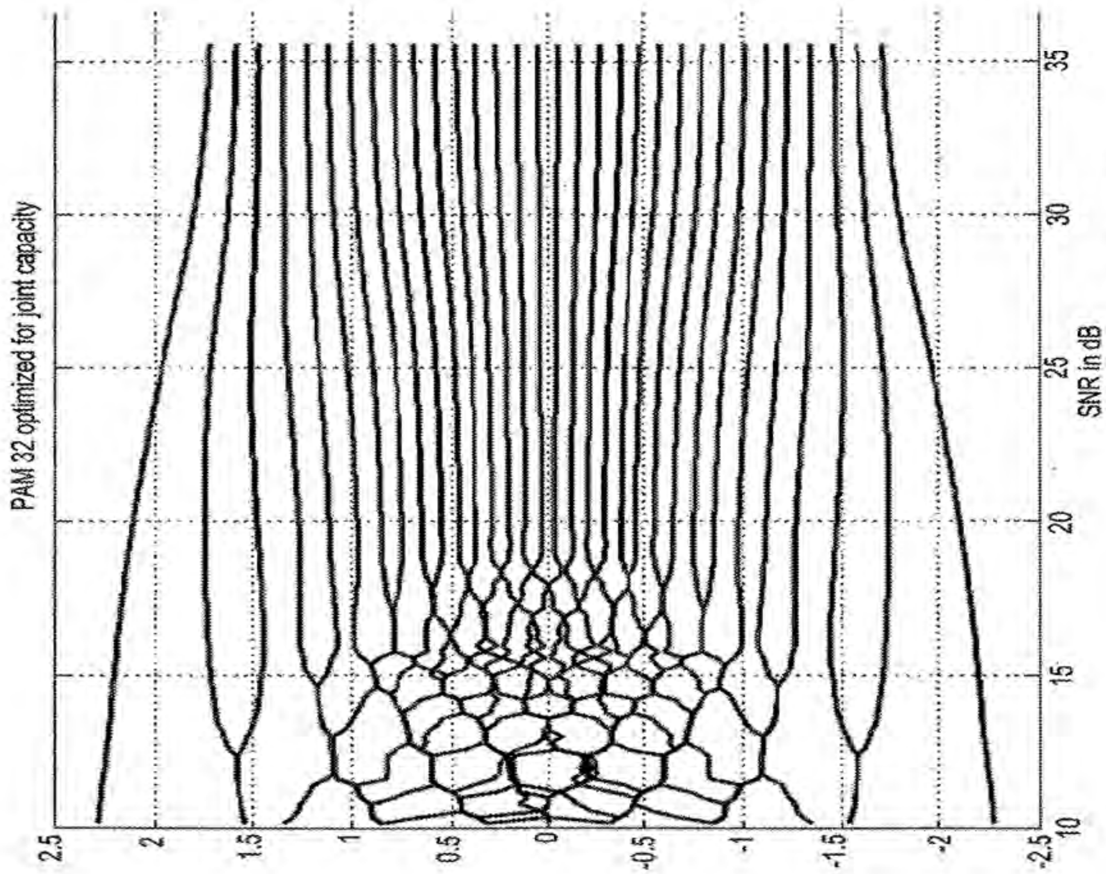

PAM 32 optimized for joint capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16a



PAM 32 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35

FIG. 16b

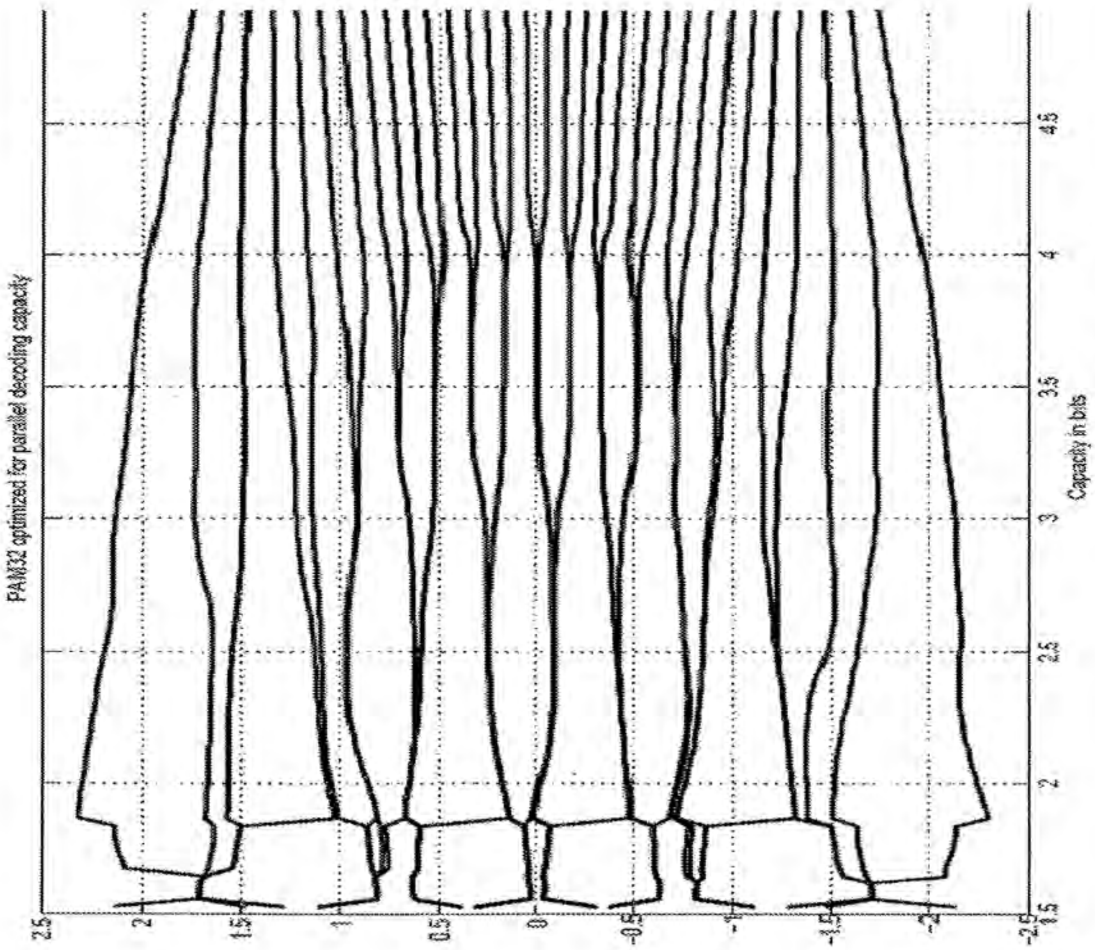

PAM32 optimized for parallel decoding capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16C

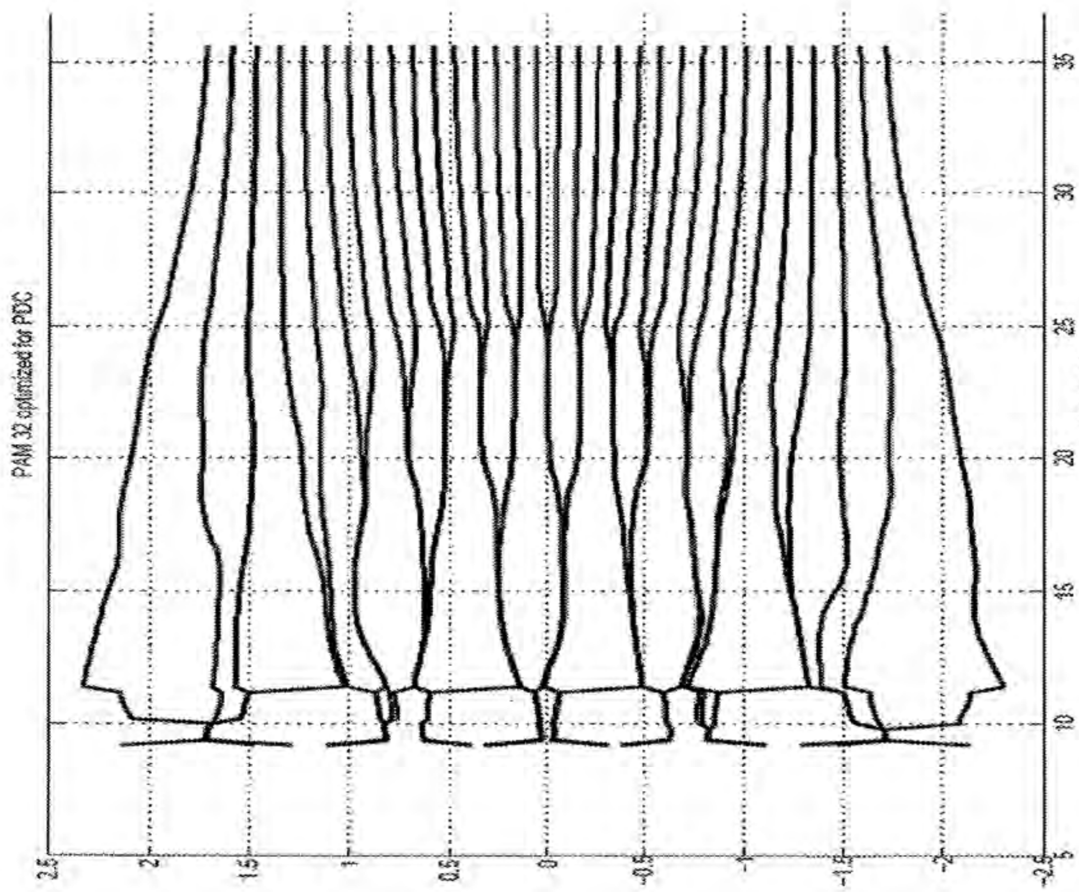

PAM 32 optimized for PDC
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25
30
35

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

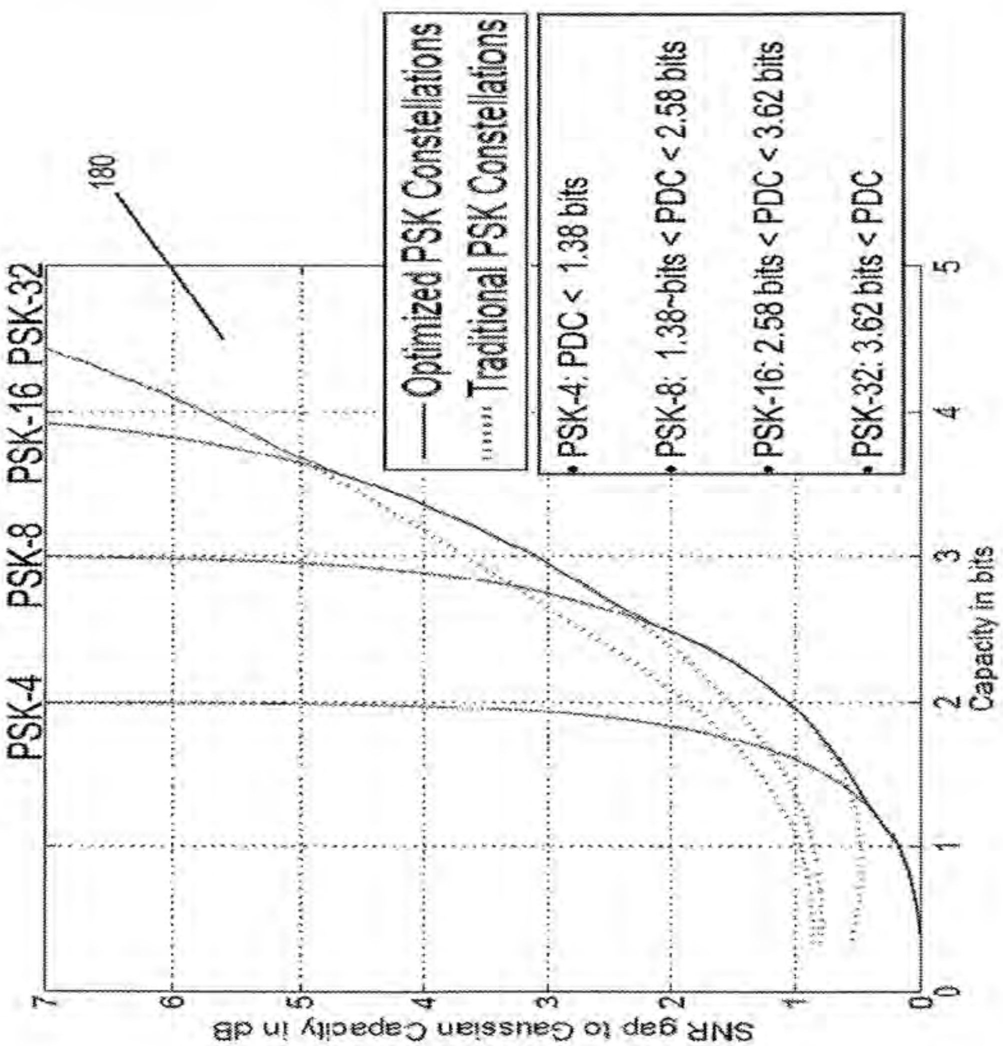

PSK-4
PSK-8
PSK-16
PSK-32
180
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38~bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

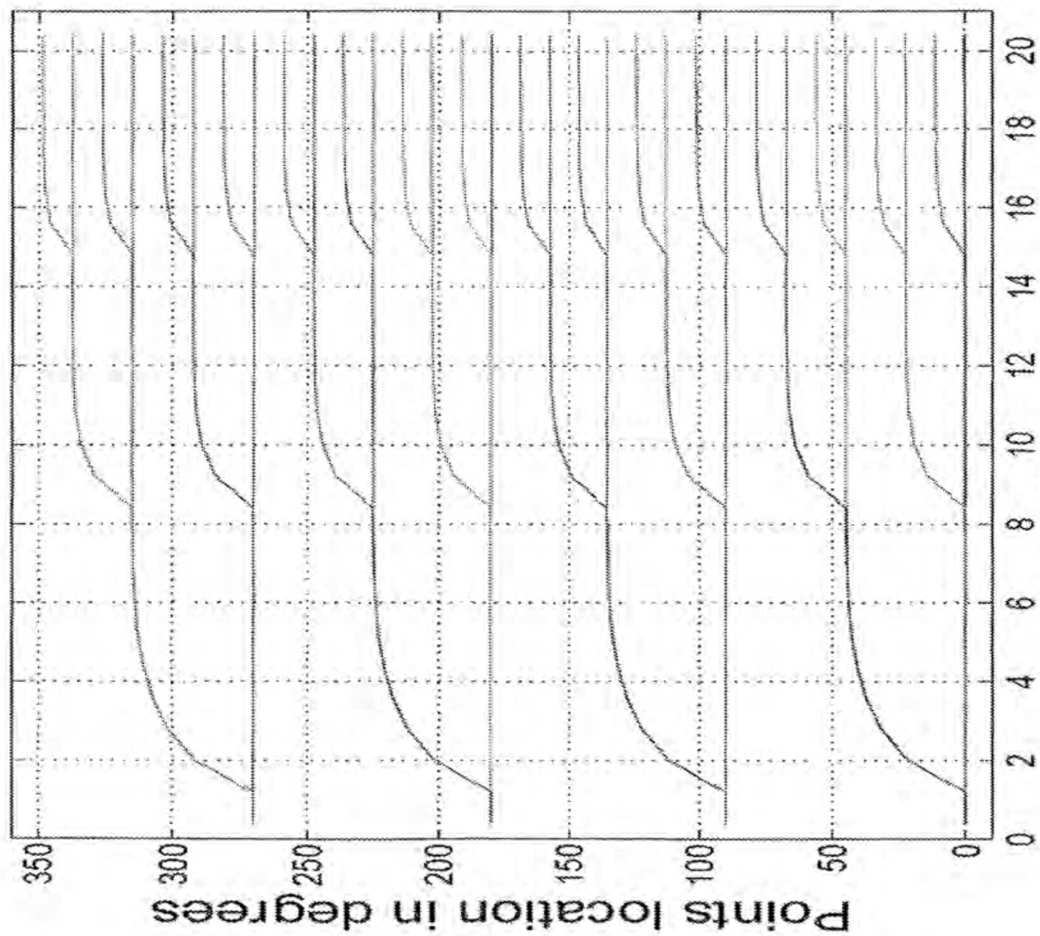

Points location in degrees
0
50
100
150
200
250
300
350
SNR in dB
0
2
4
6
8
10
12
14
16
18
20

FIG. 19

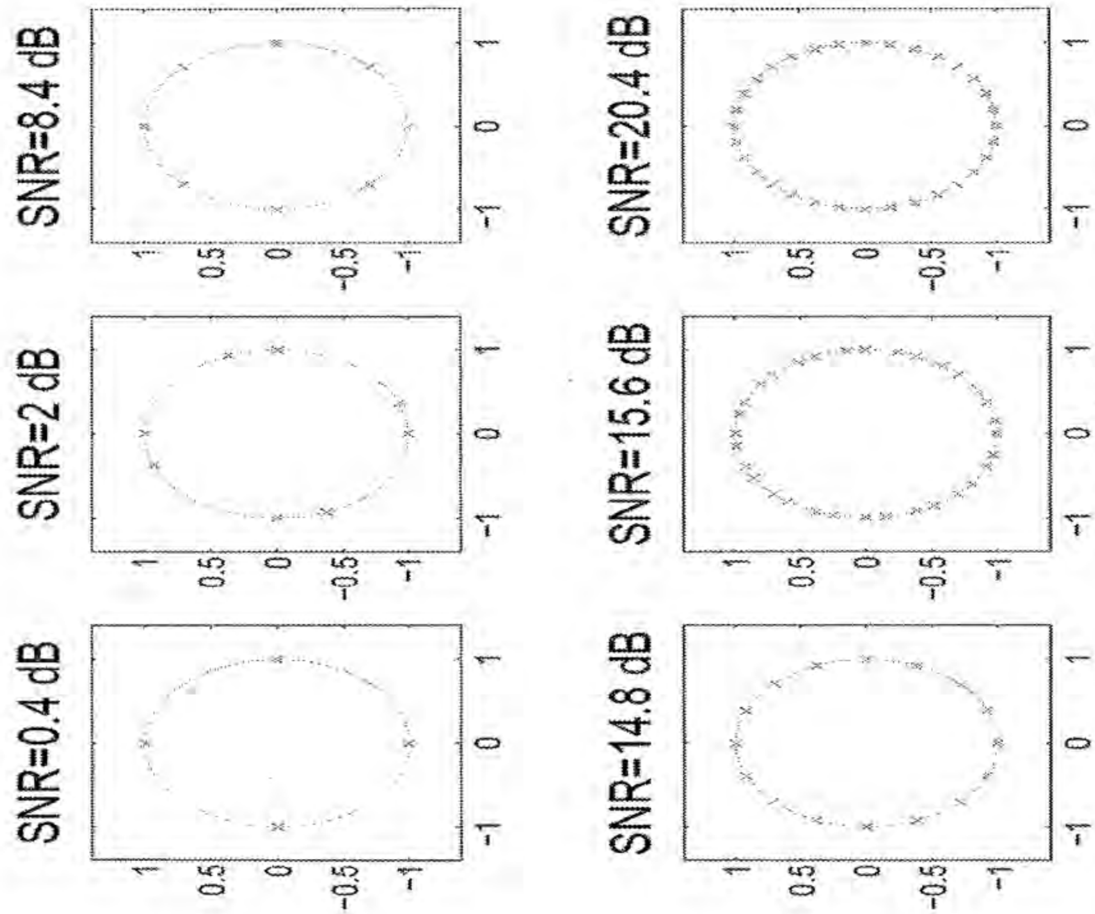


FIG. 20

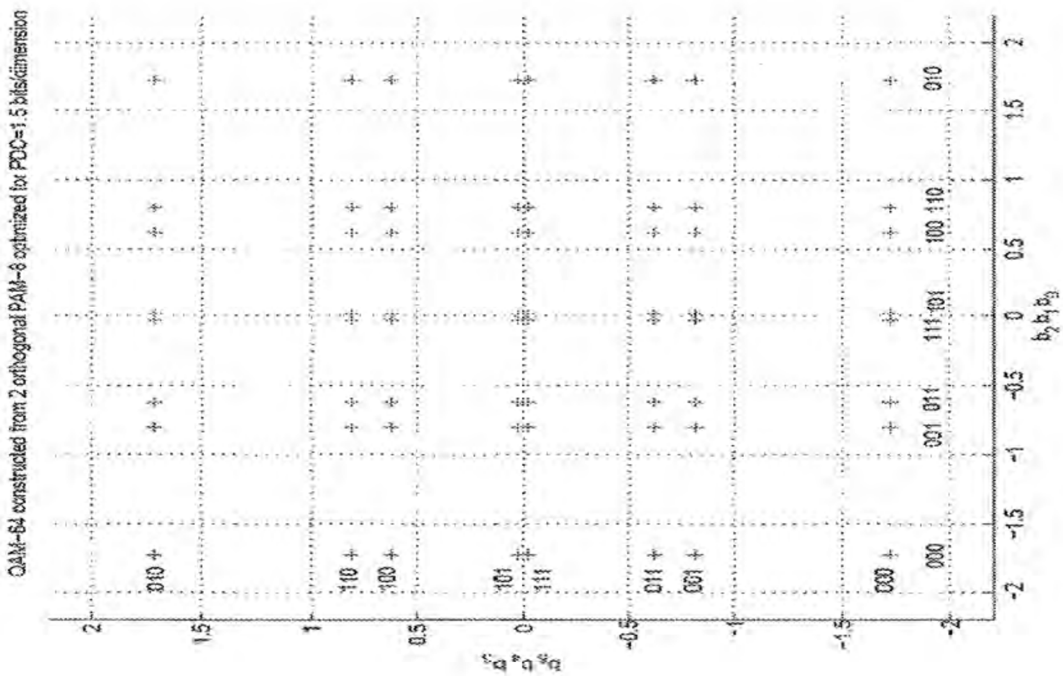

QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
$b_2 b_1 b_0$
$b_5 b_4 b_3$
010
110
100
101
111
011
001
000
001 011
111 101
100 110
010

FIG. 21

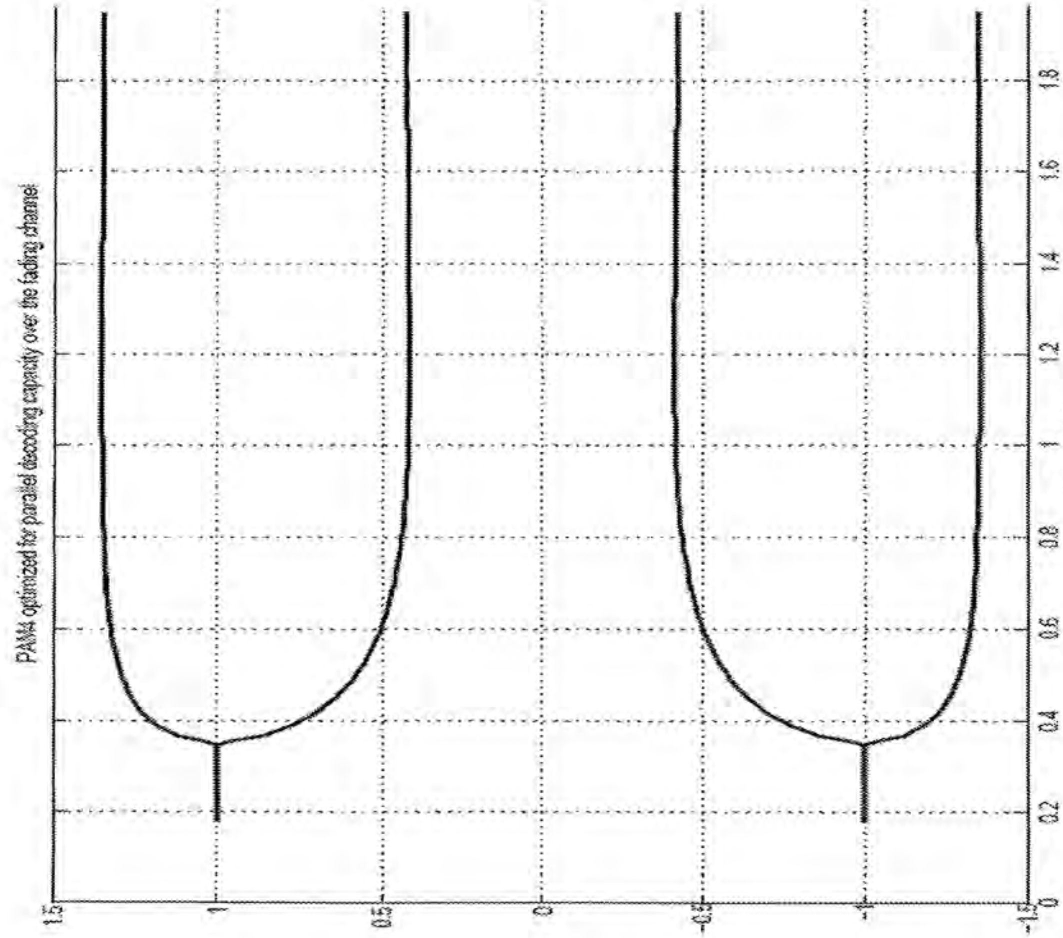

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 22a

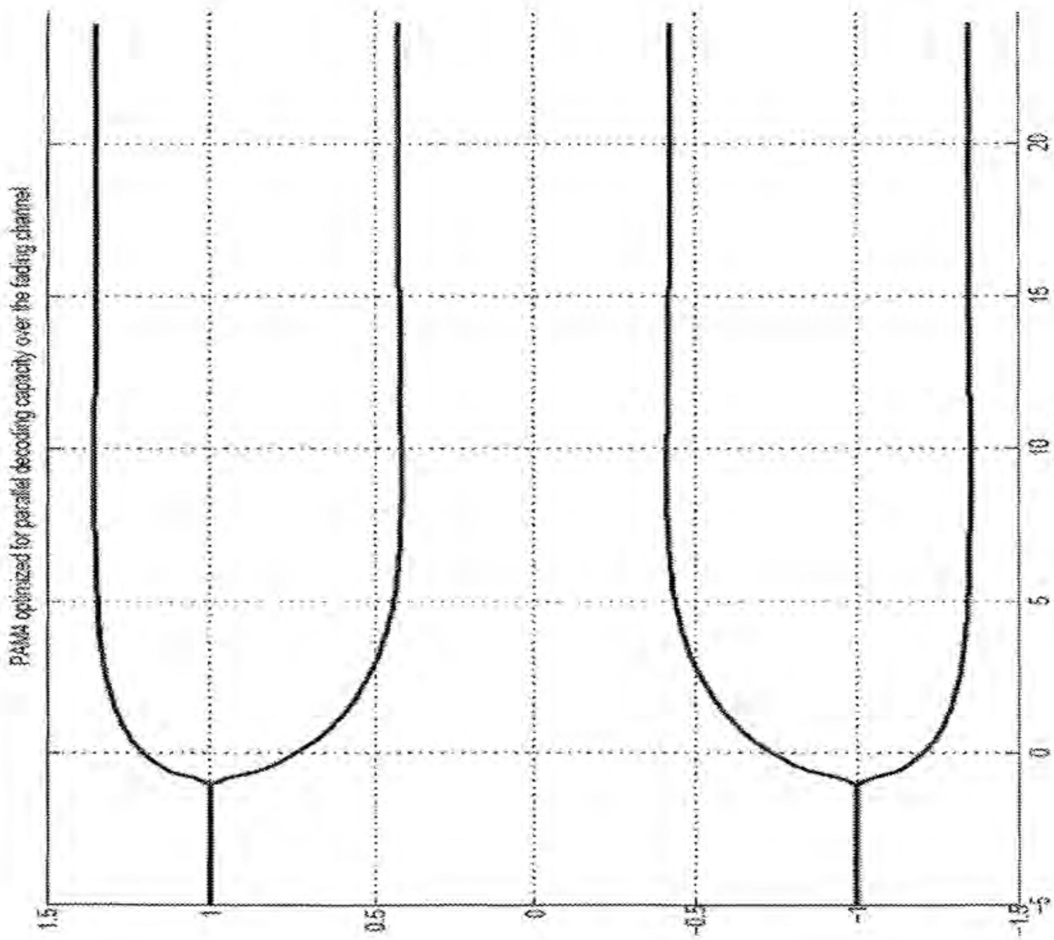

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 22b

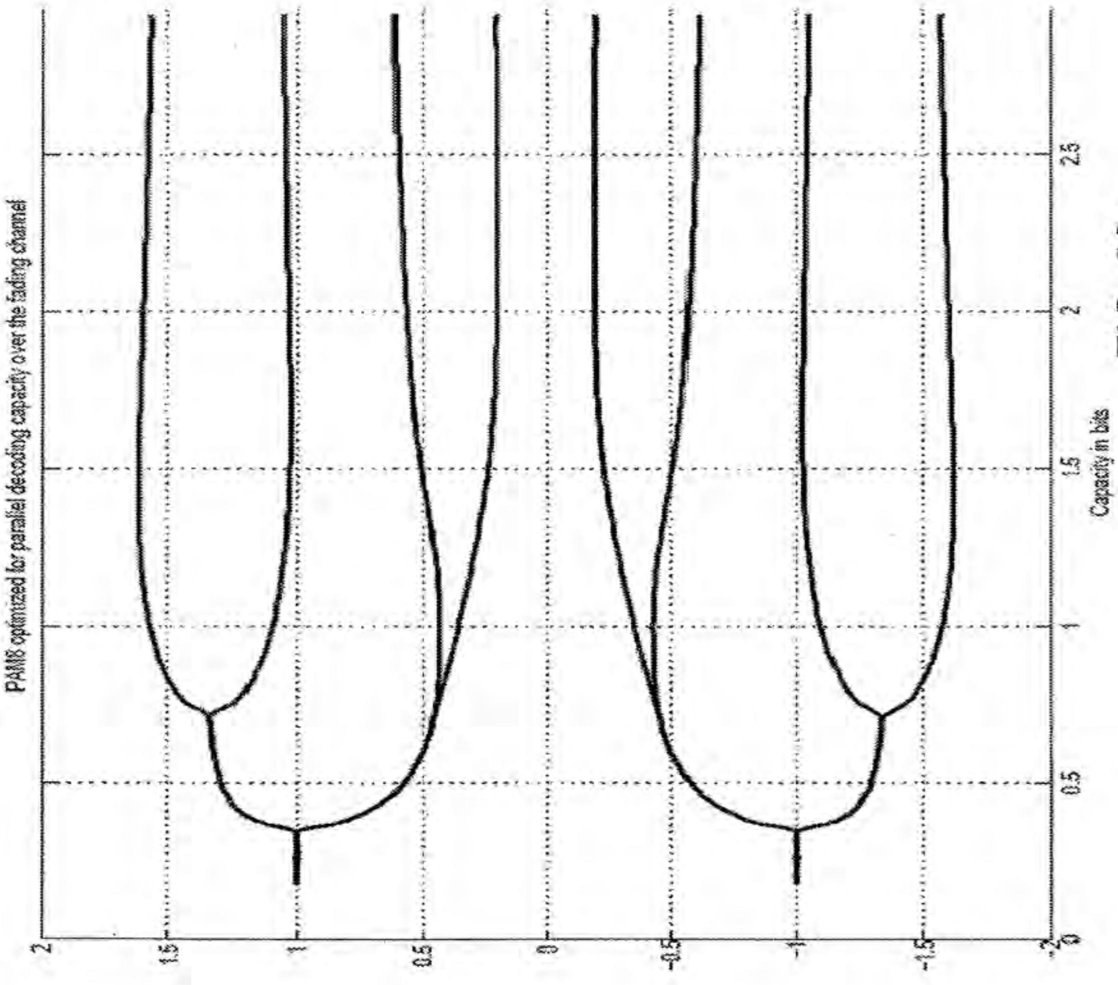

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23a

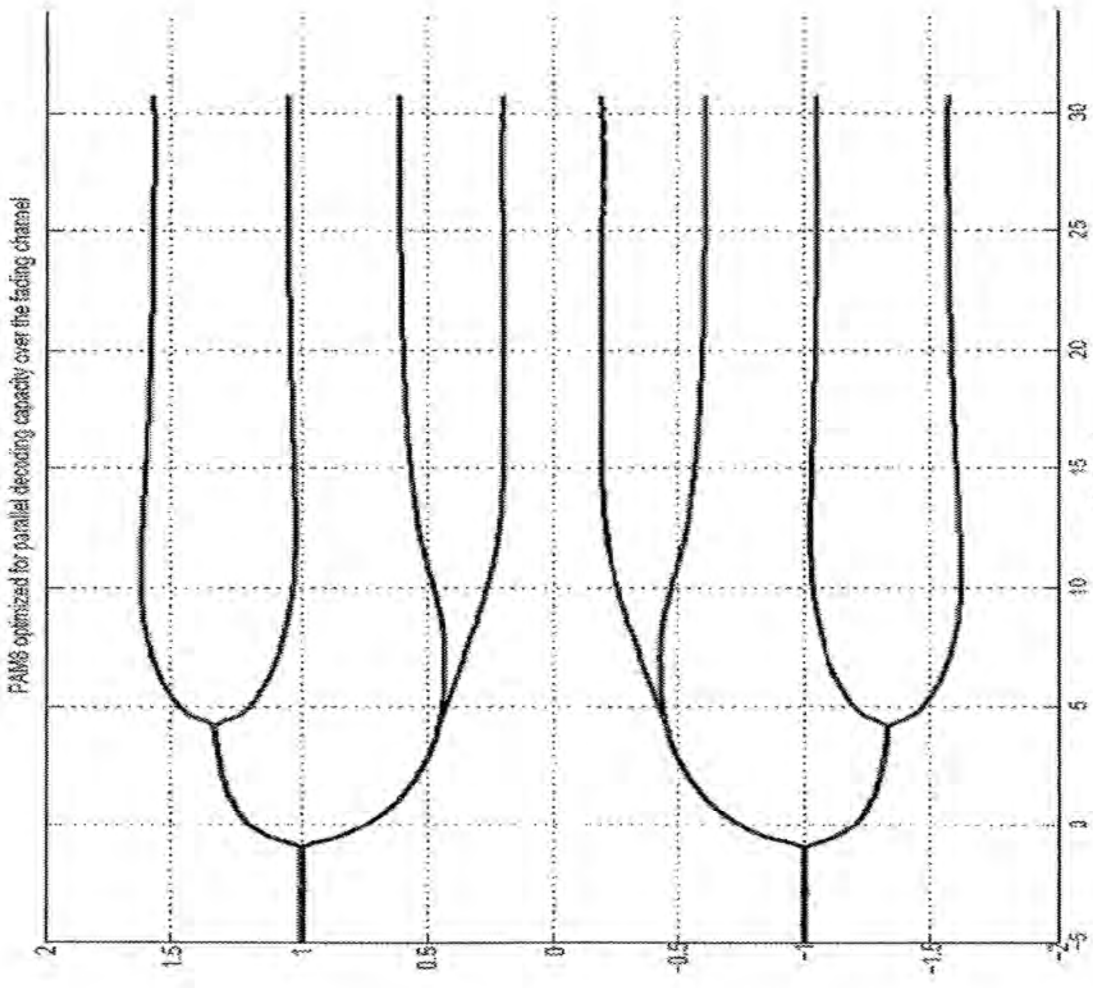

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20
25
30

FIG. 23b

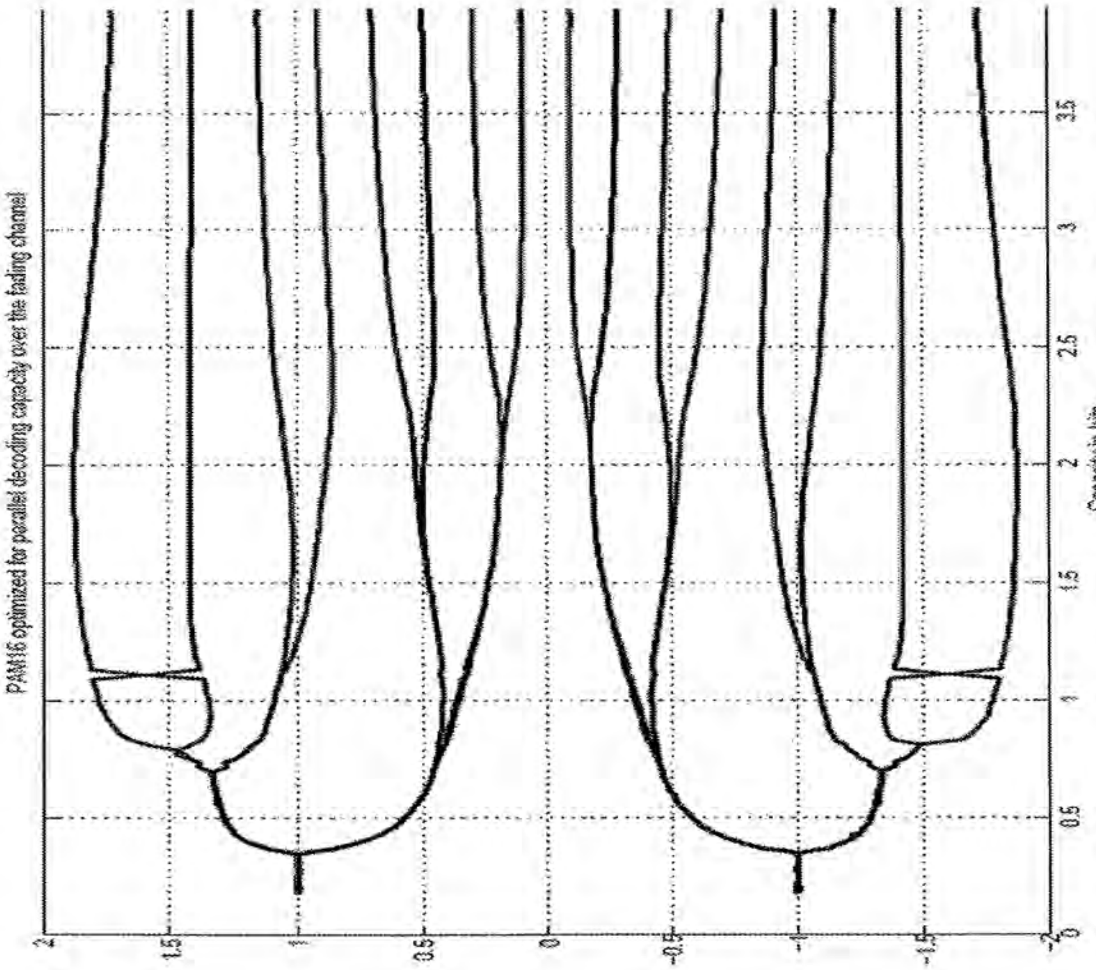

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24a

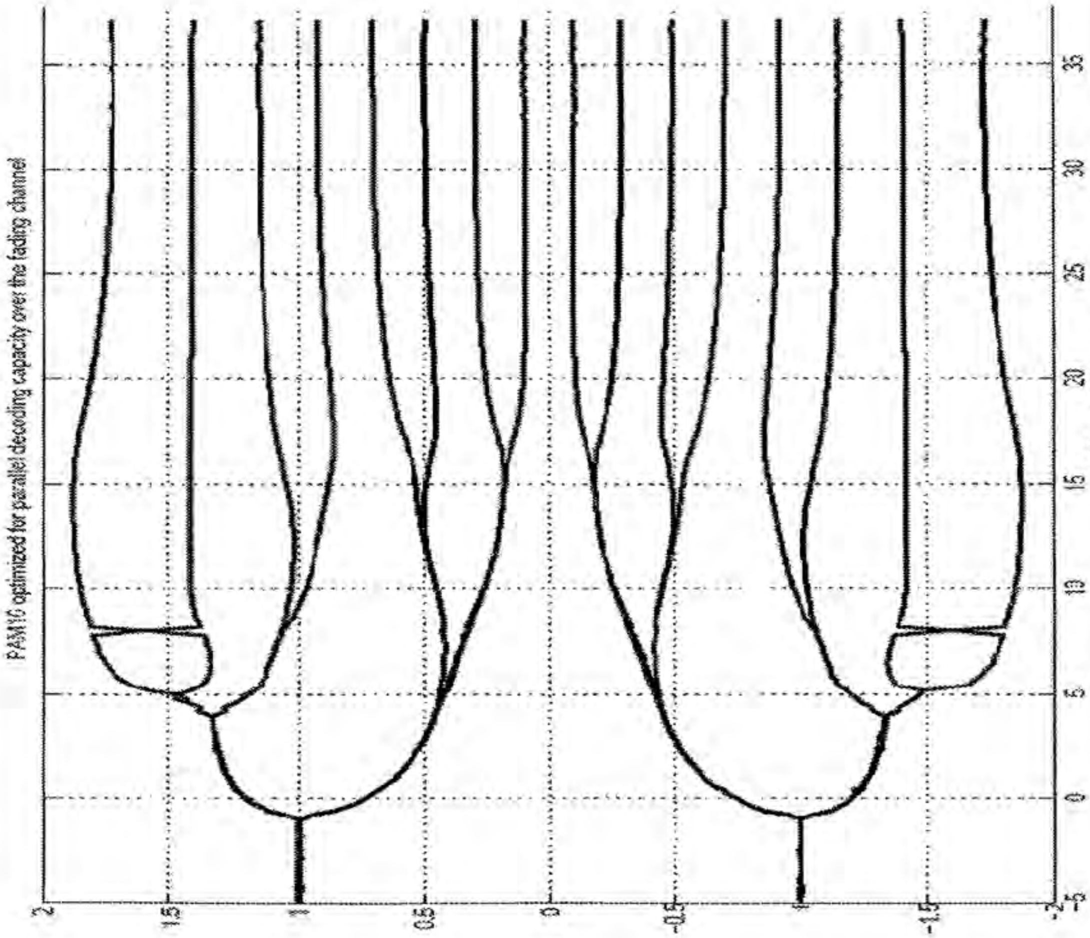

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
35
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24b

RECEIVERS INCORPORATING NON-UNIFORM CONSTELLATIONS WITH OVERLAPPING CONSTELLATION POINT LOCATIONS

RELATED APPLICATIONS

This application is a continuation of application Ser. No. 16/206,991 filed Nov. 30, 2018 and issued on Feb. 18, 2020 as U.S. Pat. No. 10,567,980, which application is a continuation of application Ser. No. 15/682,475 filed Aug. 21, 2017 and issued on Dec. 4, 2018 as U.S. Pat. No. 10,149,179, which application is a continuation of application Ser. No. 15/200,800 filed Jul. 1, 2016 and issued on Aug. 22, 2017 as U.S. Pat. No. 9,743,292, which application is a continuation of application Ser. No. 14/491,731 filed Sep. 19, 2014 and issued on Jul. 5, 2016 as U.S. Pat. No. 9,385,832, which application is a continuation of application Ser. No. 13/618,630 filed Sep. 14, 2012 and issued on Sep. 23, 2014 as U.S. Pat. No. 8,842,761, which application is a continuation of application Ser. No. 13/118,921 filed May 31, 2011 and issued on Sep. 18, 2012 as U.S. Pat. No. 8,270,511, which application is a continuation of application Ser. No. 12/156,989 filed Jun. 5, 2008 and issued on Jul. 12, 2011 as U.S. Pat. No. 7,978,777, which application claimed priority to U.S. Provisional Application 60/933,319 filed Jun. 5, 2007, the disclosures of which are incorporated herein by reference.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite

and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

FIG. **1** is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

FIG. **2** is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

FIG. **3** is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

FIG. **4***a* is a conceptual illustration of the joint capacity of a channel.

FIG. **4***b* is a conceptual illustration of the parallel decoding capacity of a channel.

FIG. **5** is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

FIG. **6***a* is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2, 4, 8, 16, 32.

FIG. **6***b* is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2, 4, 8, 16, 32.

FIG. **7** is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2, 4, 8, 16, 32 constellations.

FIG. **8***a* is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2, 4, 8, 16, 32 constellations.

FIG. **8***b* is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2, 4, 8, 16, 32 constellations.

FIG. **9** is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

FIGS. **10***a*-**10***d* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **11***a* and **11***b* are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

FIGS. **12***a*-**12***d* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **13***a* and **13***b* are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

FIGS. **14***a*-**14***d* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **15***a* and **15***b* are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

FIGS. **16***a*-**16***d* are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **17***a* and **17***b* are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

FIG. **18** is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

FIG. **19** is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

FIG. **20** is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

FIG. **21** illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

FIGS. **22***a* and **22***b* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **23***a* and **23***b* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **24***a* and **24***b* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

Communication Systems

A communication system in accordance with an embodiment of the invention is shown in FIG. 1. The communication system **10** includes a source **12** that provides user bits to a transmitter **14**. The transmitter transmits symbols over a channel to a receiver **16** using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

A transmitter in accordance with an embodiment of the invention is shown in FIG. 2. The transmitter **14** includes a coder **20** that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper **22** is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator **24**, which modulates the symbols for transmission via the channel.

A receiver in accordance with an embodiment of the invention is illustrated in FIG. 3. The receiver **16** includes a demodulator **30** that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper **32** provides the likelihoods to a decoder **34** that decodes the encoded bit stream to provide a sequence of received bits to a sink.

Geometrically Shaped Constellations

Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

Selection of a Geometrically Shaped Constellation

Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

Upgrading Existing Communication Systems

A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

Joint Capacity and Parallel Decoding Capacity

A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. 4*a*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **40**. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{l-1} I(X_i; Y)$$

where X_i is the ith bit of the l-bits transmitted symbol, and Y is the received symbol, and I(A;B) denotes the mutual information between random variables A and B.

Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4*b*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **42**. The joint capacity of the channel is given by:

$$C_{JOINT}=I(X;Y)$$

Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

Selecting a Constellation Having an Optimal Capacity

Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. **5**. The process **50** commences with the selection (**52**) of an appropriate constellation size M and a desired capacity per dimension r_I. In the illustrated embodiment, the process involves a check (**52**) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known a priori. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (**56**) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

The SNR at which the optimized M-ary constellation provides the desired capacity per dimension r_I (SNR_{out}) is determined (**57**). A determination (**58**) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (**55**). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

The process illustrated in FIG. **5** can maximize PD capacity or joint capacity of an M-ary constellation for a given SNR. Although the process illustrated in FIG. **5** shows selecting an M-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an M-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q), or with multi-stage decoding. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

Optimizing the Capacity of an M-Ary Constellation at a Given SNR

Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process **56** shown in FIG. **5**, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged.

The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. 5.

Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

Gains Achieved by Optimized Geometrically Spaced Constellations

The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G=\frac{1}{2}\log_2(1+SNR)$$

Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. **6***a* and **6***b*. FIG. **6***a* includes a chart **60** showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **62** exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. **6***b* includes a chart **64** showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **66** exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

In order to gain a better view of the differences between the curves shown in FIGS. **6***a* and **6***b* at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. **7**. It is interesting to note from the chart **70** in FIG. **7** that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

FIGS. **8***a* and **8***b* summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates).

The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. **8***a* shows the SNR gap for each constellation optimized for PD capacity. FIG. **8***b* shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

Referring to FIG. **8***a*, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. **9**. A chart **90** includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. **8***a*.

Capacity Optimized Pam Constellations

Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. **10***a*-**17***b*. The locus plots and design tables show PAM-4, 8, 16, 32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

In FIG. **10***a*, a locus plot **100** shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot **105** showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. **10***b*. In FIG. **10***c*. the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. **10***d* the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. **11***a* and FIG. **11***b* where vertical slices of FIGS. **10***ab* and **10***cd* are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities={0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4)=2$ bps, these design points represent systems with information code rates R={1/4, 3/8, 1/2, 5/8, 3/4} respectively.

FIGS. **12***ab* and **12***cd* present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. **13***a* and **13***b* provide slices from these plots at SNRs corresponding to achievable capacities r_f={0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate $R=r_f$ bps/$\log_2(8)$, resulting in R={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. **13***b* which shows a PD capacity optimized PAM-8 constellation optimized for SNR=9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. **8***a* and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (C_G) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

Similar information is presented in FIGS. **14***abcd*, and **15***ab* which provide loci plots and design tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. **16***abcd*, **17***ab* provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

Capacity Optimized PSK Constellations

Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart **180** comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. **18** where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4, 8, 16, 32 and for PD capacity optimized PSK-4, 8, 16, 32.

The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. **19**, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. **20**. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR=8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR=14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR=2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

We note now that the locus of points for PD optimized PSK-32 in FIG. **19** in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. **18** implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4, 8, 16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. **19**. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. **18**.

Adaptive Rate Design

In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation/code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper/decoder configuration in order to match the new transmit-side configuration.

Conversely, the receiver could implement a quality of performance based optimized constellation/code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation/code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper/decoder modules.

Geometrically Shaped QAM Constellations

Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

In FIG. **21** we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. **13***b*). The label-point pairs in this PAM-8 constellation are {(000, −1.72), (001, −0.81), (010, 1.72), (011, −0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, −0.02)}. Examination of FIG. **21** shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8×8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-Dimensional Constellation Optimization

Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

Capacity Optimized Constellations for Fading Channels

Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y=a(t)\cdot X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and a(t) is the fading distribution, which is a function of time.

In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

Many fading channels follow a Rayleigh distribution. FIGS. **22*a*-24*b*** are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

What is claimed is:

1. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a symbol constellation that includes constellation points at a plurality of unique point locations, where:

the plurality of unique point locations are unequally spaced;

the constellation points each have a location and a different label; and

the locations of at least two of the constellation points are the same; and

a decoder, coupled to the demapper, capable of using the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code.

2. The communication system of claim **1**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations;

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where unequally spaced symbol constellations of the first type include at least two constellation points having locations that are the same and different labels;

the receiver is capable of selecting an LDPC code rate and the unequally spaced symbol constellation as a pair from a plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs; and

each of the plurality of unequally spaced symbol constellations is only included in one of the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs.

3. The communication system of claim **2**, wherein the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs includes:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point unequally spaced symbol constellation of the first type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the first type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the first type.

4. The communication system of claim **3**, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point unequally spaced symbol constellation of the first type;

equal to or less than 15.42 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point unequally spaced symbol constellation of the first type; and

equal to or less than 21.52 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellations of the first type.

5. The communication system of claim **4**, wherein:

the plurality of unequally spaced symbol constellations include a plurality of unequally spaced symbol constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within an unequally spaced symbol constellation of the second type share locations that are the same;

the demapper is also capable of determining likelihoods using the demodulated signal and an unequally spaced symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs include:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11 dB and 9.25 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

between 9.00 dB and 15.93 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

between 18.72 dB and 22.13 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

between 24.79 dB and 28.20 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type.

6. The communication system of claim **1**, wherein:

the non uniform symbol constellation is selected from a plurality of unequally spaced symbol constellations; and

each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of unequally spaced symbol constellations is capable of providing greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of unequally spaced symbol constellations when operated at the same SNR.

7. The communication system of claim **6**, wherein the symbol constellation operating SNR is a channel SNR where the receiver is capable of using the respective unequally spaced symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

8. The communication system of claim **1**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations; and

each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

9. The communication system of claim **1**, further comprising a transmitter capable of transmitting signals via the communication channel, where the transmitter comprises:

a coder capable of receiving bits and outputting encoded bits using a Low Density Parity Check (LDPC) code;

a mapper, coupled to the coder, capable of mapping the encoded bits to symbols in the symbol constellation; and

a modulator, coupled to the mapper, capable of generating a signal for transmission via the communication channel based upon symbols selected by the mapper.

10. The communication system of claim **1**, wherein:

the symbol constellation is selected from a plurality of symbol constellations each having a same number of constellation points;

a first of the plurality of symbol constellations includes constellation points at a first set of unique point locations that are unequally spaced;

a second of the plurality of symbol constellations includes constellation points at a second set of unique point locations that are unequally spaced; and

the number of unique point locations in the first set of unique point locations is different from the number of unique point locations in the second set of unique point locations.

11. A communication system, comprising:

a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator that demodulates a received signal into a demodulated signal;

a demapper that determines likelihoods using the demodulated signal and a symbol constellation that includes constellation points at a plurality of unique point locations, where:

the plurality of unique point locations are unequally spaced;

the constellation points each have a location and a different label; and

the locations of at least two of the constellation points are the same; and

a decoder that uses the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code;

wherein the demapper is interposed between the demodulator and the decoder and the demapper receives information from the demodulator and provides information to the decoder.

12. The communication system of claim **11**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations;

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where unequally spaced symbol constellations of the first type include at least two constellation points having locations that are the same and different labels;

the receiver selects an LDPC code rate and the unequally spaced symbol constellation as a pair from a plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs; and

each of the plurality of unequally spaced symbol constellations is only included in one of the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs.

13. The communication system of claim **12**, wherein the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs includes:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point unequally spaced symbol constellation of the first type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the first type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the first type.

14. The communication system of claim **13**, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point unequally spaced symbol constellation of the first type;

equal to or less than 15.42 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point unequally spaced symbol constellation of the first type; and

equal to or less than 21.52 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellations of the first type.

15. The communication system of claim **14**, wherein:

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within an unequally spaced symbol constellation of the second type have locations that are the same;

the demapper also determines likelihoods using the demodulated signal and an unequally spaced symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs includes:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11 dB and 9.25 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

between 9.00 dB and 15.93 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

between 18.72 dB and 22.13 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

between 24.79 dB and 28.20 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type.

16. The communication system of claim **11**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations; and

each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of unequally spaced symbol constellations provides greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of unequally spaced symbol constellations when operated at the same SNR.

17. The communication system of claim **16**, wherein the symbol constellation operating SNR is a channel SNR where the receiver uses the respective unequally spaced symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

18. The communication system of claim **11**, wherein

the symbol constellation is selected from a plurality of unequally spaced symbol constellations; and each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

19. The communication system of claim **11**, further comprising a transmitter that transmits signals via the communication channel, where the transmitter comprises:

a coder that receives bits and outputs encoded bits using a Low Density Parity Check (LDPC) code;

a mapper that maps the encoded bits to symbols in the symbol constellation; and

a modulator that generates a signal for transmission via the communication channel based upon symbols selected by the mapper;

wherein the mapper is interposed between the modulator and the coder and the mapper receives information from the coder and provides information to the modulator.

20. The communication system of claim **11**, wherein:

the symbol constellation is selected from a plurality of symbol constellations each having a same number of constellation points;

a first of the plurality of symbol constellations includes constellation points at a first set of unique point locations that are unequally spaced;

a second of the plurality of symbol constellations includes constellation points at a second set of unique point locations that are unequally spaced; and

the number of unique point locations in the first set of unique point locations is different from the number of unique point locations in the second set of unique point locations.

21. A communication system, comprising a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver uses a symbol constellation to transform the received signals into received bits, and the symbol constellation includes constellation points at a plurality of unique point locations, where:

the plurality of unique point locations are unequally spaced;

the constellation points each have a location and a different label; and

the locations of at least two of the constellation points are the same.

22. The communication system of claim **21**, further comprising a transmitter that transmits signals via the communication channel, where the transmitter uses the symbol constellation to transform encoded bits into the transmitted signals.

23. The communication system of claim **21**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations;

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where unequally spaced symbol constellations of the first type include at least two constellation points having identical locations and different labels;

the receiver selects an LDPC code rate and the unequally spaced symbol constellation as a pair from a plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs; and

each of the plurality of unequally spaced symbol constellations is only included in one of the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs.

24. The communication system of claim **23**, wherein the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs includes:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the first type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the first type.

25. The communication system of claim **24**, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point unequally spaced symbol constellation of the first type;

equal to or less than 15.42 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point unequally spaced symbol constellation of the first type; and

equal to or less than 21.52 dB while using an LDPC code rate and unequally spaced symbol constellation pair that includes an LDPC code rate below 7/10 and a one-thousand-twenty-four-point unequally spaced symbol constellations of the first type.

26. The communication system of claim **25**, wherein:

the plurality of unequally spaced symbol constellations includes a plurality of unequally spaced symbol constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within an unequally spaced symbol constellation of the second type have locations that are the same;

the receiver also transforms received signals into received bits using an unequally spaced symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and unequally spaced symbol constellation pairs include:

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11 dB and 9.25 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point unequally spaced symbol constellation of the second type;

between 9.00 dB and 15.93 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point unequally spaced symbol constellation of the second type;

between 18.72 dB and 22.13 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point unequally spaced symbol constellation of the second type; and

between 24.79 dB and 28.20 dB while using at least one of the multiple LDPC code rate and unequally spaced symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point unequally spaced symbol constellation of the second type.

27. The communication system of claim **22**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations; and

each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of unequally spaced symbol constellations provides greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of unequally spaced symbol constellations when operated at the same SNR.

28. The communication system of claim **27**, wherein the symbol constellation operating SNR is a channel SNR where the receiver uses the respective unequally spaced symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

29. The communication system of claim **22**, wherein:

the symbol constellation is selected from a plurality of unequally spaced symbol constellations; and

each of the plurality of unequally spaced symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

30. The communication system of claim **22**, wherein:

the symbol constellation is selected from a plurality of symbol constellations each having a same number of constellation points;

a first of the plurality of symbol constellations includes constellation points at a first set of unique point locations that are unequally spaced;

a second of the plurality of symbol constellations includes constellation points at a second set of unique point locations that are unequally spaced; and

the number of unique point locations in the first set of unique point locations is different from the number of unique point locations in the second set of unique point locations.

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(12) **United States Patent**
Barsoum et al.

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(54) **METHODS AND APPARATUSES FOR SIGNALING WITH GEOMETRIC CONSTELLATIONS**

(71) Applicant: **Constellation Designs, LLC**, Anaheim, CA (US)

(72) Inventors: **Maged F. Barsoum**, San Jose, CA (US); **Christopher R. Jones**, Pacific Palisades, CA (US)

(73) Assignee: **Constellation Designs, LLC**, Anaheim, CA (US)

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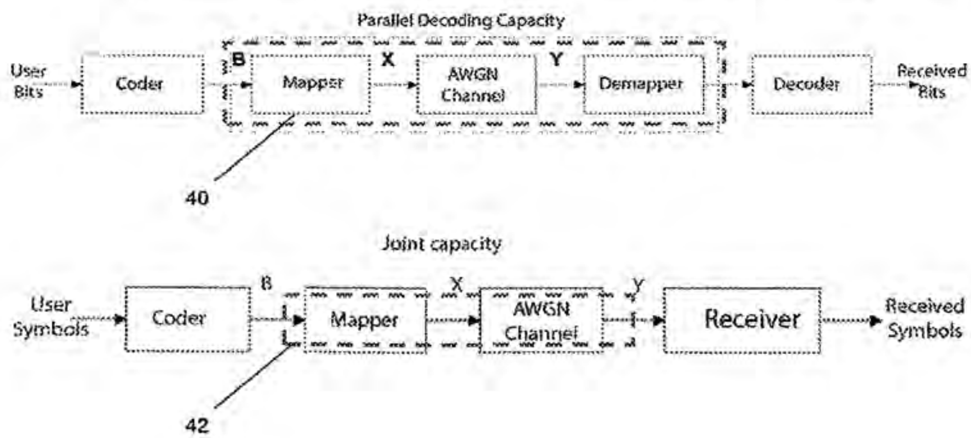
Primary Examiner — Tesfaldet Bocure
(74) *Attorney, Agent, or Firm* — KPPB LLP

(57) **ABSTRACT**

Communication systems are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In many embodiments, the communication systems use specific geometric constellations that are capacity optimized at a specific SNR. In addition, ranges within which the constellation points of a capacity optimized constellation can be perturbed and are still likely to achieve a given percentage of the optimal capacity increase compared to a constellation that maximizes d_{min} are also described. Capacity measures that are used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

47 Claims, 167 Drawing Sheets



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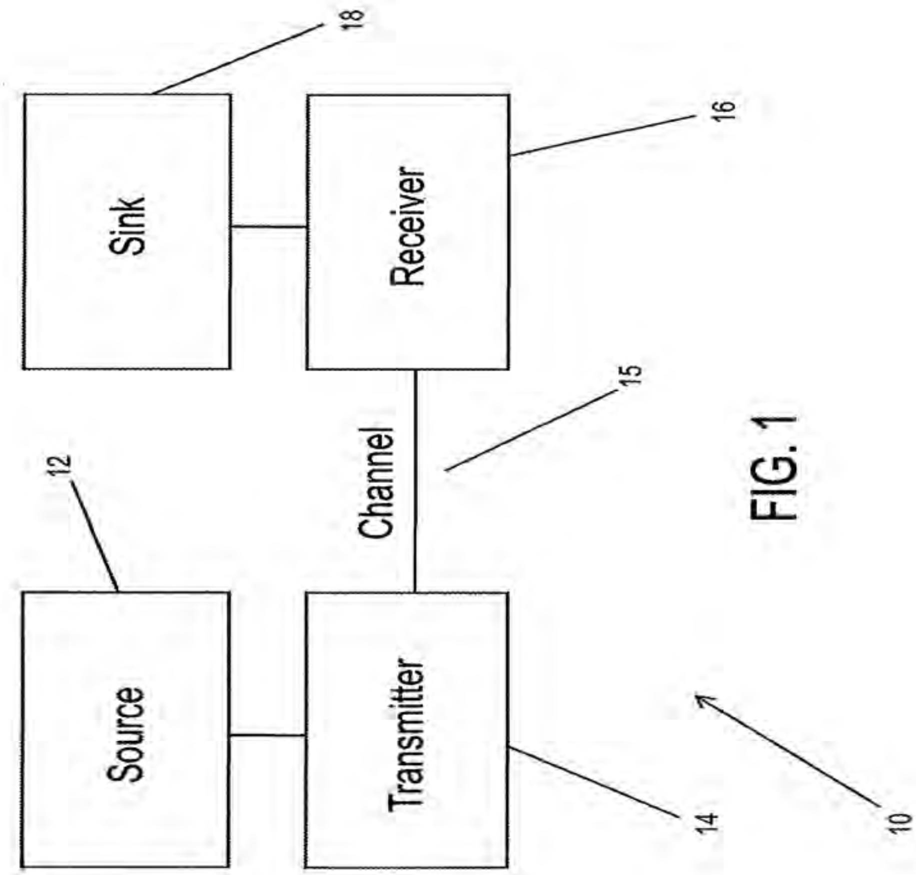


FIG. 1

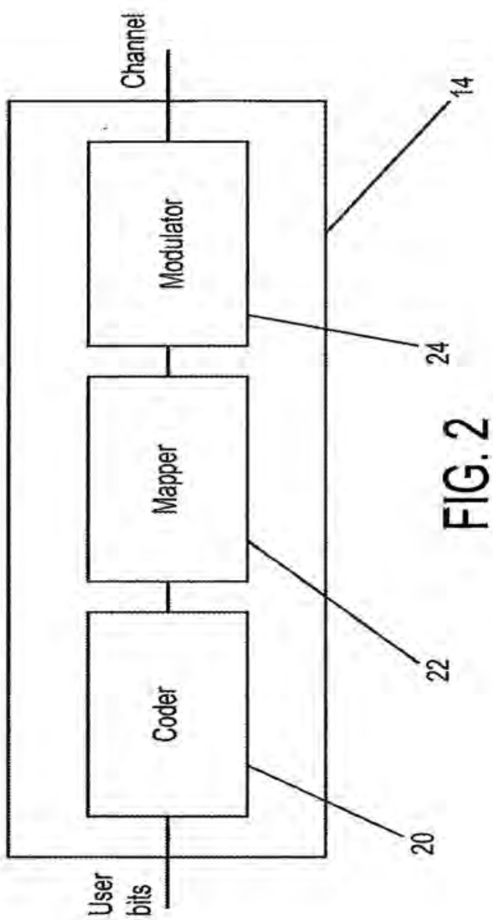

Channel
Modulator
Mapper
Coder
User bits
14
24
22
20

FIG. 2

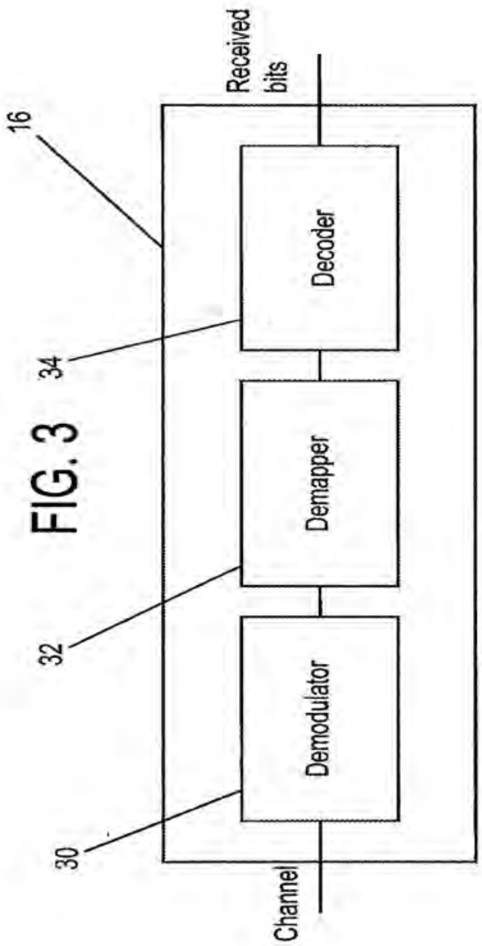
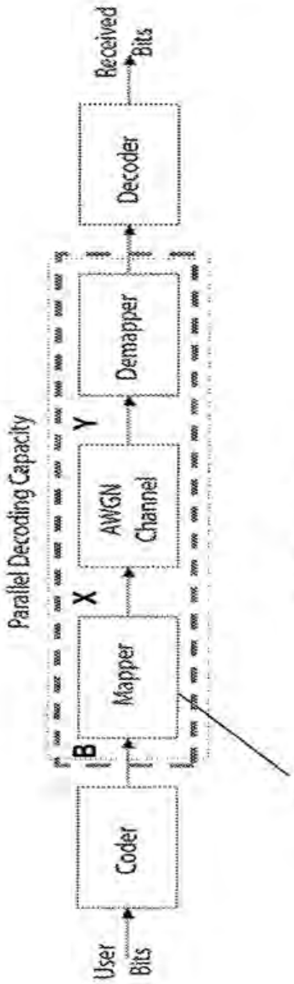
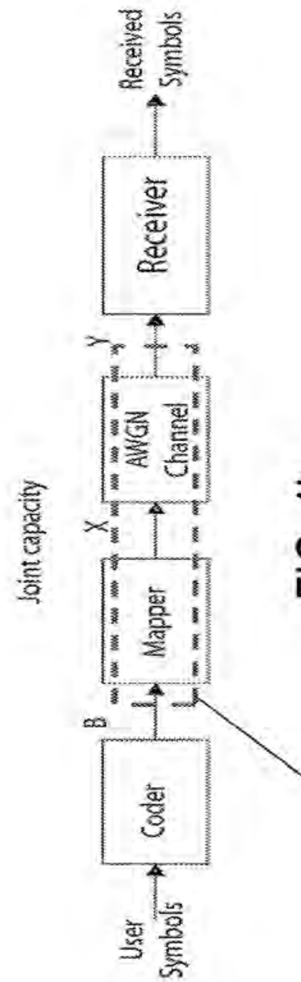

Received bits
Decoder
Demapper
Demodulator
Channel
16
34
32
30

FIG. 3



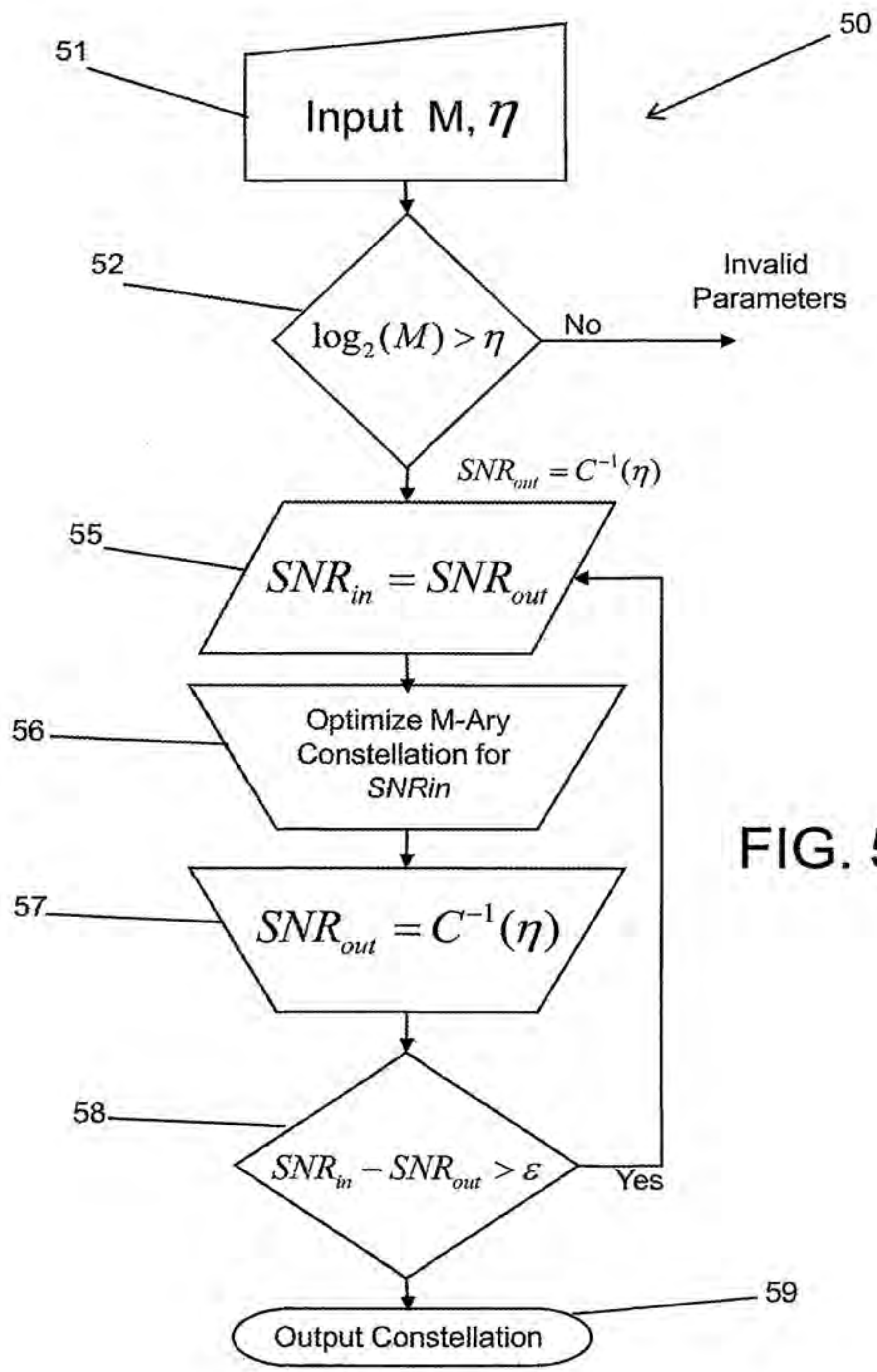
40

FIG. 4a



42

FIG. 4b



50
51
Input M, η
52
$\log_2(M) > \eta$
No
Invalid Parameters
$SNR_{out} = C^{-1}(\eta)$
55
$SNR_{in} = SNR_{out}$
56
Optimize M-Ary Constellation for SNRin
57
$SNR_{out} = C^{-1}(\eta)$
58
$SNR_{in} - SNR_{out} > \varepsilon$
Yes
59
Output Constellation

FIG. 5

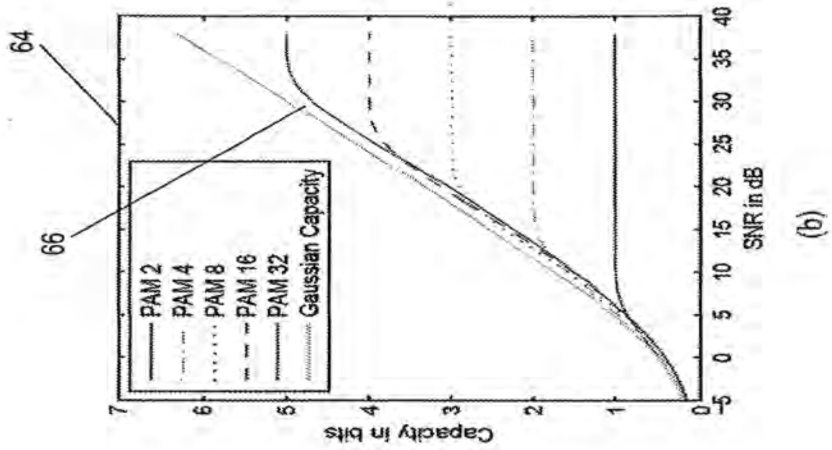

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

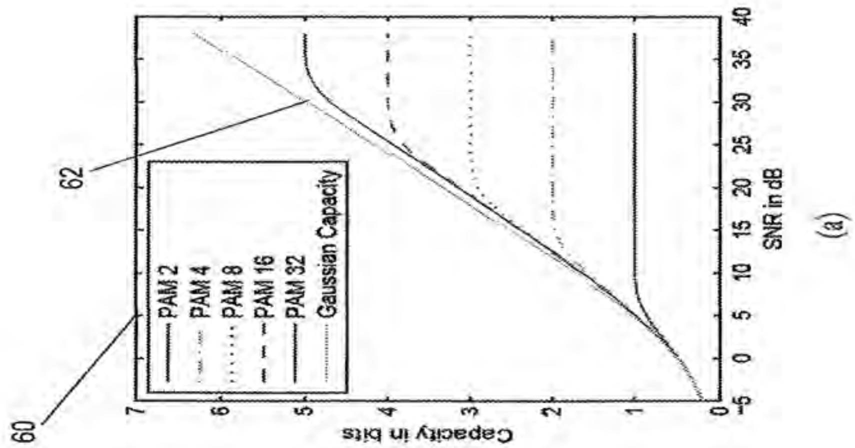

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

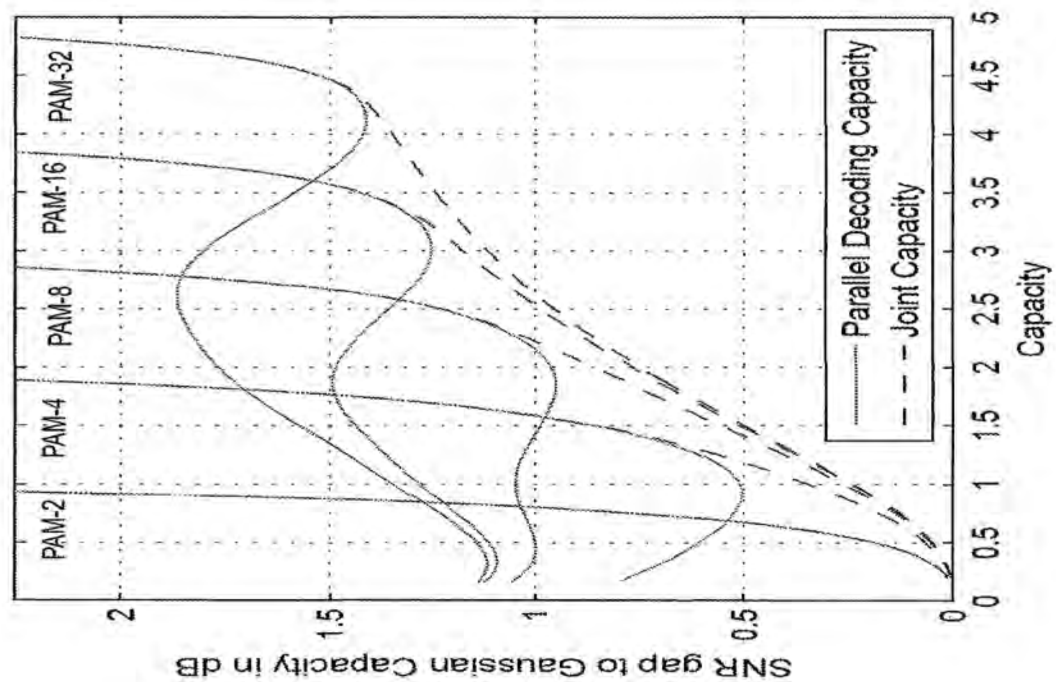

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
Parallel Decoding Capacity
Joint Capacity
Capacity
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

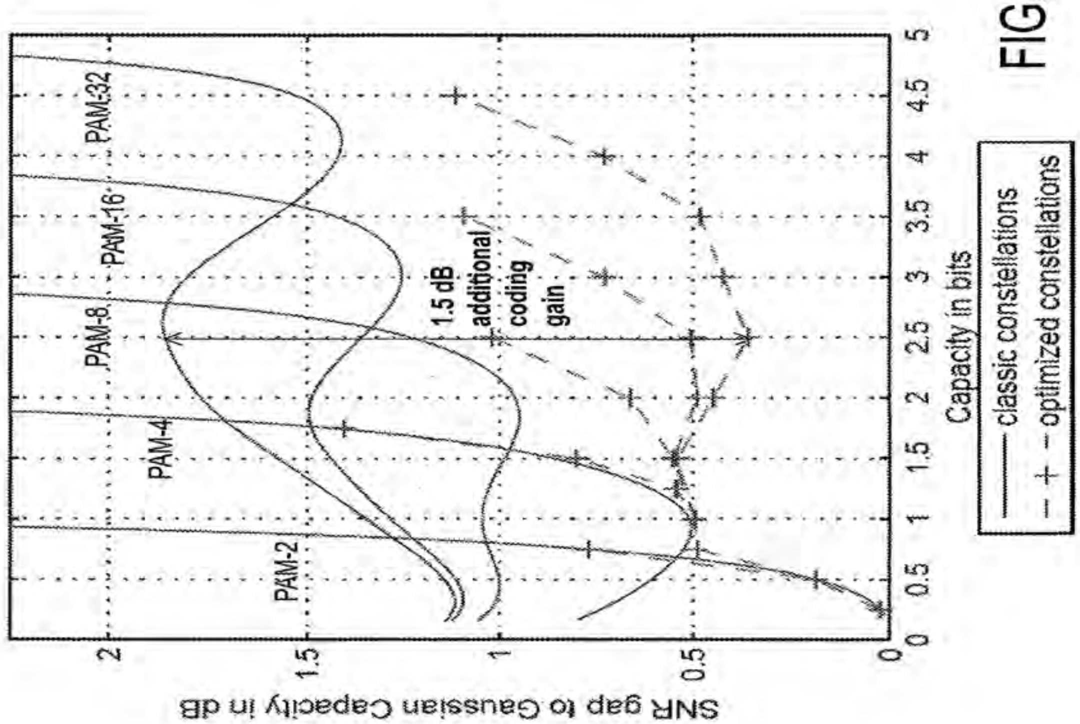

PAM-32
PAM-16
PAM-8
PAM-4
PAM-2
1.5 dB additional coding gain
SNR gap to Gaussian Capacity in dB
0
0.5
1
1.5
2
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
classic constellations
optimized constellations

FIG. 8a

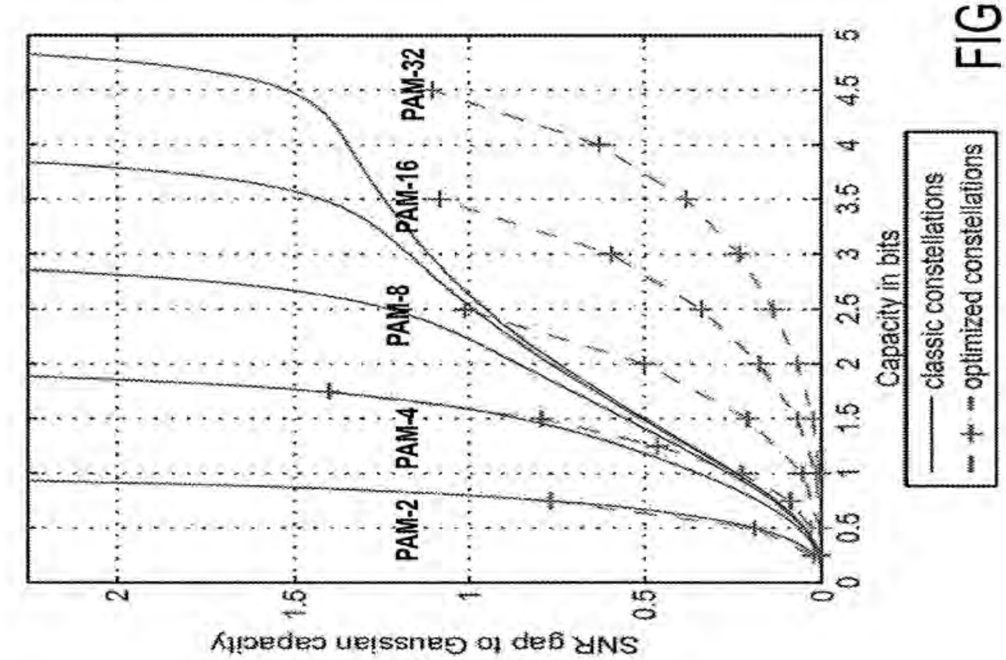

PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
SNR gap to Gaussian capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
classic constellations
optimized constellations

FIG. 8b

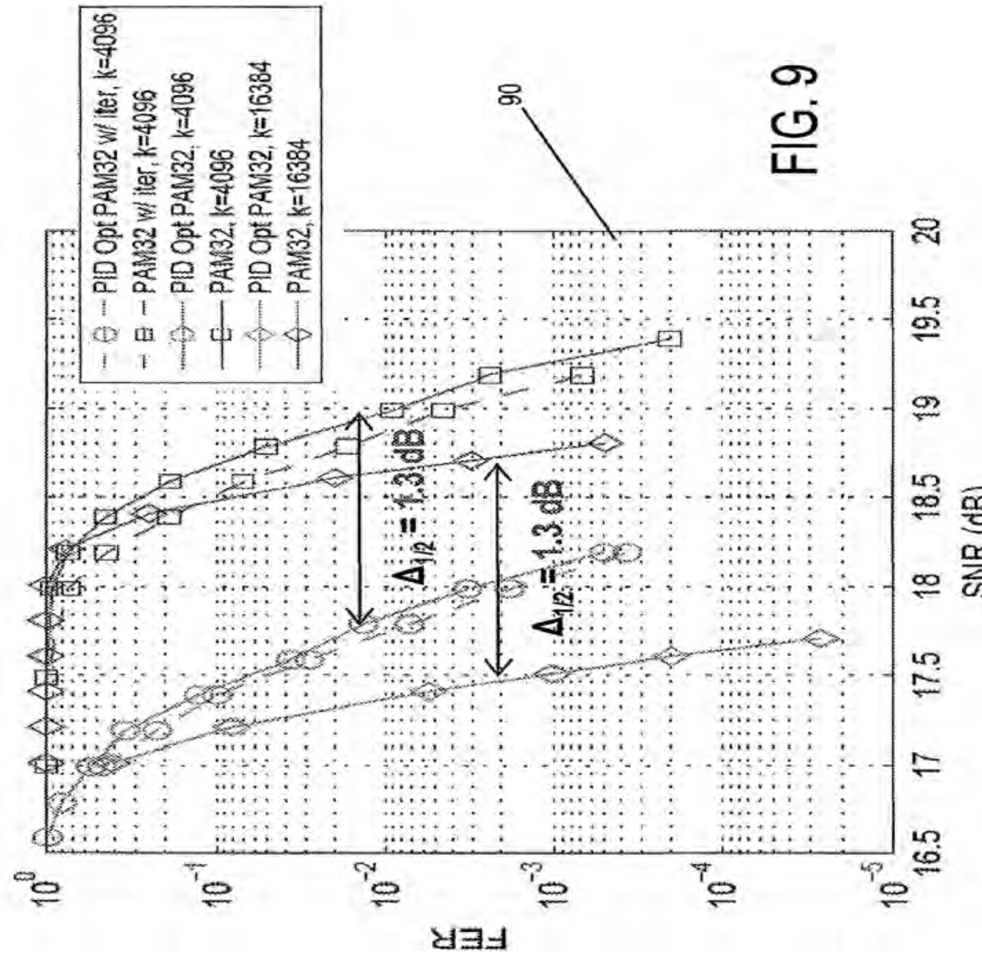

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

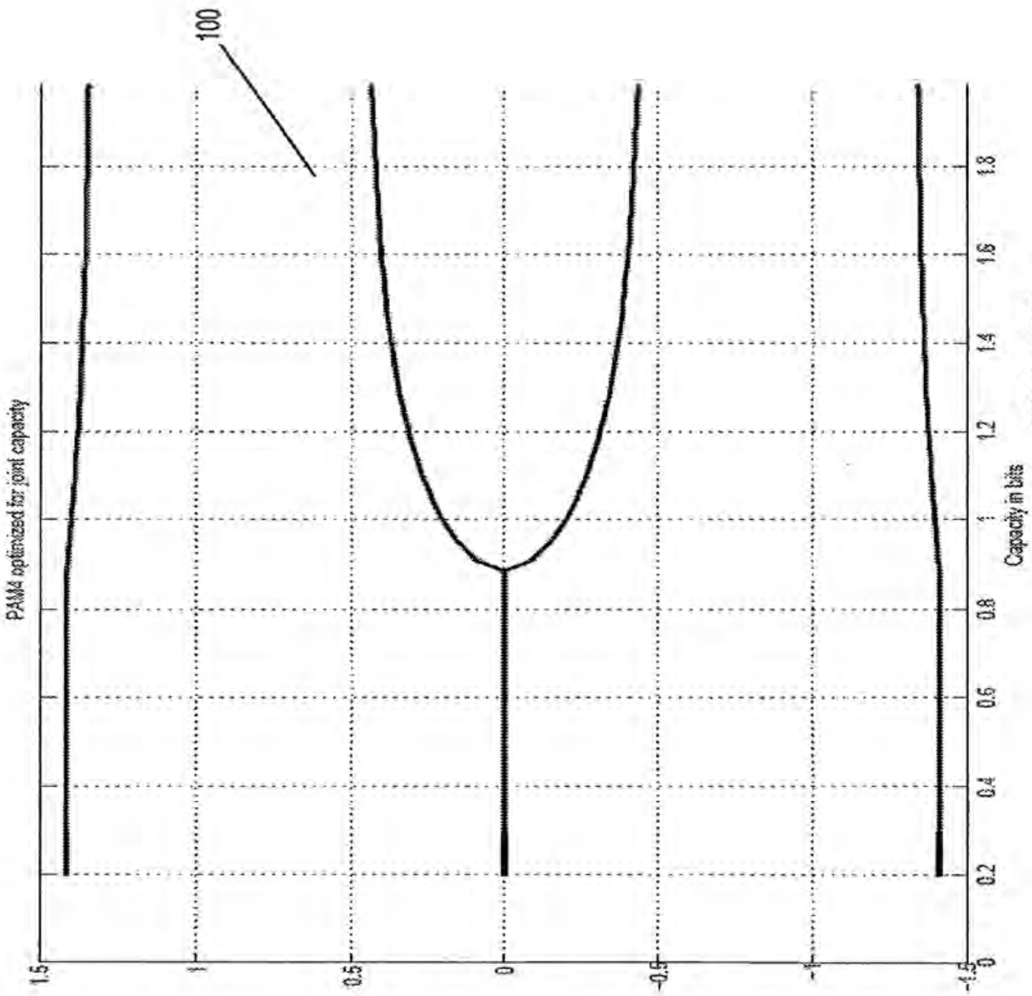

100
PAM4 optimized for joint capacity
Capacity in bits
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 10a

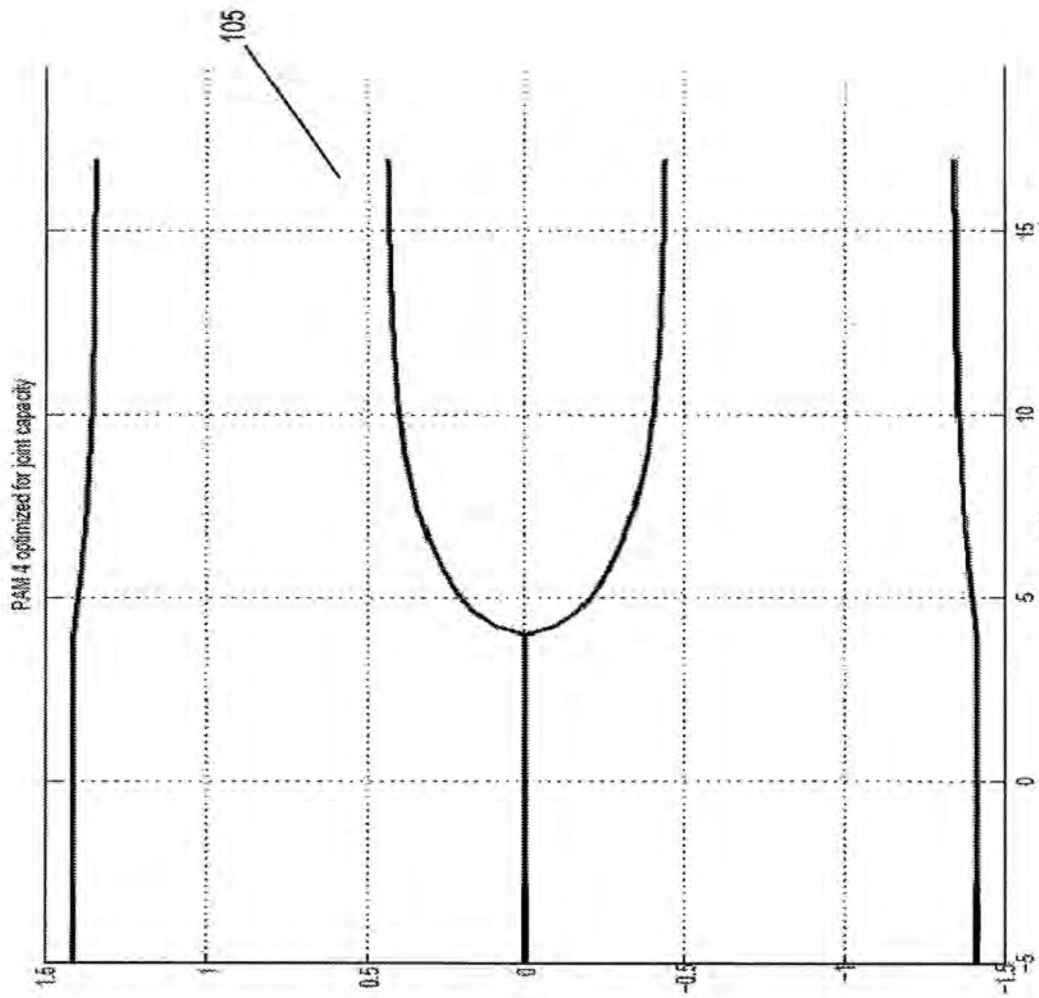

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10b

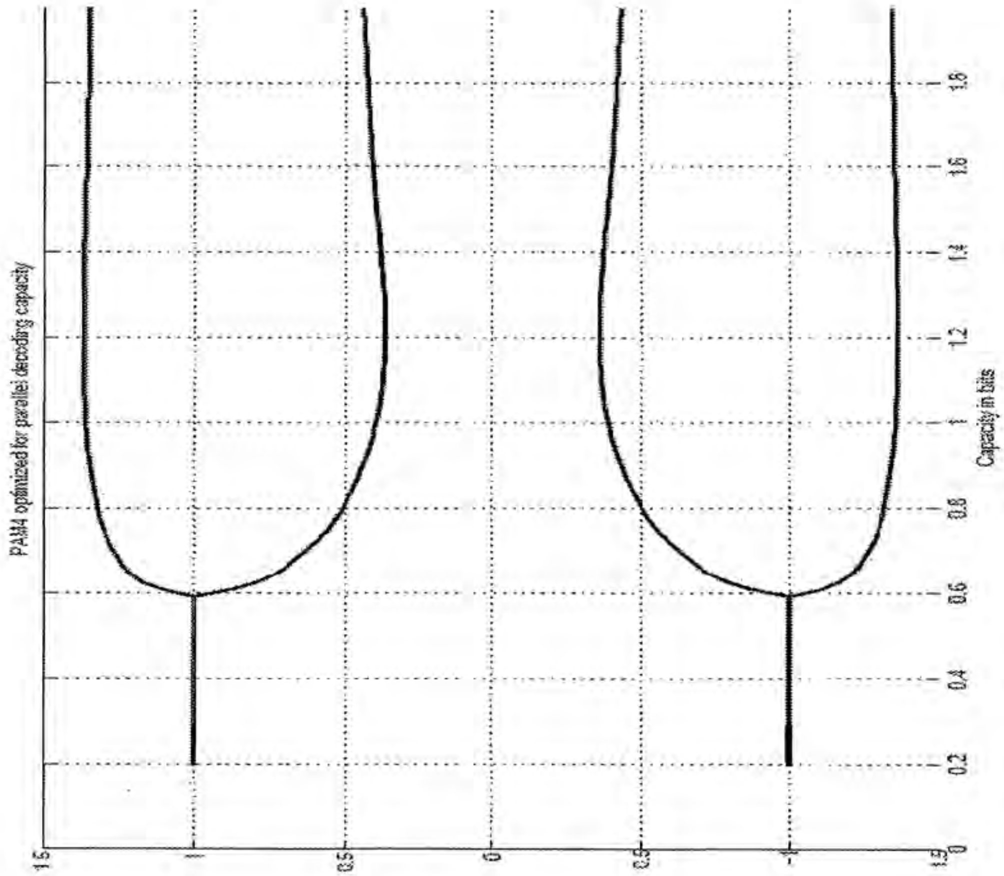

PAM4 optimized for parallel decoding capacity
Capacity in bits
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 10c

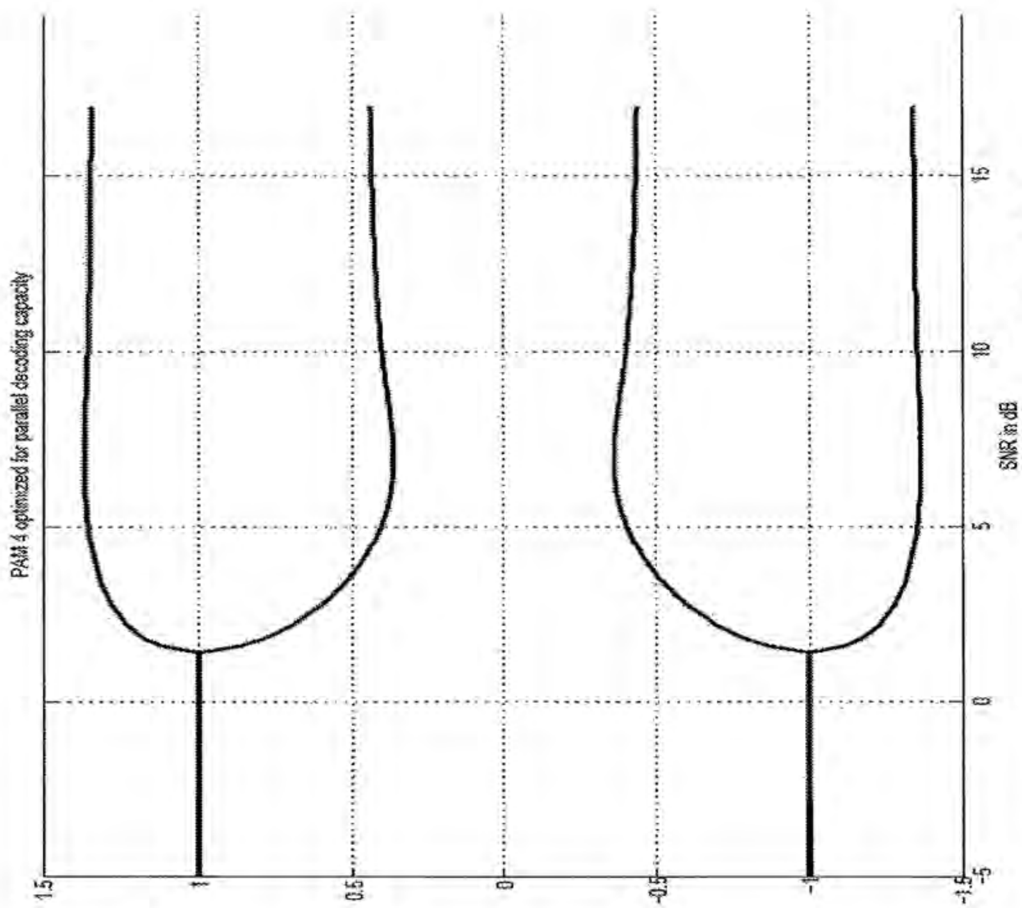

PAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

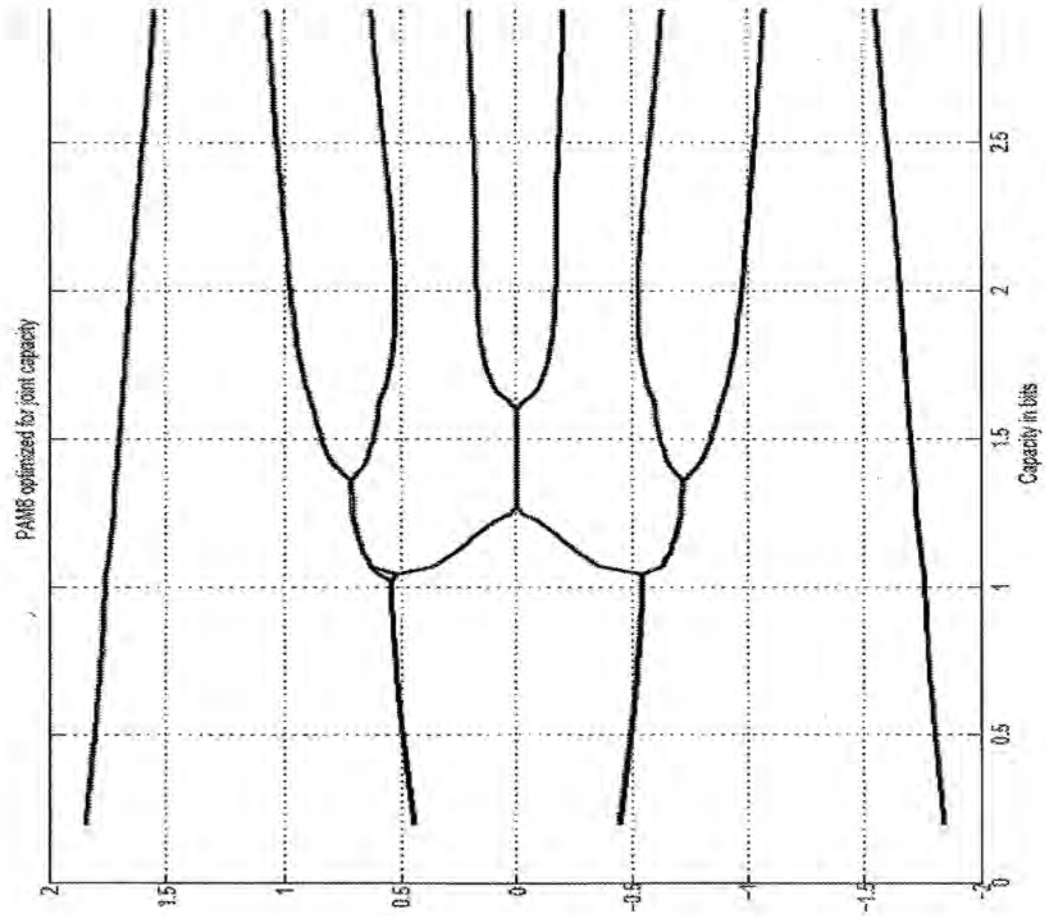

PAM8 optimized for joint capacity
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12a

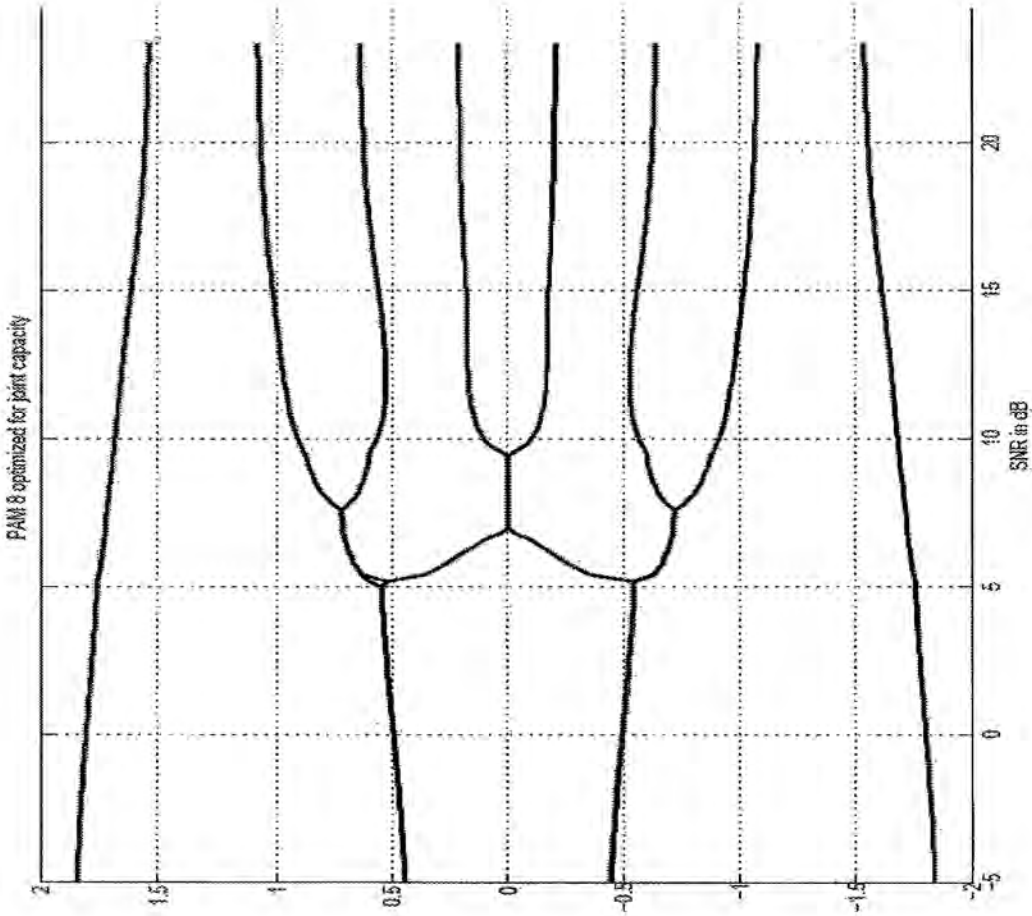

PAM 8 optimized for joint capacity
SNR in dB
-5
0
5
10
15
20
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12b

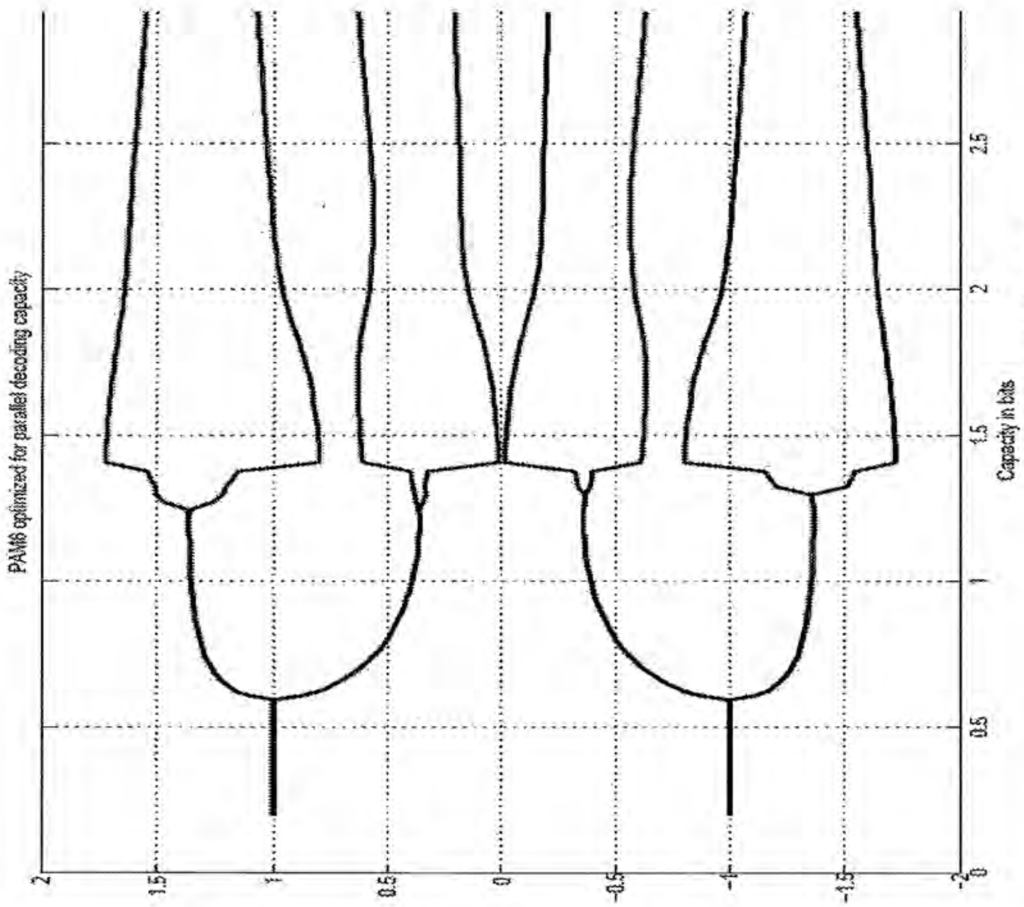

PAM6 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12c

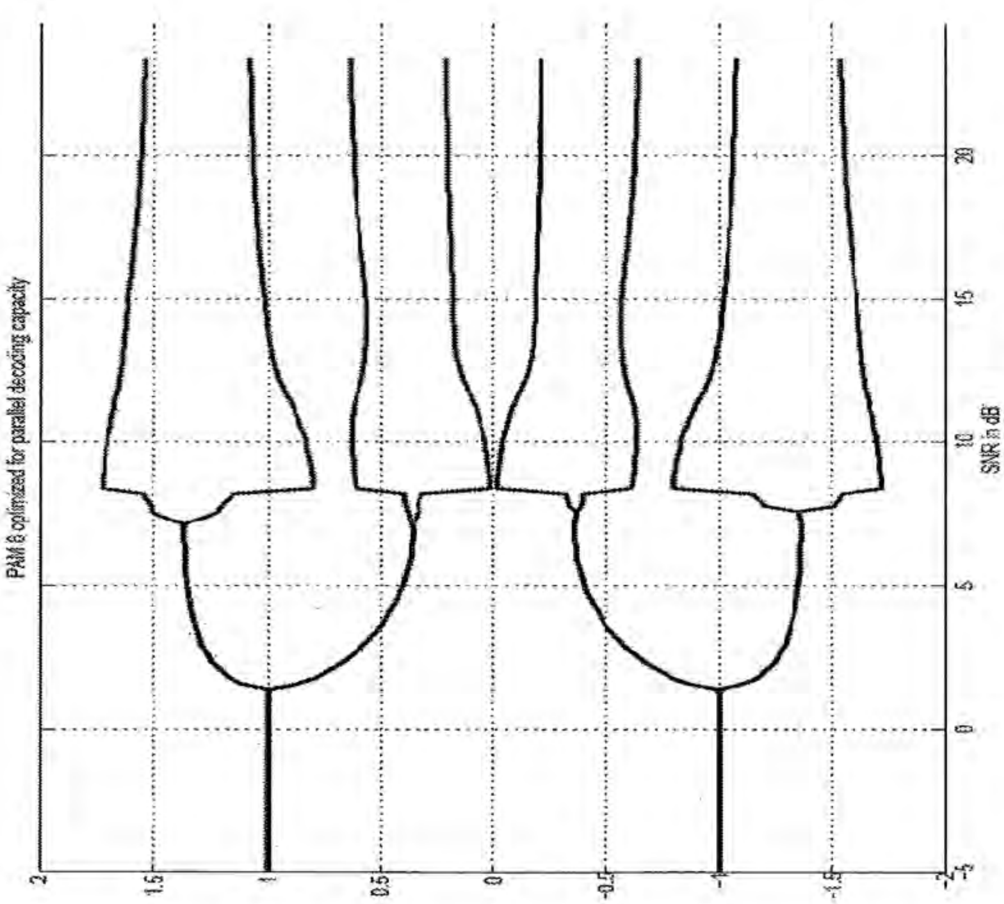

PAM 8 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5.27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

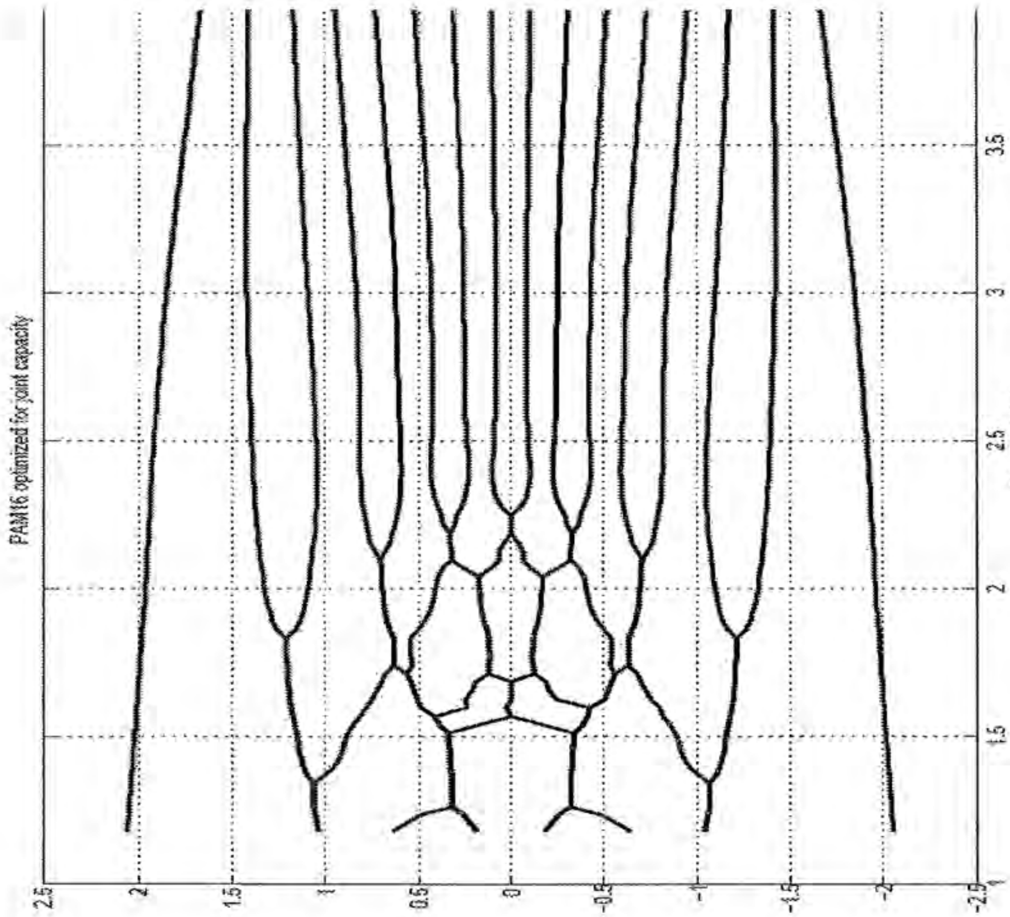

PAM16 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.5
2
2.5
3
3.5

FIG. 14a

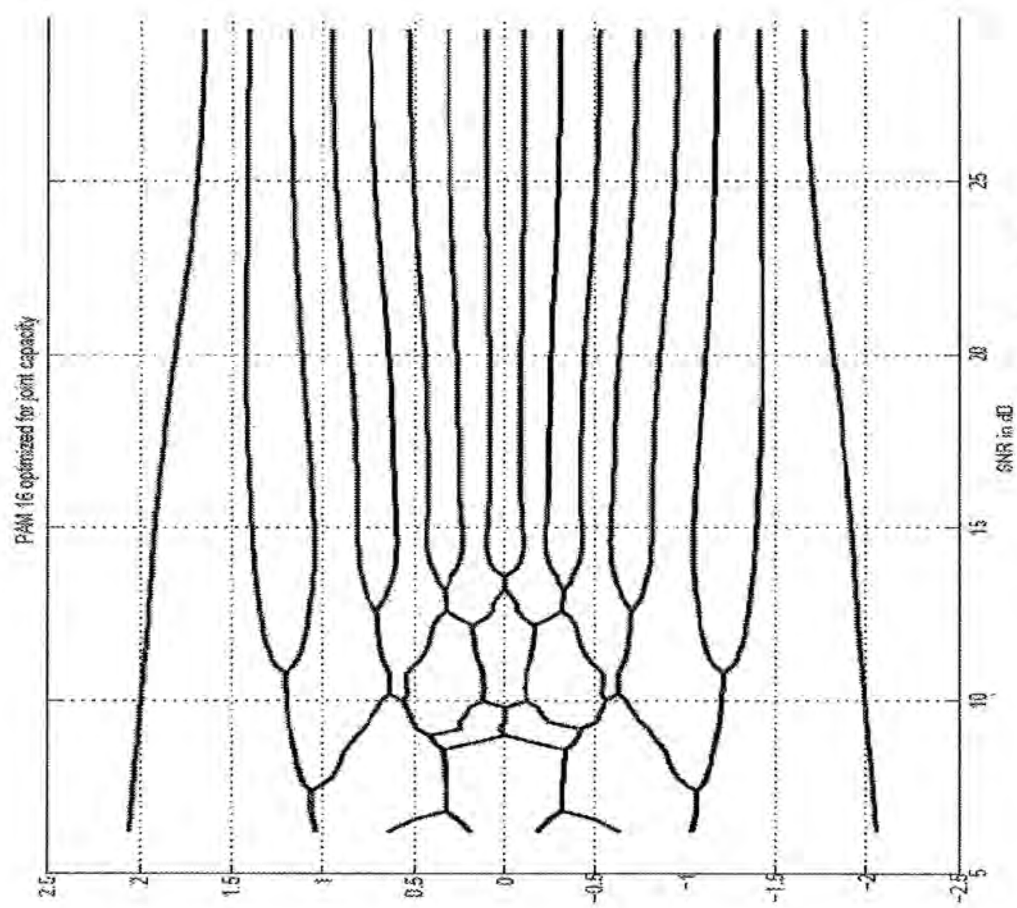

PAM 16 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14b

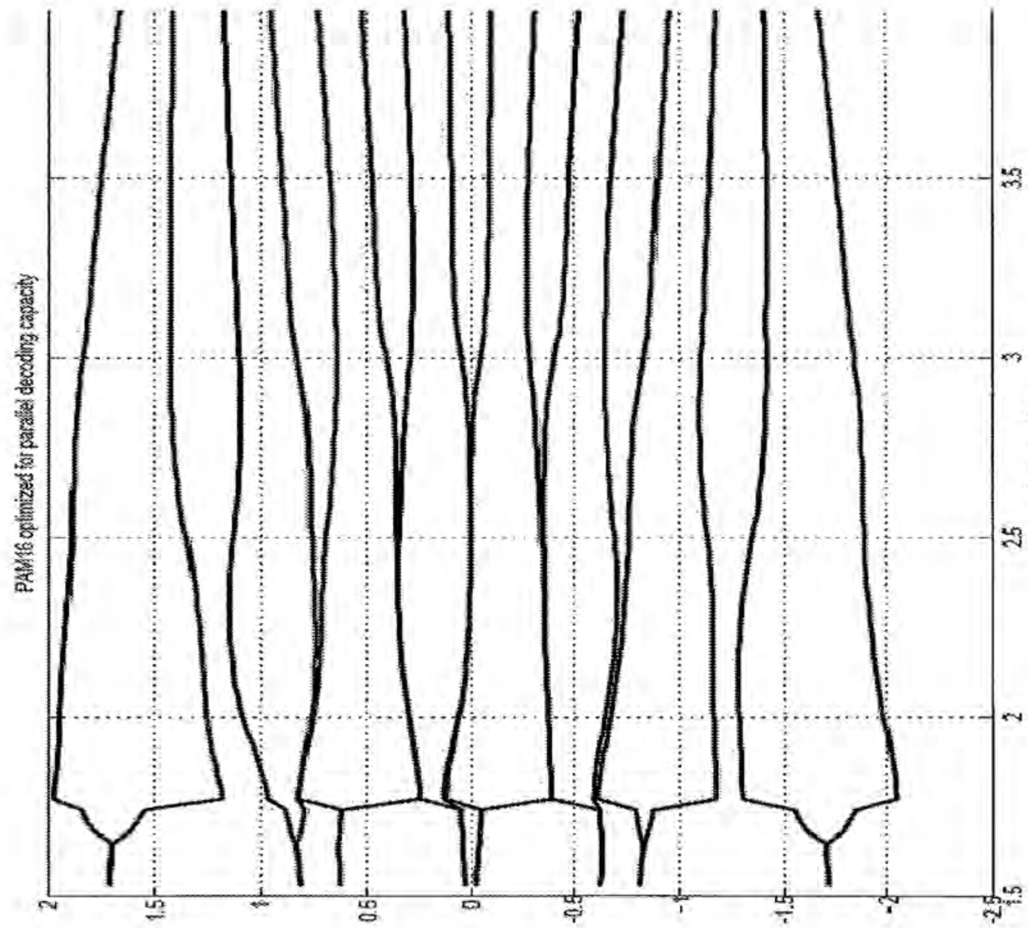

PAM16 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5

FIG. 14c

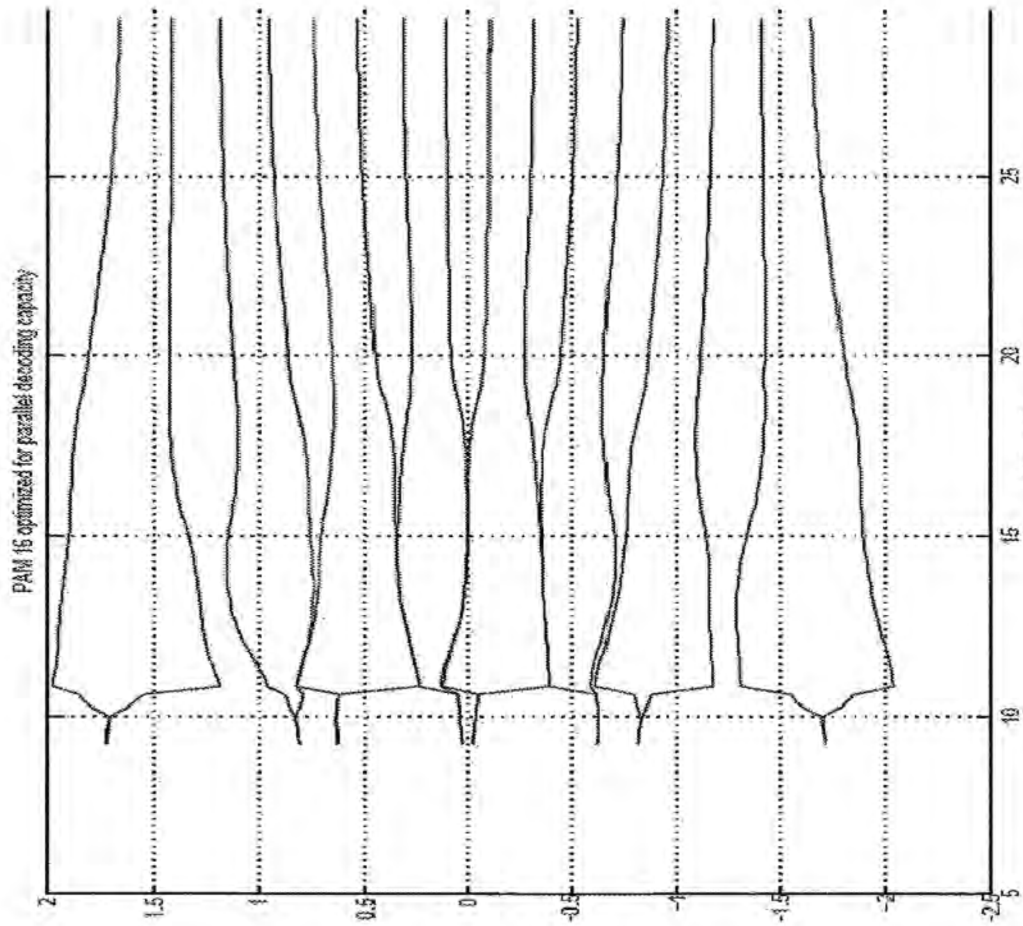

PAM 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

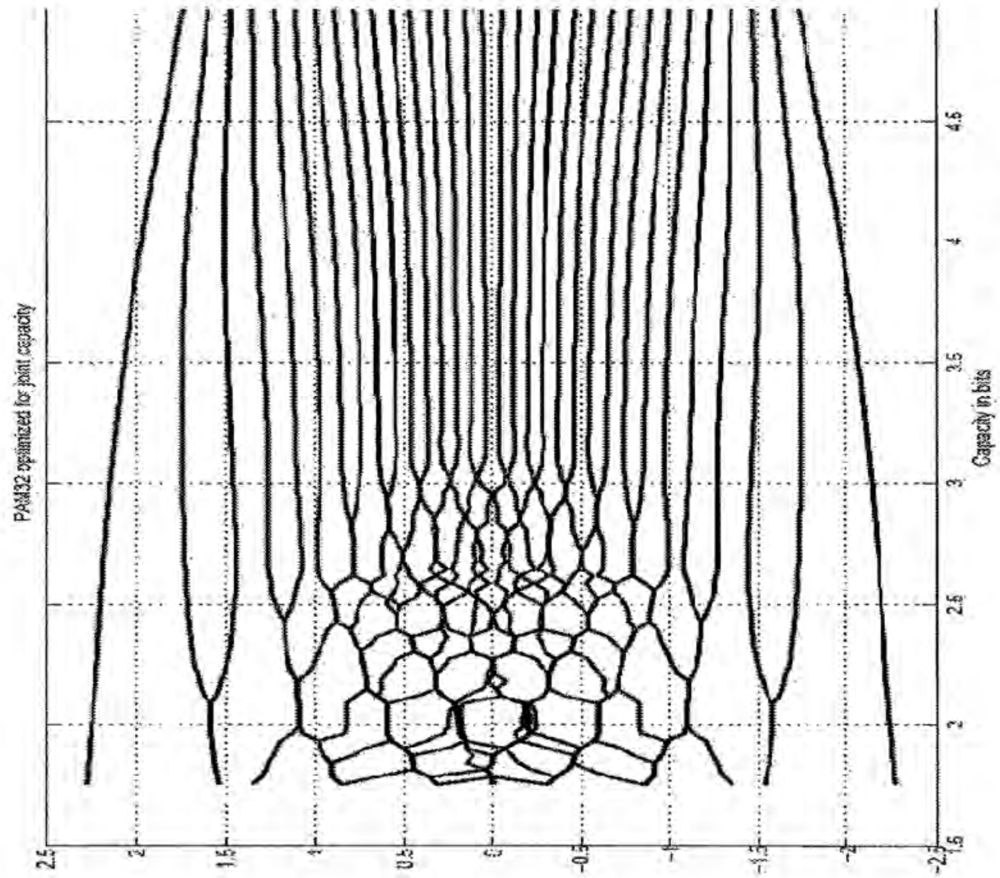

PAM32 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16a

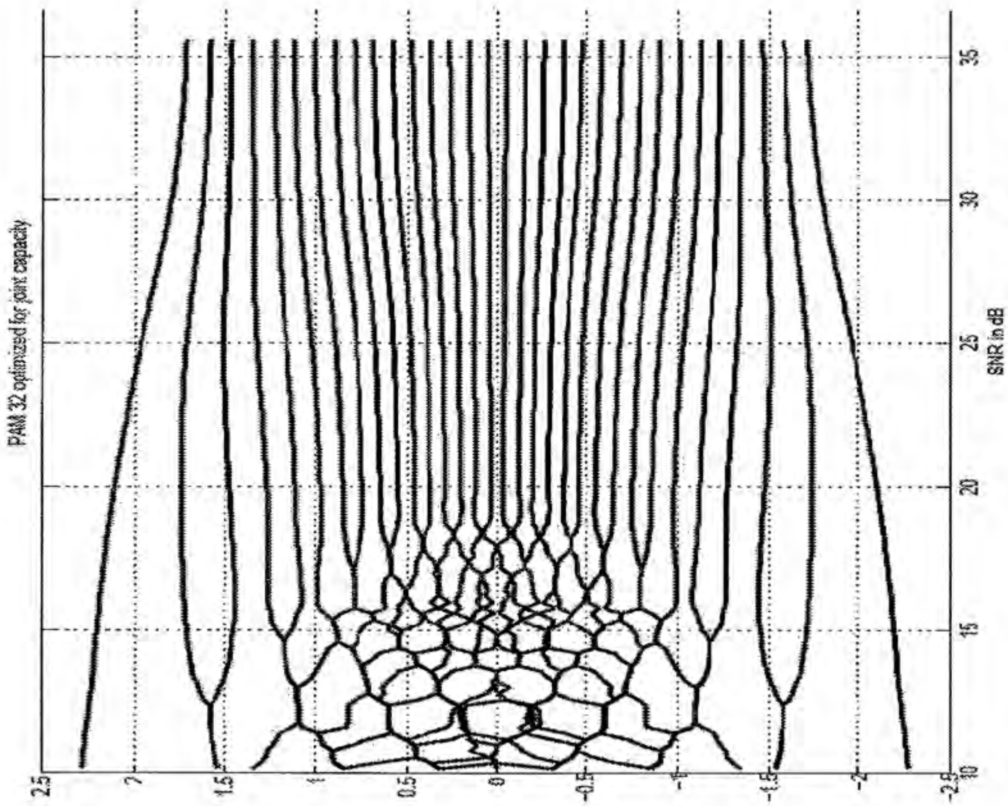


FIG. 16b

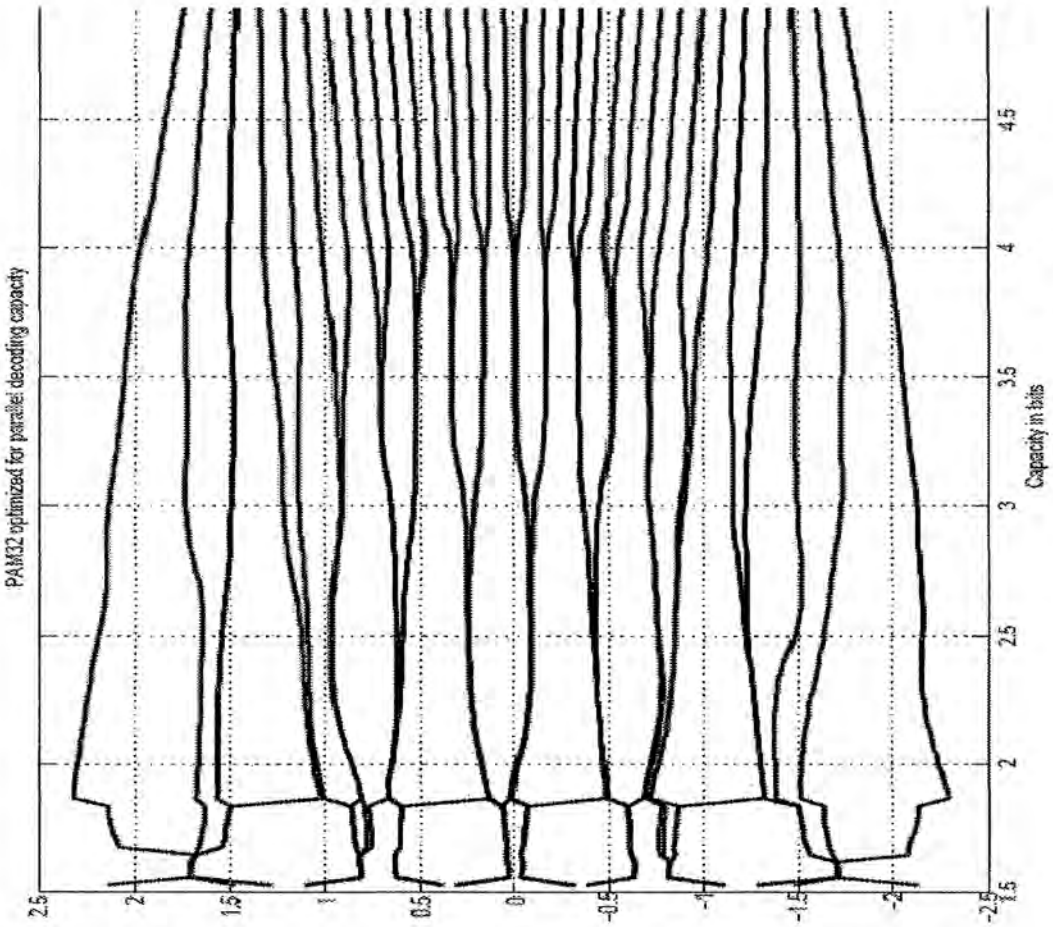

PAM32 optimized for parallel decoding capacity
Capacity in bits
1.5
2
2.5
3
3.5
4
4.5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16c

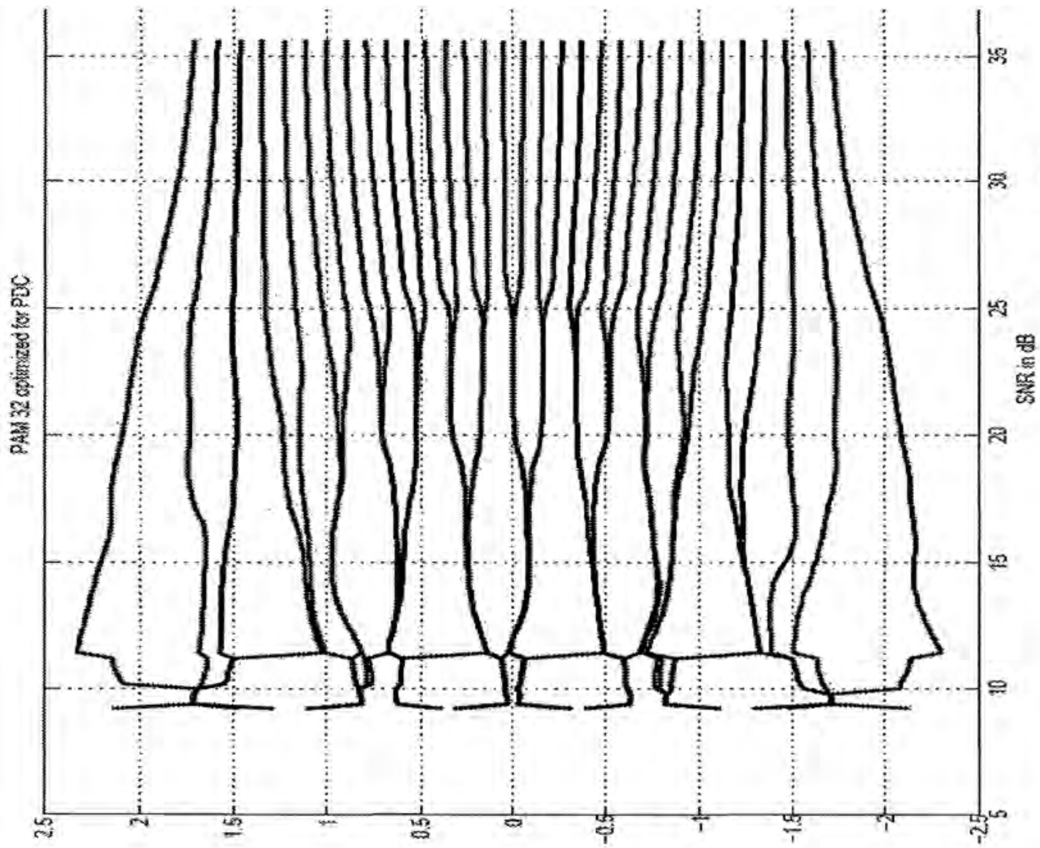

PAM 32 optimized for PDC
SNR in dB
5
10
15
20
25
30
35
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

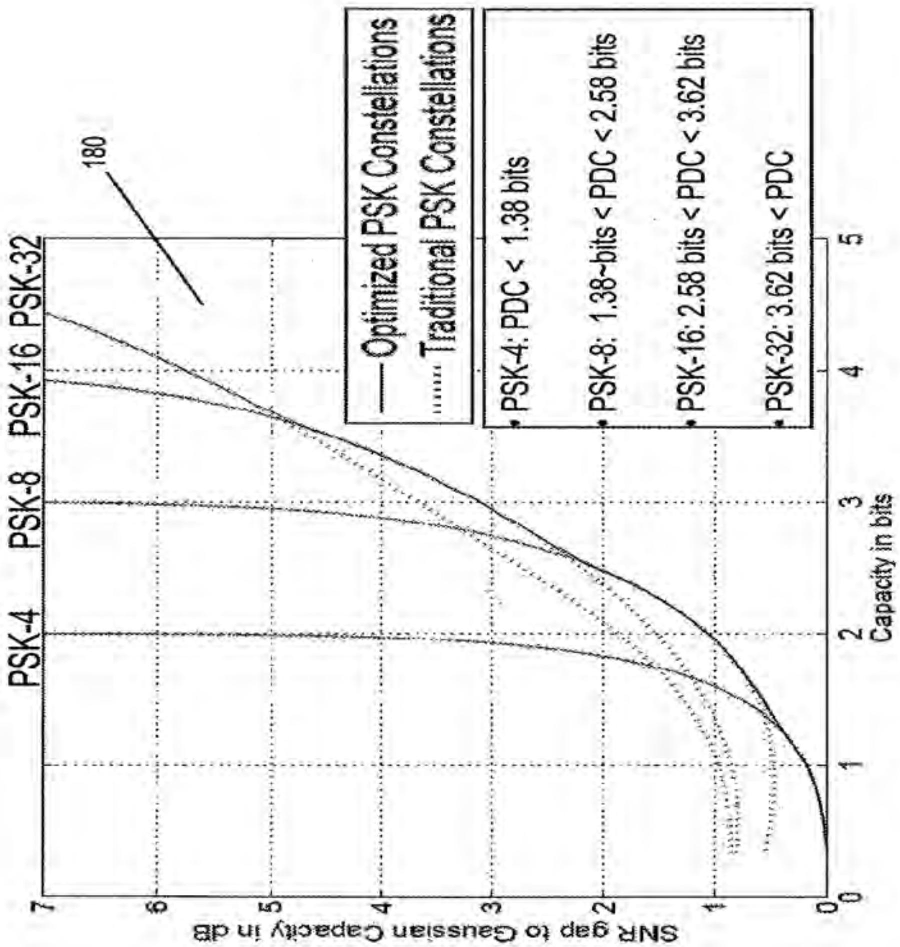

PSK-4
PSK-8
PSK-16
PSK-32
180
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38~bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

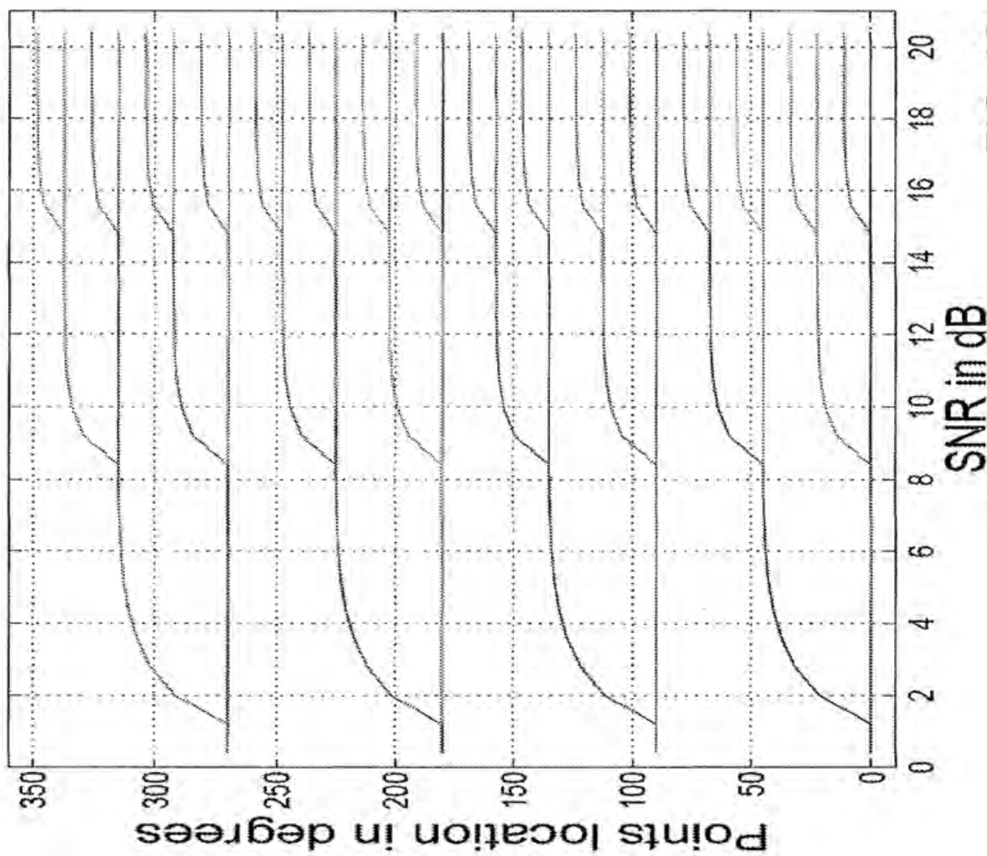

SNR in dB
0
2
4
6
8
10
12
14
16
18
20
Points location in degrees
0
50
100
150
200
250
300
350

FIG. 19

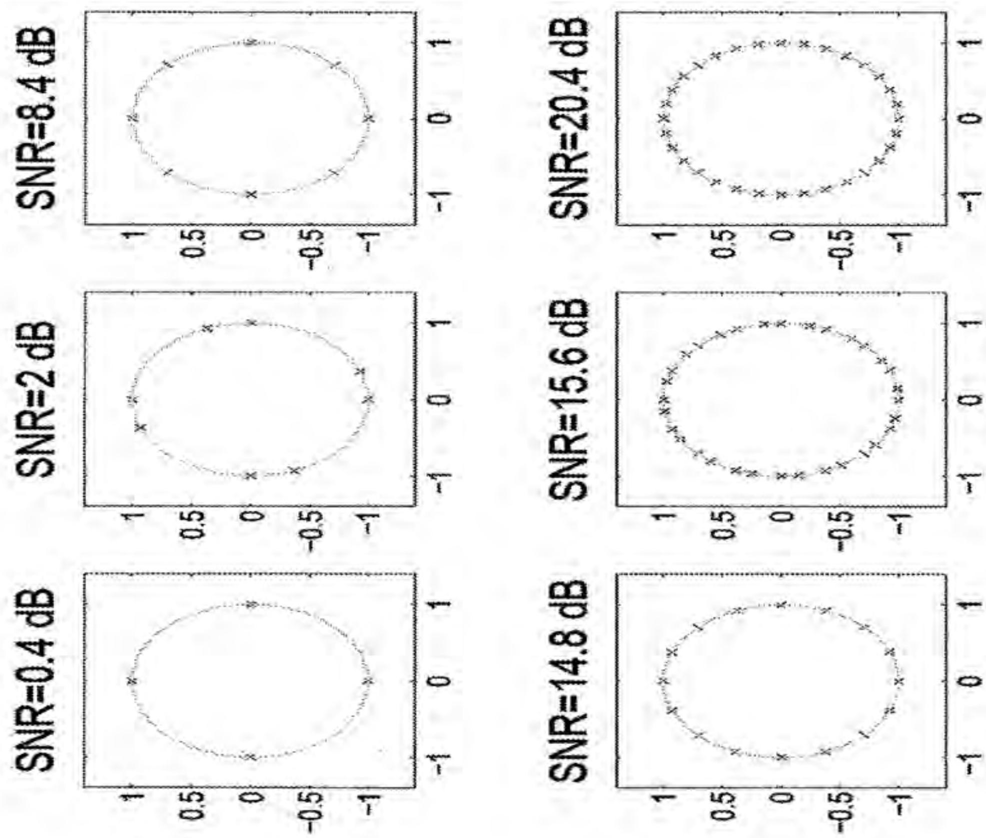


FIG. 20

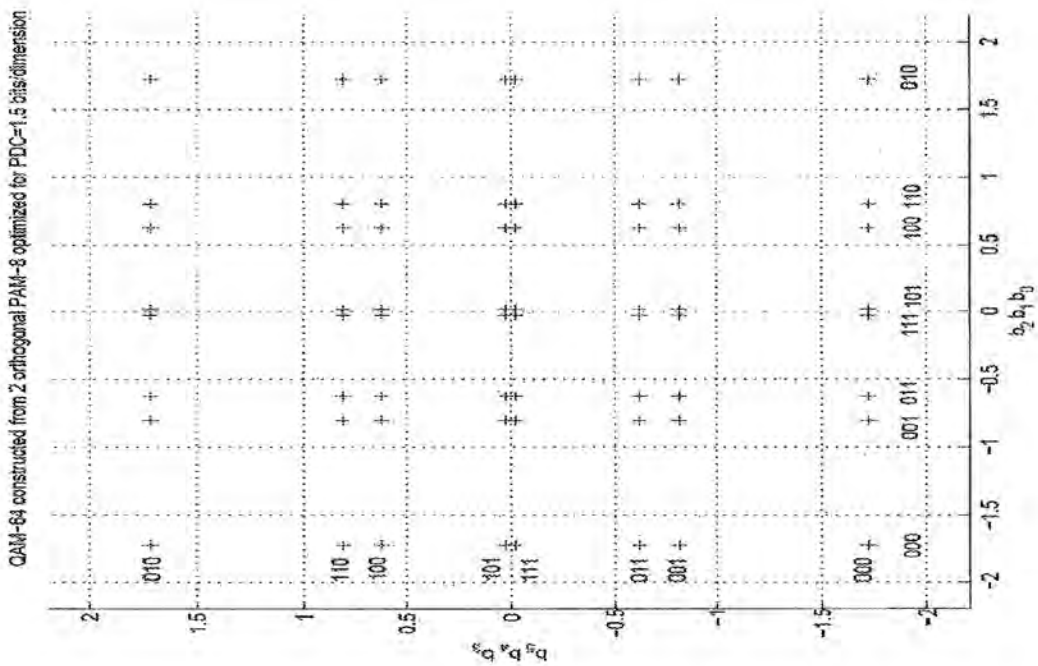

QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
$b_2 b_1 b_0$
$b_5 b_4 b_3$
010
110
100
101
111
011
001
000

FIG. 21

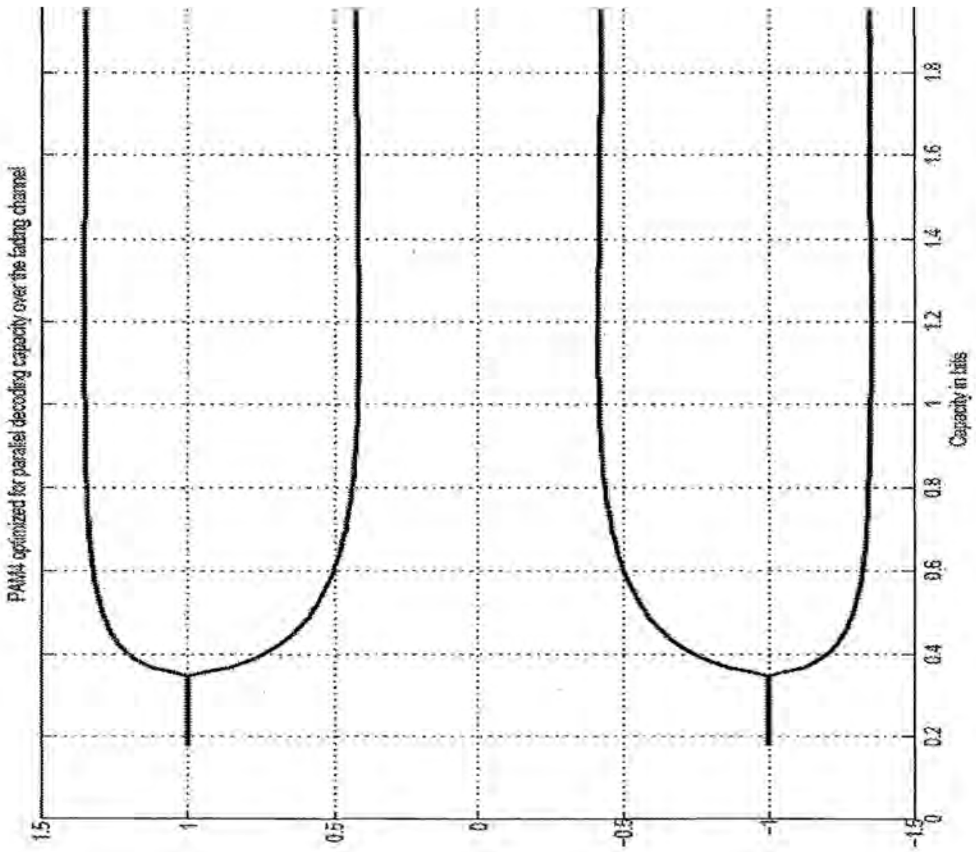

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 22a

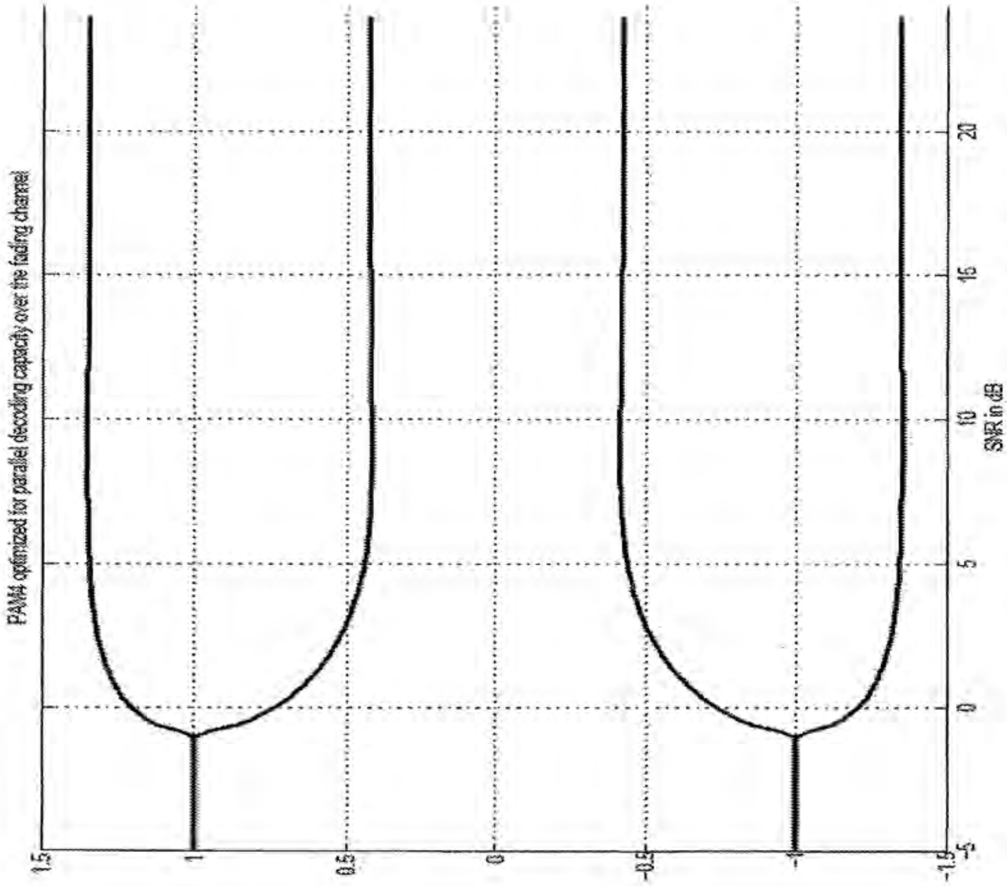

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 22b

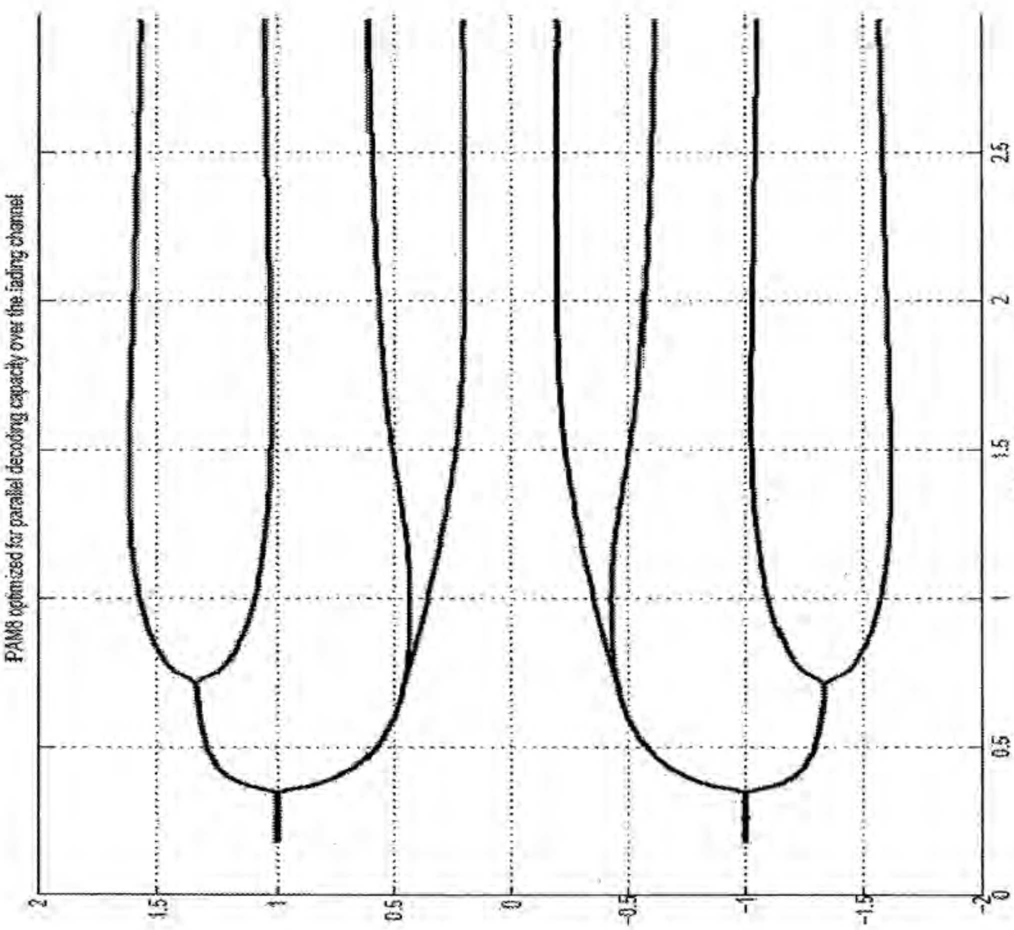

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23a

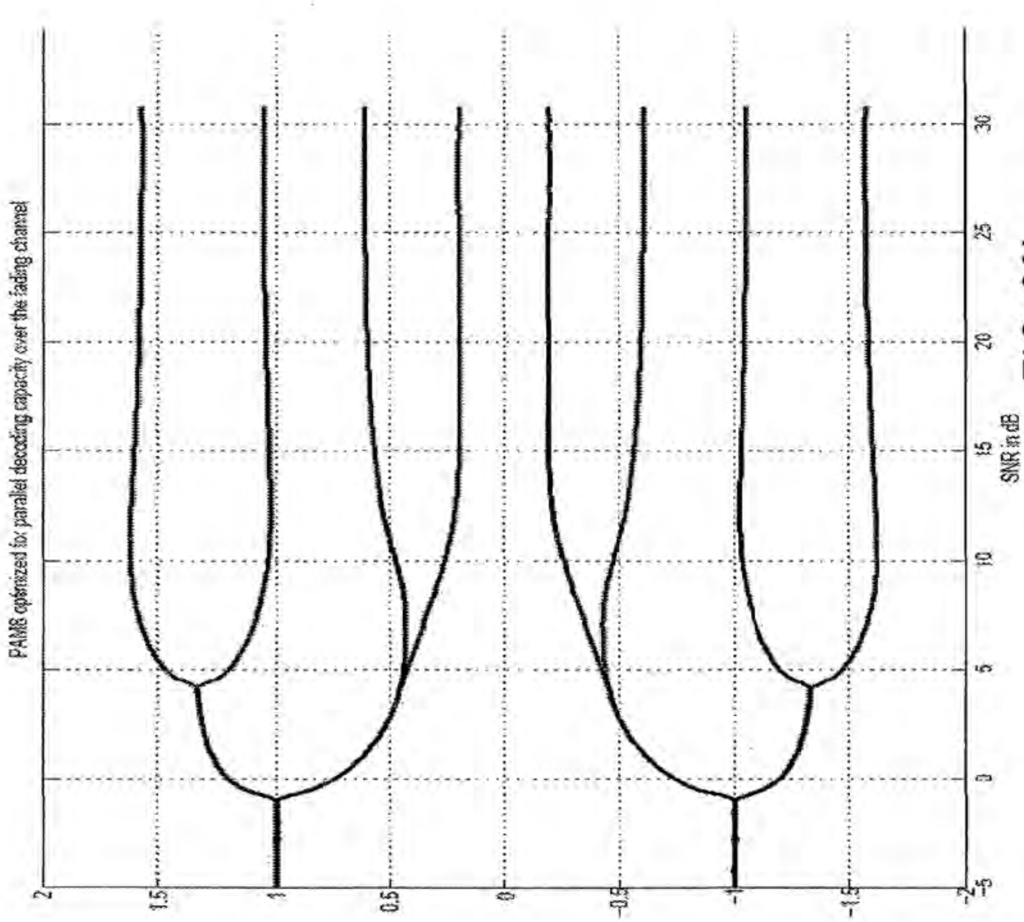

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB
-5
0
5
10
15
20
25
30
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23b

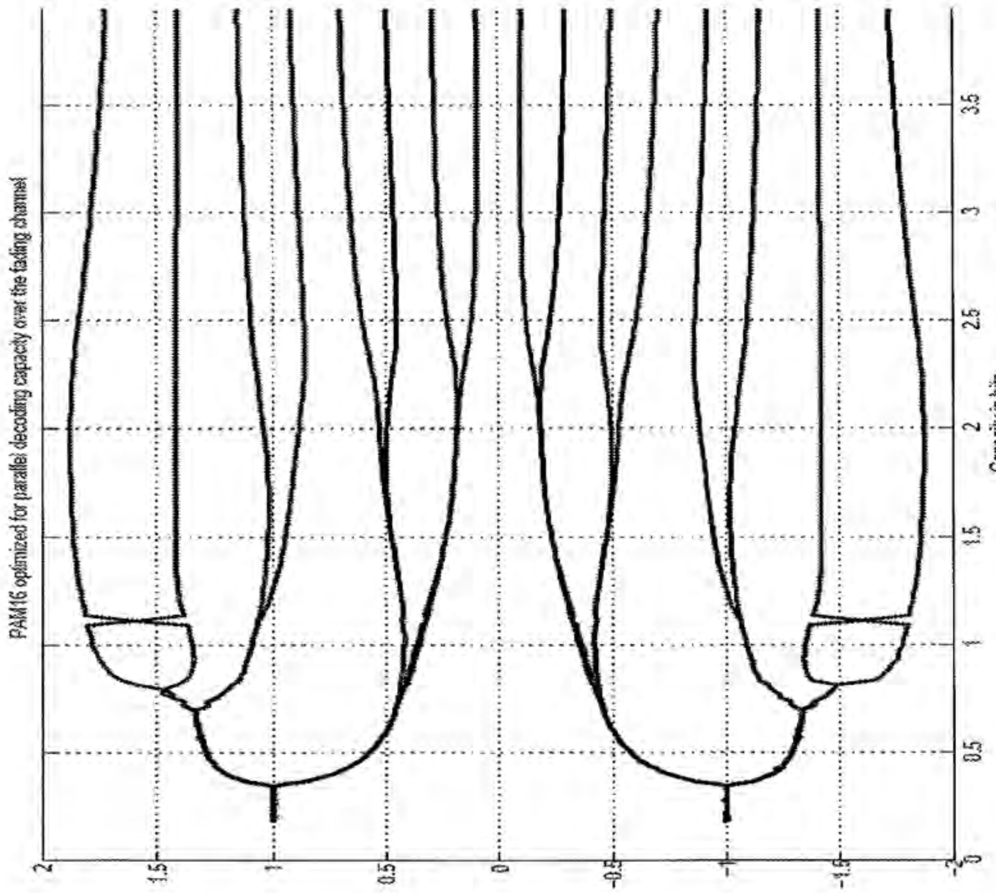

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
3
3.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 24a

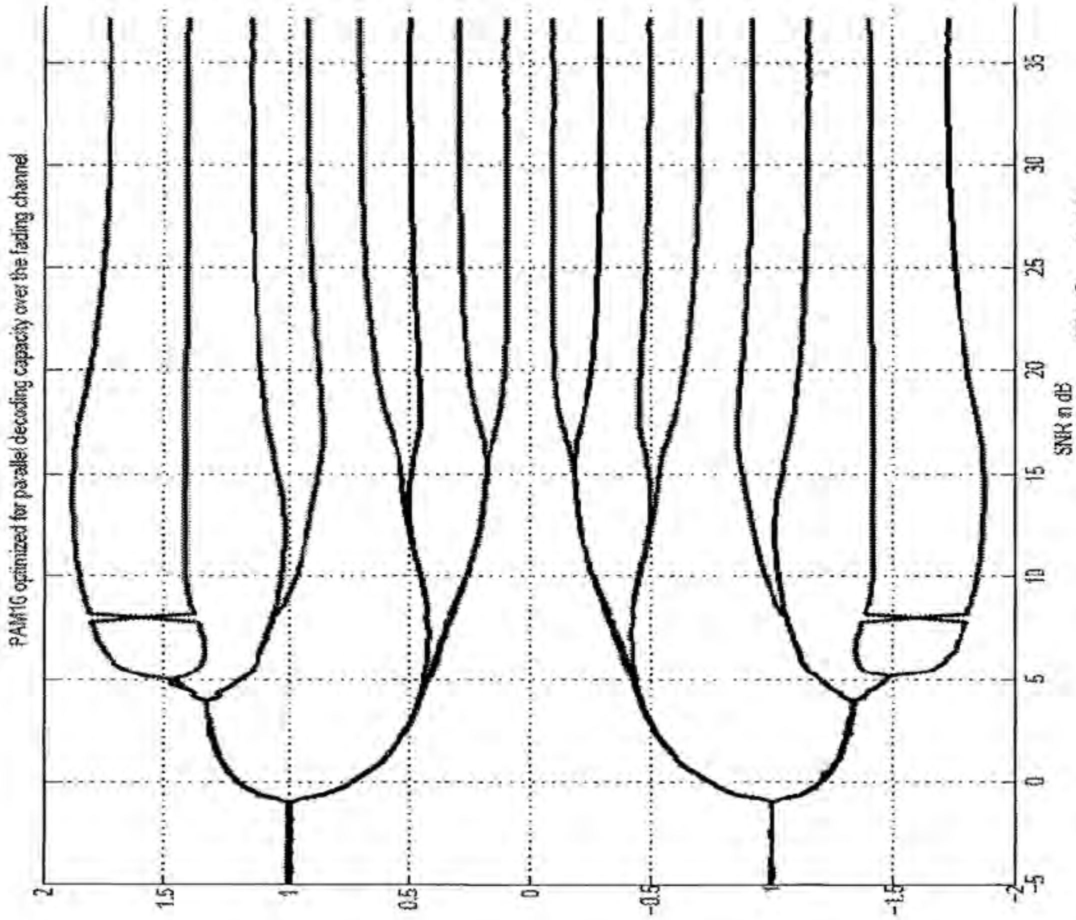

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB

FIG. 24b

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1982	0.1980	0.0002	0.0833
2	-4.8000	0.2063	0.2061	0.0002	0.0924
3	-4.6000	0.2147	0.2145	0.0002	0.1024
4	-4.4000	0.2234	0.2232	0.0003	0.1133
5	-4.2000	0.2324	0.2321	0.0003	0.1252
6	-4.0000	0.2417	0.2414	0.0003	0.1382
7	-3.8000	0.2513	0.2510	0.0004	0.1523
8	-3.6000	0.2613	0.2608	0.0004	0.1677
9	-3.4000	0.2715	0.2710	0.0005	0.1845
10	-3.2000	0.2821	0.2815	0.0006	0.2026
11	-3.0000	0.2930	0.2924	0.0006	0.2222
12	-2.8000	0.3043	0.3035	0.0007	0.2433
13	-2.6000	0.3159	0.3150	0.0008	0.2662
14	-2.4000	0.3278	0.3269	0.0010	0.2907
15	-2.2000	0.3401	0.3390	0.0011	0.3171
16	-2.0000	0.3528	0.3516	0.0012	0.3453
17	-1.8000	0.3658	0.3644	0.0014	0.3755
18	-1.6000	0.3792	0.3776	0.0015	0.4078
19	-1.4000	0.3929	0.3912	0.0017	0.4422
20	-1.2000	0.4070	0.4051	0.0019	0.4788
21	-1.0000	0.4215	0.4194	0.0022	0.5176
22	-0.8000	0.4364	0.4340	0.0024	0.5587
23	-0.6000	0.4516	0.4489	0.0027	0.6021
24	-0.4000	0.4672	0.4642	0.0030	0.6479
25	-0.2000	0.4832	0.4799	0.0033	0.6961
26	0.0000	0.4996	0.4959	0.0037	0.7468
27	0.0050	0.5000	0.4963	0.0037	0.7481
28	0.2000	0.5163	0.5122	0.0041	0.7999
29	0.4000	0.5334	0.5289	0.0045	0.8554
30	0.6000	0.5509	0.5459	0.0050	0.9133
31	0.8000	0.5688	0.5633	0.0055	0.9736
32	1.0000	0.5870	0.5810	0.0060	1.0363
33	1.2000	0.6056	0.5990	0.0066	1.1012
34	1.4000	0.6245	0.6173	0.0072	1.1684
35	1.6000	0.6439	0.6360	0.0079	1.2377
36	1.8000	0.6635	0.6550	0.0086	1.3090
37	2.0000	0.6835	0.6742	0.0093	1.3822
38	2.2000	0.7039	0.6938	0.0101	1.4572
39	2.4000	0.7246	0.7137	0.0109	1.5338
40	2.6000	0.7457	0.7338	0.0118	1.6117

Table 1

FIG. 25

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7670	0.7543	0.0128	1.6909
42	3.0000	0.7887	0.7750	0.0137	1.7709
43	3.2000	0.8107	0.7960	0.0147	1.8516
44	3.4000	0.8331	0.8173	0.0158	1.9326
45	3.6000	0.8557	0.8388	0.0169	2.0134
46	3.8000	0.8786	0.8606	0.0180	2.0938
47	4.0000	0.9018	0.8826	0.0192	2.1731
48	4.2000	0.9252	0.9049	0.0204	2.2508
49	4.4000	0.9490	0.9274	0.0216	2.3263
50	4.6000	0.9729	0.9501	0.0228	2.3989
51	4.8000	0.9971	0.9731	0.0240	2.4677
52	4.8237	1.0000	0.9758	0.0242	2.4755
53	5.0000	1.0215	0.9963	0.0252	2.5318
54	5.2000	1.0461	1.0197	0.0264	2.5905
55	5.4000	1.0710	1.0433	0.0276	2.6491
56	5.6000	1.0960	1.0672	0.0289	2.7048
57	5.8000	1.1213	1.0912	0.0301	2.7584
58	6.0000	1.1468	1.1154	0.0313	2.8099
59	6.2000	1.1724	1.1398	0.0326	2.8593
60	6.4000	1.1983	1.1644	0.0338	2.9064
61	6.6000	1.2243	1.1892	0.0351	2.9511
62	6.8000	1.2505	1.2142	0.0363	2.9934
63	7.0000	1.2769	1.2393	0.0376	3.0322
64	7.2000	1.3034	1.2646	0.0388	3.0663
65	7.4000	1.3300	1.2901	0.0399	3.0952
66	7.6000	1.3567	1.3157	0.0410	3.1187
67	7.8000	1.3836	1.3415	0.0421	3.1367
68	8.0000	1.4105	1.3674	0.0431	3.1506
69	8.2000	1.4376	1.3935	0.0440	3.1605
70	8.4000	1.4647	1.4197	0.0450	3.1665
71	8.6000	1.4919	1.4461	0.0458	3.1682
72	8.6592	1.5000	1.4539	0.0461	3.1678
73	8.8000	1.5192	1.4726	0.0466	3.1655
74	9.0000	1.5466	1.4992	0.0474	3.1584
75	9.2000	1.5740	1.5260	0.0480	3.1466
76	9.4000	1.6015	1.5529	0.0486	3.1299
77	9.6000	1.6290	1.5799	0.0491	3.1084
78	9.8000	1.6566	1.6070	0.0496	3.0839
79	10.0000	1.6843	1.6343	0.0500	3.0574
80	10.2000	1.7120	1.6616	0.0503	3.0290

Table 2

FIG. 26

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
81	10.4000	1.7398	1.6891	0.0506	2.9986
82	10.6000	1.7676	1.7167	0.0509	2.9662
83	10.8000	1.7955	1.7443	0.0511	2.9317
84	11.0000	1.8234	1.7721	0.0513	2.8951
85	11.2000	1.8514	1.8000	0.0514	2.8562
86	11.4000	1.8794	1.8279	0.0515	2.8149
87	11.6000	1.9074	1.8560	0.0514	2.7710
88	11.8000	1.9354	1.8841	0.0513	2.7244
89	12.0000	1.9634	1.9123	0.0512	2.6750
90	12.2000	1.9915	1.9406	0.0509	2.6226
91	12.2611	2.0000	1.9492	0.0508	2.6059
92	12.4000	2.0195	1.9689	0.0505	2.5670
93	12.6000	2.0474	1.9973	0.0501	2.5084
94	12.8000	2.0754	2.0258	0.0496	2.4466
95	13.0000	2.1033	2.0544	0.0489	2.3816
96	13.2000	2.1311	2.0829	0.0482	2.3135
97	13.4000	2.1589	2.1116	0.0473	2.2424
98	13.6000	2.1867	2.1402	0.0464	2.1683
99	13.8000	2.2143	2.1689	0.0454	2.0915
100	14.0000	2.2419	2.1976	0.0442	2.0121
101	14.2000	2.2693	2.2263	0.0430	1.9306
102	14.4000	2.2967	2.2550	0.0417	1.8471
103	14.6000	2.3239	2.2837	0.0402	1.7622
104	14.8000	2.3510	2.3122	0.0388	1.6762
105	15.0000	2.3779	2.3407	0.0372	1.5896
106	15.2000	2.4047	2.3691	0.0356	1.5030
107	15.4000	2.4313	2.3973	0.0340	1.4168
108	15.6000	2.4576	2.4253	0.0323	1.3317
109	15.8000	2.4837	2.4531	0.0306	1.2481
110	15.9256	2.5000	2.4704	0.0296	1.1965
111	16.0000	2.5096	2.4806	0.0289	1.1664
112	16.2000	2.5351	2.5078	0.0273	1.0871
113	16.4000	2.5603	2.5347	0.0256	1.0105
114	16.6000	2.5851	2.5611	0.0240	0.9369
115	16.8000	2.6095	2.5871	0.0224	0.8664
116	17.0000	2.6334	2.6125	0.0209	0.7992
117	17.2000	2.6568	2.6374	0.0194	0.7354
118	17.4000	2.6796	2.6616	0.0180	0.6750
119	17.6000	2.7017	2.6851	0.0166	0.6179
120	17.8000	2.7233	2.7080	0.0153	0.5642

Table 3

FIG. 27

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.7441	2.7300	0.0140	0.5137
122	18.2000	2.7641	2.7513	0.0128	0.4664
123	18.4000	2.7834	2.7717	0.0117	0.4222
124	18.6000	2.8018	2.7912	0.0106	0.3811
125	18.8000	2.8194	2.8097	0.0096	0.3428
126	19.0000	2.8360	2.8274	0.0087	0.3072
127	19.2000	2.8518	2.8440	0.0078	0.2744
128	19.4000	2.8667	2.8597	0.0070	0.2442
129	19.6000	2.8806	2.8743	0.0062	0.2164
130	19.8000	2.8936	2.8880	0.0055	0.1910
131	20.0000	2.9056	2.9007	0.0049	0.1678
132	20.2000	2.9167	2.9125	0.0043	0.1468
133	20.4000	2.9269	2.9232	0.0037	0.1277
134	20.6000	2.9363	2.9330	0.0032	0.1104
135	20.8000	2.9448	2.9420	0.0028	0.0951
136	21.0000	2.9524	2.9500	0.0024	0.0813
137	21.2000	2.9593	2.9572	0.0020	0.0692
138	21.4000	2.9654	2.9636	0.0017	0.0584
139	21.6000	2.9708	2.9693	0.0015	0.0491
140	21.8000	2.9755	2.9743	0.0012	0.0409
141	22.0000	2.9797	2.9786	0.0010	0.0338
142	22.2000	2.9832	2.9824	0.0008	0.0278
143	22.4000	2.9863	2.9856	0.0007	0.0226
144	22.6000	2.9889	2.9884	0.0005	0.0183
145	22.8000	2.9911	2.9907	0.0004	0.0145
146	23.0000	2.9929	2.9926	0.0003	0.0115
147	23.2000	2.9944	2.9942	0.0003	0.0092
148	23.4000	2.9957	2.9955	0.0002	0.0071

Table 4

FIG. 28

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-8.450	-2.051	-2.051	-2.048	2.035	2.057	2.057	8.450
2	-8.446	-2.061	-2.059	-2.046	2.055	2.055	2.055	8.446
3	-8.441	-2.064	-2.064	-2.057	2.061	2.061	2.062	8.441
4	-8.436	-2.067	-2.069	-2.068	2.067	2.067	2.069	8.436
5	-8.432	-2.075	-2.076	-2.072	2.074	2.075	2.074	8.432
6	-8.426	-2.082	-2.081	-2.081	2.081	2.081	2.081	8.426
7	-8.421	-2.089	-2.089	-2.089	2.089	2.089	2.089	8.421
8	-8.416	-2.097	-2.097	-2.095	2.096	2.096	2.097	8.416
9	-8.410	-2.104	-2.104	-2.104	2.102	2.105	2.105	8.410
10	-8.404	-2.112	-2.112	-2.112	2.111	2.112	2.112	8.404
11	-8.397	-2.120	-2.120	-2.120	2.120	2.120	2.120	8.397
12	-8.391	-2.129	-2.129	-2.129	2.129	2.129	2.129	8.391
13	-8.384	-2.138	-2.138	-2.138	2.138	2.138	2.138	8.384
14	-8.377	-2.147	-2.147	-2.147	2.147	2.147	2.147	8.377
15	-8.370	-2.156	-2.156	-2.156	2.156	2.156	2.156	8.370
16	-8.362	-2.166	-2.166	-2.166	2.166	2.166	2.166	8.362
17	-8.354	-2.176	-2.176	-2.176	2.176	2.176	2.176	8.354
18	-8.347	-2.186	-2.186	-2.186	2.186	2.186	2.186	8.347
19	-8.338	-2.196	-2.196	-2.196	2.196	2.196	2.196	8.338
20	-8.330	-2.207	-2.207	-2.207	2.207	2.207	2.207	8.330
21	-8.322	-2.218	-2.218	-2.218	2.217	2.218	2.218	8.322
22	-8.313	-2.228	-2.228	-2.228	2.228	2.228	2.228	8.313
23	-8.304	-2.239	-2.239	-2.239	2.239	2.239	2.239	8.304
24	-8.295	-2.250	-2.250	-2.250	2.250	2.250	2.250	8.295
25	-8.286	-2.261	-2.261	-2.261	2.261	2.261	2.261	8.286
26	-8.277	-2.273	-2.273	-2.272	2.273	2.273	2.273	8.277
27	-8.277	-2.273	-2.273	-2.273	2.273	2.273	2.272	8.277
28	-8.268	-2.284	-2.284	-2.284	2.284	2.284	2.284	8.268
29	-8.258	-2.295	-2.295	-2.295	2.295	2.295	2.295	8.258
30	-8.249	-2.306	-2.306	-2.306	2.306	2.306	2.306	8.249
31	-8.240	-2.317	-2.317	-2.317	2.317	2.317	2.317	8.240
32	-8.230	-2.328	-2.328	-2.328	2.328	2.328	2.328	8.230
33	-8.221	-2.339	-2.339	-2.339	2.339	2.339	2.339	8.221
34	-8.212	-2.350	-2.350	-2.350	2.350	2.350	2.350	8.212
35	-8.202	-2.361	-2.361	-2.361	2.361	2.361	2.361	8.202
36	-8.193	-2.372	-2.372	-2.372	2.372	2.372	2.372	8.193
37	-8.184	-2.382	-2.382	-2.382	2.382	2.382	2.382	8.184
38	-8.175	-2.392	-2.392	-2.392	2.392	2.392	2.392	8.175
39	-8.166	-2.402	-2.402	-2.402	2.402	2.402	2.402	8.166
40	-8.157	-2.412	-2.412	-2.412	2.412	2.412	2.412	8.157

Table 5

FIG. 29

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-8.149	-2.422	-2.422	-2.422	2.422	2.422	2.422	8.149
42	-8.141	-2.431	-2.431	-2.431	2.431	2.431	2.431	8.141
43	-8.132	-2.440	-2.440	-2.440	2.440	2.440	2.440	8.132
44	-8.124	-2.449	-2.449	-2.449	2.449	2.449	2.449	8.124
45	-8.117	-2.458	-2.458	-2.458	2.458	2.458	2.458	8.117
46	-8.109	-2.466	-2.466	-2.466	2.466	2.466	2.466	8.109
47	-8.102	-2.474	-2.474	-2.474	2.474	2.474	2.474	8.102
48	-8.095	-2.481	-2.481	-2.481	2.481	2.481	2.481	8.095
49	-8.088	-2.489	-2.489	-2.489	2.489	2.489	2.489	8.088
50	-8.082	-2.496	-2.496	-2.496	2.496	2.496	2.496	8.082
51	-8.075	-2.503	-2.503	-2.503	2.503	2.503	2.503	8.075
52	-8.075	-2.503	-2.503	-2.503	2.503	2.503	2.503	8.075
53	-8.069	-2.509	-2.509	-2.509	2.509	2.509	2.509	8.069
54	-8.063	-2.513	-2.513	-2.513	2.391	2.391	2.761	8.061
55	-8.031	-2.900	-2.900	-1.639	1.639	2.900	2.900	8.031
56	-8.012	-2.987	-2.987	-1.403	1.403	2.987	2.987	8.012
57	-7.993	-3.055	-3.055	-1.199	1.199	3.055	3.055	7.993
58	-7.976	-3.111	-3.111	-1.012	1.012	3.111	3.111	7.976
59	-7.960	-3.158	-3.158	-0.832	0.832	3.158	3.158	7.960
60	-7.945	-3.199	-3.199	-0.649	0.649	3.199	3.199	7.945
61	-7.930	-3.234	-3.234	-0.445	0.445	3.234	3.234	7.930
62	-7.916	-3.265	-3.265	-0.123	0.123	3.265	3.265	7.916
63	-7.906	-3.279	-3.279	0.000	0.000	3.279	3.279	7.906
64	-7.896	-3.291	-3.291	0.000	0.000	3.291	3.291	7.896
65	-7.885	-3.303	-3.303	0.000	0.000	3.303	3.303	7.885
66	-7.875	-3.316	-3.316	0.000	0.000	3.316	3.316	7.875
67	-7.862	-3.486	-3.168	0.000	0.000	3.168	3.486	7.862
68	-7.847	-3.621	-3.052	0.000	0.000	3.052	3.621	7.847
69	-7.833	-3.711	-2.979	0.000	0.000	2.979	3.711	7.833
70	-7.819	-3.782	-2.925	0.000	0.000	2.925	3.782	7.819
71	-7.806	-3.843	-2.881	0.000	0.000	2.881	3.843	7.806
72	-7.802	-3.860	-2.870	0.000	0.000	2.870	3.860	7.802
73	-7.793	-3.897	-2.844	0.000	0.000	2.844	3.897	7.793
74	-7.780	-3.945	-2.812	0.000	0.000	2.812	3.945	7.780
75	-7.768	-3.990	-2.783	0.000	0.000	2.783	3.990	7.768
76	-7.756	-4.031	-2.756	0.000	0.000	2.756	4.031	7.756
77	-7.744	-4.075	-2.720	-0.164	0.164	2.720	4.075	7.744
78	-7.730	-4.131	-2.656	-0.362	0.362	2.656	4.131	7.730
79	-7.717	-4.178	-2.603	-0.471	0.471	2.603	4.178	7.717
80	-7.704	-4.219	-2.560	-0.549	0.549	2.560	4.219	7.704

Table 6

FIG. 30

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-7.692	-4.255	-2.524	-0.608	0.608	2.524	4.255	7.692
82	-7.680	-4.286	-2.494	-0.654	0.654	2.494	4.286	7.680
83	-7.669	-4.314	-2.471	-0.690	0.690	2.471	4.314	7.669
84	-7.657	-4.340	-2.452	-0.718	0.718	2.452	4.340	7.657
85	-7.647	-4.363	-2.438	-0.740	0.740	2.438	4.363	7.647
86	-7.636	-4.385	-2.428	-0.757	0.757	2.428	4.385	7.636
87	-7.625	-4.405	-2.422	-0.770	0.770	2.422	4.405	7.625
88	-7.614	-4.424	-2.418	-0.780	0.780	2.418	4.424	7.614
89	-7.603	-4.441	-2.417	-0.787	0.787	2.417	4.441	7.603
90	-7.592	-4.458	-2.419	-0.792	0.792	2.419	4.458	7.592
91	-7.589	-4.464	-2.420	-0.793	0.793	2.420	4.464	7.589
92	-7.581	-4.475	-2.423	-0.795	0.795	2.423	4.475	7.581
93	-7.570	-4.491	-2.428	-0.797	0.797	2.428	4.491	7.570
94	-7.558	-4.507	-2.435	-0.799	0.799	2.435	4.507	7.558
95	-7.546	-4.522	-2.444	-0.800	0.800	2.444	4.522	7.546
96	-7.533	-4.538	-2.453	-0.800	0.800	2.453	4.538	7.533
97	-7.521	-4.553	-2.464	-0.801	0.801	2.464	4.553	7.521
98	-7.508	-4.568	-2.475	-0.802	0.802	2.475	4.568	7.508
99	-7.494	-4.583	-2.488	-0.804	0.804	2.488	4.583	7.494
100	-7.481	-4.597	-2.501	-0.805	0.805	2.501	4.597	7.481
101	-7.467	-4.612	-2.514	-0.808	0.808	2.514	4.612	7.467
102	-7.453	-4.626	-2.529	-0.811	0.811	2.529	4.626	7.453
103	-7.439	-4.641	-2.543	-0.815	0.815	2.543	4.641	7.439
104	-7.424	-4.655	-2.559	-0.819	0.819	2.559	4.655	7.424
105	-7.409	-4.669	-2.574	-0.824	0.824	2.574	4.669	7.409
106	-7.395	-4.682	-2.590	-0.830	0.830	2.590	4.682	7.395
107	-7.380	-4.695	-2.607	-0.836	0.836	2.607	4.695	7.380
108	-7.365	-4.708	-2.623	-0.842	0.842	2.623	4.708	7.365
109	-7.350	-4.721	-2.640	-0.849	0.849	2.640	4.721	7.350
110	-7.340	-4.729	-2.651	-0.853	0.853	2.651	4.729	7.340
111	-7.336	-4.733	-2.656	-0.855	0.855	2.656	4.733	7.336
112	-7.322	-4.745	-2.672	-0.862	0.862	2.672	4.745	7.322
113	-7.308	-4.756	-2.687	-0.869	0.869	2.687	4.756	7.308
114	-7.294	-4.767	-2.703	-0.875	0.875	2.703	4.767	7.294
115	-7.281	-4.778	-2.717	-0.882	0.882	2.717	4.778	7.281
116	-7.269	-4.788	-2.731	-0.888	0.888	2.731	4.788	7.269
117	-7.256	-4.798	-2.744	-0.893	0.893	2.744	4.798	7.256
118	-7.245	-4.807	-2.757	-0.899	0.899	2.757	4.807	7.245
119	-7.233	-4.816	-2.769	-0.904	0.904	2.769	4.816	7.233
120	-7.223	-4.824	-2.781	-0.909	0.909	2.781	4.824	7.223

Table 7

FIG. 31

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-7.213	-4.832	-2.792	-0.914	0.914	2.792	4.832	7.213
122	-7.203	-4.840	-2.802	-0.918	0.918	2.802	4.840	7.203
123	-7.194	-4.847	-2.812	-0.922	0.922	2.812	4.847	7.194
124	-7.185	-4.854	-2.821	-0.926	0.926	2.821	4.854	7.185
125	-7.176	-4.861	-2.830	-0.930	0.930	2.830	4.861	7.176
126	-7.168	-4.867	-2.838	-0.933	0.933	2.838	4.867	7.168
127	-7.161	-4.873	-2.846	-0.937	0.937	2.846	4.873	7.161
128	-7.154	-4.879	-2.853	-0.940	0.940	2.853	4.879	7.154
129	-7.147	-4.885	-2.860	-0.943	0.943	2.860	4.885	7.147
130	-7.140	-4.890	-2.867	-0.945	0.945	2.867	4.890	7.140
131	-7.134	-4.895	-2.873	-0.948	0.948	2.873	4.895	7.134
132	-7.128	-4.900	-2.879	-0.950	0.950	2.879	4.900	7.128
133	-7.122	-4.904	-2.885	-0.953	0.953	2.885	4.904	7.122
134	-7.116	-4.909	-2.890	-0.955	0.955	2.890	4.909	7.116
135	-7.111	-4.913	-2.895	-0.957	0.957	2.895	4.913	7.111
136	-7.106	-4.917	-2.900	-0.959	0.959	2.900	4.917	7.106
137	-7.102	-4.920	-2.904	-0.961	0.961	2.904	4.920	7.102
138	-7.097	-4.924	-2.909	-0.963	0.963	2.909	4.924	7.097
139	-7.093	-4.927	-2.913	-0.965	0.965	2.913	4.927	7.093
140	-7.089	-4.931	-2.917	-0.966	0.966	2.917	4.931	7.089
141	-7.084	-4.934	-2.921	-0.968	0.968	2.921	4.934	7.084
142	-7.081	-4.937	-2.924	-0.969	0.969	2.924	4.937	7.081
143	-7.077	-4.940	-2.928	-0.971	0.971	2.928	4.940	7.077
144	-7.074	-4.942	-2.931	-0.972	0.972	2.931	4.942	7.074
145	-7.071	-4.945	-2.934	-0.973	0.973	2.934	4.945	7.071
146	-7.068	-4.947	-2.937	-0.974	0.974	2.937	4.947	7.068
147	-7.064	-4.950	-2.941	-0.976	0.976	2.941	4.950	7.064
148	-7.062	-4.952	-2.942	-0.977	0.977	2.942	4.952	7.062

Table 8

FIG. 32

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.12	0.11	0.1	0.09	0.06	0
2	-4.8	0.12	0.11	0.1	0.09	0.06	0
3	-4.6	0.13	0.12	0.11	0.09	0.08	0
4	-4.4	0.13	0.12	0.1	0.09	0.07	0
5	-4.2	0.14	0.13	0.11	0.1	0.07	0
6	-4	0.15	0.14	0.12	0.11	0.08	0
7	-3.8	0.15	0.14	0.12	0.11	0.08	0
8	-3.6	0.17	0.15	0.14	0.12	0.09	0
9	-3.4	0.18	0.16	0.15	0.13	0.09	0
10	-3.2	0.18	0.16	0.15	0.13	0.1	0
11	-3	0.2	0.18	0.16	0.14	0.1	0
12	-2.8	0.19	0.17	0.16	0.14	0.1	0
13	-2.6	0.21	0.19	0.17	0.15	0.11	0
14	-2.4	0.23	0.21	0.19	0.17	0.12	0
15	-2.2	0.23	0.2	0.18	0.17	0.12	0
16	-2	0.25	0.22	0.2	0.18	0.13	0
17	-1.8	0.27	0.24	0.22	0.2	0.13	0
18	-1.6	0.26	0.24	0.21	0.19	0.14	0
19	-1.4	0.29	0.26	0.23	0.21	0.15	0
20	-1.2	0.28	0.25	0.23	0.2	0.15	0
21	-1	0.3	0.27	0.25	0.22	0.16	0
22	-0.8	0.33	0.3	0.27	0.24	0.18	0
23	-0.6	0.32	0.29	0.26	0.23	0.17	0
24	-0.4	0.35	0.31	0.28	0.25	0.18	0
25	-0.2	0.34	0.3	0.27	0.25	0.2	0
26	0	0.33	0.3	0.27	0.24	0.19	0
27	0.01	0.33	0.3	0.27	0.24	0.19	0
28	0.2	0.32	0.29	0.26	0.23	0.21	0
29	0.4	0.31	0.28	0.25	0.23	0.2	0
30	0.6	0.3	0.27	0.24	0.22	0.2	0
31	0.8	0.29	0.26	0.23	0.21	0.19	0
32	1	0.28	0.25	0.23	0.2	0.18	0
33	1.2	0.27	0.24	0.22	0.2	0.18	0
34	1.4	0.26	0.24	0.21	0.19	0.17	0
35	1.6	0.25	0.23	0.2	0.18	0.17	0
36	1.8	0.24	0.22	0.2	0.18	0.16	0
37	2	0.23	0.21	0.19	0.17	0.15	0
38	2.2	0.22	0.2	0.18	0.16	0.15	0
39	2.4	0.22	0.19	0.17	0.16	0.14	0
40	2.6	0.21	0.19	0.17	0.15	0.14	0

Table 9

FIG. 33

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
41	2.8	0.2	0.18	0.16	0.14	0.13	0
42	3	0.19	0.17	0.15	0.14	0.13	0
43	3.2	0.18	0.16	0.15	0.13	0.12	0
44	3.4	0.17	0.16	0.14	0.13	0.11	0
45	3.6	0.17	0.15	0.14	0.12	0.11	0
46	3.8	0.16	0.14	0.13	0.12	0.11	0
47	4	0.15	0.14	0.12	0.11	0.1	0
48	4.2	0.15	0.13	0.12	0.11	0.1	0
49	4.4	0.14	0.13	0.11	0.1	0.09	0
50	4.6	0.13	0.12	0.11	0.1	0.09	0
51	4.8	0.13	0.11	0.1	0.09	0.08	0
52	4.82	0.13	0.11	0.1	0.09	0.08	0
53	5	0.12	0.11	0.1	0.09	0.08	0
54	5.2	0.2	0.18	0.16	0.15	0.13	0
55	5.4	0.57	0.51	0.46	0.41	0.27	0
56	5.6	0.56	0.51	0.45	0.41	0.3	0
57	5.8	0.53	0.47	0.43	0.38	0.28	0
58	6	0.56	0.51	0.46	0.41	0.3	0
59	6.2	0.54	0.49	0.44	0.39	0.29	0
60	6.4	0.55	0.49	0.44	0.4	0.29	0
61	6.6	0.55	0.43	0.39	0.35	0.31	0
62	6.8	0.56	0.34	0.31	0.28	0.25	0
63	7	0.56	0.3	0.27	0.25	0.22	0
64	7.2	0.56	0.3	0.27	0.24	0.22	0
65	7.4	0.56	0.29	0.26	0.23	0.21	0
66	7.6	0.57	0.28	0.25	0.23	0.2	0
67	7.8	0.53	0.27	0.24	0.22	0.2	0
68	8	0.54	0.26	0.24	0.21	0.19	0
69	8.2	0.53	0.26	0.23	0.21	0.19	0
70	8.4	0.53	0.25	0.23	0.2	0.18	0
71	8.6	0.53	0.24	0.22	0.2	0.18	0
72	8.66	0.53	0.24	0.22	0.2	0.18	0
73	8.8	0.48	0.24	0.21	0.19	0.17	0
74	9	0.47	0.23	0.21	0.19	0.17	0
75	9.2	0.47	0.22	0.2	0.18	0.16	0
76	9.4	0.47	0.22	0.2	0.18	0.16	0
77	9.6	0.46	0.25	0.23	0.2	0.18	0
78	9.8	0.35	0.29	0.26	0.24	0.21	0
79	10	0.26	0.23	0.21	0.19	0.17	0
80	10.2	0.2	0.18	0.16	0.14	0.13	0

Table 10

FIG. 34

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	0.15	0.13	0.12	0.11	0.1	0
82	10.6	0.11	0.1	0.09	0.08	0.07	0
83	10.8	0.08	0.07	0.06	0.06	0.05	0
84	11	0.05	0.05	0.04	0.04	0.04	0
85	11.2	0.04	0.03	0.03	0.03	0.02	0
86	11.4	0.02	0.02	0.02	0.02	0.01	0
87	11.6	0.01	0.01	0.01	0.01	0.01	0
88	11.8	0.01	0.01	0	0	0	0
89	12	0.03	0.02	0.01	0.01	0.01	0
90	12.2	0.04	0.03	0.02	0.02	0.02	0
91	12.26	0.04	0.04	0.03	0.02	0.02	0
92	12.4	0.05	0.05	0.03	0.03	0.02	0
93	12.6	0.06	0.06	0.04	0.03	0.03	0
94	12.8	0.08	0.07	0.05	0.04	0.04	0
95	13	0.09	0.08	0.05	0.05	0.04	0
96	13.2	0.1	0.09	0.06	0.05	0.05	0
97	13.4	0.12	0.1	0.07	0.06	0.06	0
98	13.6	0.13	0.12	0.08	0.07	0.07	0
99	13.8	0.14	0.13	0.09	0.08	0.08	0
100	14	0.15	0.14	0.1	0.09	0.08	0
101	14.2	0.17	0.15	0.11	0.1	0.09	0
102	14.4	0.18	0.16	0.13	0.11	0.1	0
103	14.6	0.19	0.17	0.14	0.13	0.11	0
104	14.8	0.21	0.19	0.15	0.14	0.11	0
105	15	0.2	0.18	0.16	0.14	0.1	0
106	15.2	0.21	0.19	0.17	0.14	0.1	0
107	15.4	0.18	0.16	0.14	0.13	0.09	0
108	15.6	0.17	0.15	0.14	0.12	0.09	0
109	15.8	0.16	0.14	0.13	0.12	0.08	0
110	15.93	0.16	0.15	0.13	0.12	0.09	0
111	16	0.15	0.13	0.12	0.11	0.08	0
112	16.2	0.14	0.13	0.11	0.1	0.07	0
113	16.4	0.13	0.12	0.11	0.1	0.07	0
114	16.6	0.13	0.11	0.1	0.09	0.07	0
115	16.8	0.12	0.11	0.1	0.09	0.06	0
116	17	0.12	0.1	0.09	0.08	0.06	0
117	17.2	0.11	0.1	0.09	0.08	0.06	0
118	17.4	0.1	0.09	0.08	0.08	0.06	0
119	17.6	0.1	0.09	0.08	0.07	0.05	0
120	17.8	0.09	0.09	0.08	0.07	0.05	0

Table 11

FIG. 35

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.09	0.08	0.07	0.07	0.05	0
122	18.2	0.09	0.08	0.07	0.06	0.05	0
123	18.4	0.08	0.07	0.07	0.06	0.04	0
124	18.6	0.08	0.07	0.06	0.06	0.04	0
125	18.8	0.07	0.07	0.06	0.05	0.04	0
126	19	0.07	0.06	0.06	0.05	0.04	0
127	19.2	0.07	0.06	0.05	0.05	0.04	0
128	19.4	0.06	0.06	0.05	0.05	0.03	0
129	19.6	0.06	0.06	0.05	0.04	0.03	0
130	19.8	0.06	0.05	0.05	0.04	0.03	0
131	20	0.06	0.05	0.05	0.04	0.03	0
132	20.2	0.05	0.05	0.04	0.04	0.03	0
133	20.4	0.05	0.05	0.04	0.04	0.02	0
134	20.6	0.05	0.04	0.04	0.04	0.02	0
135	20.8	0.04	0.04	0.03	0.03	0.02	0
136	21	0.04	0.04	0.04	0.03	0.02	0
137	21.2	0.04	0.04	0.03	0.03	0.02	0
138	21.4	0.04	0.03	0.03	0.03	0.02	0
139	21.6	0.03	0.03	0.03	0.03	0.02	0
140	21.8	0.03	0.03	0.03	0.02	0.02	0
141	22	0.03	0.03	0.03	0.02	0.02	0
142	22.2	0.03	0.03	0.02	0.02	0.02	0
143	22.4	0.03	0.03	0.02	0.02	0.02	0
144	22.6	0.03	0.02	0.02	0.02	0.01	0
145	22.8	0.03	0.02	0.02	0.02	0.01	0
146	23	0.02	0.02	0.02	0.02	0.01	0
147	23.2	0.02	0.02	0.02	0.02	0.01	0
148	23.4	0.02	0.02	0.02	0.02	0.01	0

Table 12

FIG. 36

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1977	0.1598	0.0379	23.7349
2	-4.8000	0.2058	0.1666	0.0391	23.4911
3	-4.6000	0.2141	0.1737	0.0404	23.2414
4	-4.4000	0.2227	0.1811	0.0416	22.9855
5	-4.2000	0.2316	0.1887	0.0429	22.7233
6	-4.0000	0.2407	0.1966	0.0441	22.4546
7	-3.8000	0.2502	0.2048	0.0454	22.1792
8	-3.6000	0.2600	0.2133	0.0467	21.8969
9	-3.4000	0.2700	0.2220	0.0480	21.6075
10	-3.2000	0.2804	0.2311	0.0493	21.3107
11	-3.0000	0.2910	0.2405	0.0505	21.0063
12	-2.8000	0.3020	0.2502	0.0518	20.6939
13	-2.6000	0.3133	0.2602	0.0530	20.3732
14	-2.4000	0.3248	0.2706	0.0542	20.0439
15	-2.2000	0.3367	0.2813	0.0554	19.7054
16	-2.0000	0.3489	0.2923	0.0566	19.3575
17	-1.8000	0.3613	0.3037	0.0577	18.9996
18	-1.6000	0.3741	0.3154	0.0588	18.6313
19	-1.4000	0.3872	0.3274	0.0598	18.2519
20	-1.2000	0.4005	0.3398	0.0607	17.8609
21	-1.0000	0.4141	0.3526	0.0615	17.4577
22	-0.8000	0.4280	0.3657	0.0623	17.0415
23	-0.6000	0.4421	0.3791	0.0630	16.6117
24	-0.4000	0.4565	0.3930	0.0635	16.1675
25	-0.2000	0.4711	0.4072	0.0640	15.7080
26	0.0000	0.4859	0.4217	0.0642	15.2325
27	0.1871	0.5000	0.4356	0.0644	14.7724
28	0.2000	0.5010	0.4366	0.0644	14.7399
29	0.4000	0.5162	0.4519	0.0643	14.2295
30	0.6000	0.5316	0.4675	0.0641	13.7002
31	0.8000	0.5471	0.4835	0.0636	13.1510
32	1.0000	0.5628	0.4999	0.0629	12.5809
33	1.2000	0.5786	0.5166	0.0619	11.9887
34	1.4000	0.5944	0.5337	0.0607	11.3736
35	1.6000	0.6104	0.5511	0.0593	10.7597
36	1.8000	0.6270	0.5689	0.0581	10.2106
37	2.0000	0.6442	0.5871	0.0571	9.7259
38	2.2000	0.6619	0.6056	0.0563	9.3017
39	2.4000	0.6802	0.6244	0.0558	8.9342
40	2.6000	0.6991	0.6437	0.0555	8.6193

Table 13

FIG. 37

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7186	0.6632	0.0554	8.3527
42	3.0000	0.7386	0.6831	0.0555	8.1300
43	3.2000	0.7593	0.7034	0.0559	7.9468
44	3.4000	0.7804	0.7240	0.0565	7.7983
45	3.6000	0.8021	0.7449	0.0572	7.6801
46	3.8000	0.8243	0.7662	0.0581	7.5872
47	4.0000	0.8470	0.7878	0.0592	7.5148
48	4.2000	0.8702	0.8098	0.0604	7.4581
49	4.4000	0.8938	0.8321	0.0617	7.4120
50	4.6000	0.9178	0.8548	0.0630	7.3718
51	4.8000	0.9421	0.8778	0.0644	7.3328
52	5.0000	0.9668	0.9011	0.0657	7.2903
53	5.2000	0.9917	0.9248	0.0670	7.2406
54	5.2657	1.0000	0.9326	0.0674	7.2219
55	5.4000	1.0169	0.9488	0.0681	7.1793
56	5.6000	1.0423	0.9732	0.0691	7.1031
57	5.8000	1.0678	0.9979	0.0699	7.0087
58	6.0000	1.0934	1.0229	0.0705	6.8934
59	6.2000	1.1191	1.0483	0.0708	6.7550
60	6.4000	1.1448	1.0740	0.0708	6.5919
61	6.6000	1.1704	1.1000	0.0704	6.4032
62	6.8000	1.1960	1.1263	0.0697	6.1889
63	7.0000	1.2216	1.1530	0.0686	5.9490
64	7.2000	1.2470	1.1799	0.0671	5.6843
65	7.4000	1.2724	1.2072	0.0652	5.4035
66	7.6000	1.2980	1.2348	0.0632	5.1173
67	7.8000	1.3238	1.2626	0.0612	4.8477
68	8.0000	1.3500	1.2907	0.0593	4.5905
69	8.2000	1.3780	1.3191	0.0589	4.4669
70	8.4000	1.4086	1.3477	0.0609	4.5190
71	8.6000	1.4392	1.3766	0.0626	4.5447
72	8.8000	1.4696	1.4057	0.0639	4.5449
73	9.0000	1.4999	1.4350	0.0649	4.5208
74	9.0008	1.5000	1.4351	0.0649	4.5207
75	9.2000	1.5300	1.4645	0.0655	4.4740
76	9.4000	1.5600	1.4942	0.0658	4.4056
77	9.6000	1.5899	1.5241	0.0658	4.3170
78	9.8000	1.6195	1.5541	0.0654	4.2101
79	10.0000	1.6490	1.5843	0.0647	4.0864
80	10.2000	1.6783	1.6145	0.0637	3.9475

Table 14

FIG. 38

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7074	1.6449	0.0624	3.7953
82	10.6000	1.7363	1.6754	0.0609	3.6327
83	10.8000	1.7650	1.7060	0.0591	3.4625
84	11.0000	1.7937	1.7366	0.0571	3.2884
85	11.2000	1.8223	1.7672	0.0551	3.1155
86	11.4000	1.8509	1.7979	0.0530	2.9491
87	11.6000	1.8797	1.8286	0.0511	2.7943
88	11.8000	1.9086	1.8593	0.0494	2.6548
89	12.0000	1.9378	1.8899	0.0478	2.5317
90	12.2000	1.9671	1.9206	0.0466	2.4250
91	12.4000	1.9966	1.9511	0.0455	2.3327
92	12.4227	2.0000	1.9546	0.0454	2.3230
93	12.6000	2.0263	1.9816	0.0446	2.2522
94	12.8000	2.0559	2.0121	0.0439	2.1810
95	13.0000	2.0856	2.0424	0.0432	2.1167
96	13.2000	2.1153	2.0726	0.0426	2.0569
97	13.4000	2.1448	2.1028	0.0421	1.9998
98	13.6000	2.1742	2.1328	0.0415	1.9437
99	13.8000	2.2035	2.1627	0.0408	1.8875
100	14.0000	2.2326	2.1925	0.0401	1.8301
101	14.2000	2.2614	2.2221	0.0394	1.7711
102	14.4000	2.2901	2.2516	0.0385	1.7096
103	14.6000	2.3184	2.2809	0.0375	1.6457
104	14.8000	2.3465	2.3100	0.0365	1.5792
105	15.0000	2.3743	2.3390	0.0353	1.5104
106	15.2000	2.4018	2.3678	0.0341	1.4395
107	15.4000	2.4290	2.3963	0.0328	1.3670
108	15.6000	2.4559	2.4246	0.0314	1.2934
109	15.8000	2.4825	2.4526	0.0299	1.2192
110	15.9336	2.5000	2.4711	0.0289	1.1703
111	16.0000	2.5087	2.4802	0.0284	1.1457
112	16.2000	2.5344	2.5076	0.0269	1.0722
113	16.4000	2.5598	2.5345	0.0253	1.0000
114	16.6000	2.5848	2.5610	0.0238	0.9296
115	16.8000	2.6093	2.5870	0.0223	0.8615
116	17.0000	2.6332	2.6124	0.0208	0.7960
117	17.2000	2.6567	2.6373	0.0193	0.7333
118	17.4000	2.6795	2.6616	0.0179	0.6737
119	17.6000	2.7017	2.6851	0.0166	0.6171
120	17.8000	2.7232	2.7080	0.0153	0.5637

Table 15

FIG. 39

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
121	18.0000	2.7440	2.7300	0.0140	0.5134
122	18.2000	2.7641	2.7513	0.0128	0.4663
123	18.4000	2.7834	2.7717	0.0117	0.4222
124	18.6000	2.8018	2.7912	0.0106	0.3810

Table 16

FIG. 40

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.583	4.583
2	-4.584	-4.584	-4.581	-4.582	4.584	4.582	4.582	4.582
3	-4.584	-4.582	-4.582	-4.582	4.584	4.584	4.579	4.584
4	-4.584	-4.582	-4.582	-4.582	4.583	4.583	4.582	4.583
5	-4.583	-4.583	-4.583	-4.583	4.586	4.586	4.579	4.579
6	-4.585	-4.582	-4.582	-4.582	4.584	4.584	4.580	4.583
7	-4.587	-4.587	-4.578	-4.578	4.583	4.583	4.582	4.583
8	-4.585	-4.585	-4.580	-4.580	4.583	4.583	4.583	4.583
9	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.583
10	-4.585	-4.584	-4.581	-4.581	4.583	4.583	4.583	4.583
11	-4.583	-4.583	-4.583	-4.583	4.584	4.584	4.581	4.581
12	-4.583	-4.583	-4.582	-4.582	4.585	4.582	4.582	4.582
13	-4.587	-4.581	-4.581	-4.581	4.587	4.582	4.581	4.581
14	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.582	4.583
15	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.583
16	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.583	4.583
17	-4.587	-4.582	-4.580	-4.581	4.587	4.582	4.581	4.582
18	-4.583	-4.583	-4.582	-4.583	4.584	4.584	4.581	4.581
19	-4.583	-4.583	-4.582	-4.582	4.584	4.584	4.581	4.582
20	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
21	-4.583	-4.583	-4.582	-4.583	4.583	4.584	4.582	4.582
22	-4.584	-4.584	-4.580	-4.583	4.587	4.588	4.576	4.579
23	-4.589	-4.586	-4.578	-4.578	4.588	4.581	4.581	4.581
24	-4.584	-4.584	-4.581	-4.581	4.583	4.583	4.582	4.583
25	-4.585	-4.582	-4.582	-4.582	4.584	4.583	4.582	4.582
26	-4.583	-4.583	-4.583	-4.583	4.583	4.583	4.582	4.583
27	-4.584	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
28	-4.583	-4.583	-4.583	-4.583	4.583	4.583	4.582	4.582
29	-4.583	-4.582	-4.582	-4.582	4.583	4.583	4.583	4.583
30	-4.587	-4.587	-4.578	-4.579	4.587	4.587	4.579	4.579
31	-4.584	-4.583	-4.582	-4.582	4.584	4.584	4.581	4.582
32	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
33	-4.584	-4.584	-4.578	-4.584	4.584	4.584	4.580	4.582
34	-4.584	-4.584	-4.578	-4.584	4.584	4.584	4.578	4.584
35	-5.110	-5.110	-3.986	-3.986	5.110	5.110	3.986	3.986
36	-5.355	-5.354	-3.651	-3.651	5.355	5.355	3.650	3.650
37	-5.515	-5.514	-3.404	-3.404	5.515	5.515	3.404	3.404
38	-5.638	-5.638	-3.196	-3.196	5.640	5.637	3.196	3.196
39	-5.729	-5.729	-3.029	-3.029	5.729	5.729	3.029	3.029
40	-5.808	-5.807	-2.876	-2.876	5.808	5.808	2.876	2.876

Table 17

FIG. 41

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-5.874	-5.874	-2.739	-2.739	5.874	5.872	2.740	2.740
42	-5.929	-5.929	-2.617	-2.617	5.929	5.929	2.617	2.617
43	-5.978	-5.975	-2.506	-2.506	5.978	5.976	2.505	2.505
44	-6.018	-6.018	-2.405	-2.405	6.018	6.018	2.405	2.405
45	-6.055	-6.054	-2.312	-2.312	6.054	6.054	2.312	2.312
46	-6.087	-6.085	-2.227	-2.227	6.087	6.086	2.227	2.227
47	-6.113	-6.113	-2.151	-2.151	6.113	6.113	2.151	2.151
48	-6.138	-6.138	-2.081	-2.081	6.138	6.137	2.081	2.081
49	-6.159	-6.159	-2.017	-2.017	6.159	6.159	2.017	2.017
50	-6.179	-6.176	-1.960	-1.960	6.178	6.178	1.959	1.959
51	-6.194	-6.193	-1.907	-1.907	6.194	6.194	1.907	1.907
52	-6.208	-6.208	-1.860	-1.860	6.208	6.208	1.860	1.860
53	-6.221	-6.220	-1.819	-1.819	6.221	6.219	1.819	1.820
54	-6.224	-6.224	-1.807	-1.807	6.224	6.224	1.807	1.807
55	-6.231	-6.230	-1.783	-1.783	6.231	6.231	1.783	1.783
56	-6.240	-6.240	-1.752	-1.752	6.242	6.237	1.752	1.752
57	-6.247	-6.247	-1.725	-1.725	6.247	6.247	1.725	1.725
58	-6.254	-6.253	-1.703	-1.703	6.253	6.253	1.703	1.703
59	-6.260	-6.256	-1.686	-1.686	6.260	6.256	1.686	1.686
60	-6.261	-6.261	-1.673	-1.673	6.261	6.261	1.673	1.673
61	-6.263	-6.263	-1.665	-1.665	6.263	6.263	1.665	1.665
62	-6.264	-6.264	-1.661	-1.661	6.264	6.264	1.661	1.661
63	-6.265	-6.265	-1.660	-1.660	6.265	6.264	1.660	1.660
64	-6.264	-6.264	-1.663	-1.663	6.291	6.237	1.658	1.667
65	-6.734	-5.796	-1.570	-1.737	6.244	6.244	1.674	1.674
66	-6.781	-5.697	-1.582	-1.750	6.781	5.697	1.582	1.750
67	-6.914	-5.533	-1.550	-1.781	6.915	5.533	1.549	1.781
68	-7.016	-5.403	-1.517	-1.809	7.017	5.403	1.517	1.809
69	-7.923	-3.658	7.923	-2.800	2.800	0.035	3.659	-0.035
70	-7.913	-3.668	7.913	-2.816	2.816	0.052	3.668	-0.052
71	-7.902	-3.680	7.902	-2.831	2.831	0.068	3.680	-0.068
72	-7.890	-3.694	7.890	-2.845	2.845	0.084	3.694	-0.084
73	-7.878	-3.710	7.878	-2.859	2.859	0.099	3.710	-0.099
74	-7.878	-3.710	7.878	-2.859	2.859	0.099	3.710	-0.099
75	-7.864	-3.728	7.864	-2.872	2.872	0.115	3.728	-0.115
76	-7.850	-3.749	7.850	-2.883	2.883	0.131	3.749	-0.131
77	-7.835	-3.772	7.835	-2.893	2.893	0.148	3.772	-0.148
78	-7.819	-3.798	7.819	-2.902	2.902	0.166	3.798	-0.166
79	-7.802	-3.827	7.802	-2.908	2.908	0.185	3.827	-0.185
80	-7.784	-3.859	7.784	-2.911	2.911	0.206	3.859	-0.206

Table 18

FIG. 42

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-7.765	-3.896	7.765	-2.911	2.911	0.230	3.896	-0.230
82	-7.745	-3.937	7.745	-2.907	2.907	0.257	3.937	-0.257
83	-7.724	-3.983	7.724	-2.897	2.897	0.290	3.983	-0.290
84	-7.701	-4.036	7.701	-2.880	2.880	0.328	4.036	-0.328
85	-7.677	-4.095	7.677	-2.857	2.857	0.374	4.095	-0.374
86	-7.651	-4.157	7.651	-2.828	2.829	0.425	4.157	-0.425
87	-7.625	-4.220	7.625	-2.797	2.797	0.479	4.220	-0.479
88	-7.600	-4.280	7.600	-2.765	2.765	0.533	4.280	-0.533
89	-7.576	-4.334	7.576	-2.734	2.734	0.585	4.334	-0.585
90	-7.555	-4.382	7.555	-2.707	2.707	0.632	4.382	-0.632
91	-7.536	-4.423	7.536	-2.682	2.682	0.672	4.423	-0.672
92	-7.534	-4.428	7.534	-2.680	2.680	0.677	4.428	-0.677
93	-7.519	-4.459	7.519	-2.662	2.662	0.707	4.459	-0.707
94	-7.504	-4.489	7.504	-2.644	2.644	0.736	4.489	-0.736
95	-7.491	-4.515	7.491	-2.630	2.630	0.761	4.515	-0.761
96	-7.479	-4.538	7.479	-2.618	2.618	0.780	4.538	-0.780
97	-7.469	-4.558	7.469	-2.609	2.609	0.796	4.558	-0.796
98	-7.459	-4.576	7.459	-2.603	2.603	0.808	4.576	-0.808
99	-7.449	-4.593	7.449	-2.598	2.599	0.818	4.593	-0.817
100	-7.439	-4.608	7.439	-2.597	2.597	0.825	4.608	-0.825
101	-7.430	-4.623	7.430	-2.597	2.597	0.831	4.623	-0.830
102	-7.420	-4.636	7.420	-2.599	2.599	0.835	4.636	-0.835
103	-7.410	-4.650	7.410	-2.603	2.603	0.838	4.650	-0.838
104	-7.399	-4.663	7.399	-2.608	2.608	0.841	4.663	-0.841
105	-7.389	-4.675	7.388	-2.615	2.615	0.844	4.675	-0.844
106	-7.377	-4.688	7.377	-2.624	2.624	0.847	4.688	-0.847
107	-7.366	-4.700	7.366	-2.634	2.633	0.850	4.700	-0.850
108	-7.354	-4.712	7.354	-2.644	2.644	0.854	4.712	-0.854
109	-7.342	-4.724	7.342	-2.656	2.656	0.858	4.724	-0.858
110	-7.334	-4.732	-0.862	-2.661	7.334	4.732	0.862	2.661
111	-7.330	-4.736	-0.863	-2.666	7.329	4.736	0.863	2.666
112	-7.317	-4.747	-0.868	-2.679	7.317	4.747	0.868	2.679
113	-7.304	-4.758	-0.873	-2.693	7.304	4.758	0.873	2.693
114	-7.292	-4.768	-0.878	-2.707	7.292	4.768	0.878	2.707
115	-7.279	-4.778	-0.884	-2.720	7.279	4.778	0.884	2.720
116	-7.267	-4.788	-0.889	-2.733	7.267	4.788	0.889	2.733
117	-7.255	-4.798	-0.894	-2.746	7.256	4.798	0.894	2.746
118	-7.244	-4.807	-0.899	-2.758	7.244	4.807	0.899	2.758
119	-7.233	-4.816	-0.904	-2.770	7.233	4.816	0.904	2.770
120	-7.223	-4.824	-0.909	-2.781	7.223	4.824	0.909	2.781

Table 19

FIG. 43

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-7.212	-4.832	-0.914	-2.792	7.212	4.832	0.914	2.792
122	-7.203	-4.840	-0.918	-2.802	7.203	4.840	0.918	2.802
123	-7.194	-4.847	-0.922	-2.812	7.194	4.847	0.922	2.812
124	-7.185	-4.854	-0.926	-2.821	7.185	4.854	0.926	2.821

Table 20

FIG. 44

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
65	7.4	0.62	0.22	0.2	0.18	0.16	0
66	7.6	0.55	0.21	0.19	0.17	0.15	0
67	7.8	0.57	0.26	0.23	0.21	0.19	0
68	8	0.5	0.3	0.27	0.24	0.22	0
69	8.2	0.46	0.34	0.3	0.27	0.25	0
70	8.4	0.45	0.33	0.3	0.27	0.24	0
71	8.6	0.44	0.33	0.3	0.27	0.24	0
72	8.8	0.48	0.32	0.29	0.26	0.24	0
73	9	0.47	0.32	0.29	0.26	0.23	0
74	9	0.47	0.32	0.29	0.26	0.23	0
75	9.2	0.46	0.31	0.28	0.25	0.23	0
76	9.4	0.45	0.3	0.27	0.25	0.22	0
77	9.6	0.43	0.3	0.27	0.24	0.22	0
78	9.8	0.42	0.29	0.26	0.23	0.21	0
79	10	0.41	0.28	0.25	0.23	0.21	0
80	10.2	0.4	0.27	0.25	0.22	0.2	0
81	10.4	0.39	0.27	0.24	0.22	0.19	0
82	10.6	0.42	0.26	0.23	0.21	0.19	0
83	10.8	0.39	0.26	0.23	0.21	0.19	0
84	11	0.37	0.25	0.23	0.2	0.18	0
85	11.2	0.37	0.25	0.22	0.2	0.18	0
86	11.4	0.33	0.25	0.22	0.2	0.18	0
87	11.6	0.29	0.24	0.22	0.2	0.18	0
88	11.8	0.24	0.22	0.2	0.18	0.16	0
89	12	0.22	0.2	0.18	0.16	0.14	0
90	12.2	0.2	0.18	0.17	0.15	0.13	0
91	12.4	0.19	0.17	0.15	0.13	0.12	0
92	12.42	0.19	0.17	0.15	0.13	0.12	0
93	12.6	0.18	0.16	0.14	0.12	0.11	0
94	12.8	0.17	0.16	0.14	0.13	0.11	0
95	13	0.17	0.15	0.13	0.12	0.11	0
96	13.2	0.16	0.14	0.13	0.12	0.11	0
97	13.4	0.16	0.14	0.13	0.11	0.1	0
98	13.6	0.15	0.14	0.13	0.11	0.1	0
99	13.8	0.16	0.15	0.13	0.12	0.11	0
100	14	0.17	0.16	0.14	0.13	0.11	0
101	14.2	0.19	0.17	0.15	0.14	0.12	0
102	14.4	0.2	0.18	0.16	0.14	0.12	0
103	14.6	0.21	0.19	0.17	0.15	0.11	0
104	14.8	0.2	0.18	0.16	0.14	0.1	0

Table 21

FIG. 45

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
105	15	0.19	0.17	0.15	0.14	0.1	0
106	15.2	0.19	0.18	0.16	0.14	0.1	0
107	15.4	0.18	0.16	0.15	0.13	0.1	0
108	15.6	0.17	0.15	0.14	0.13	0.09	0
109	15.8	0.16	0.14	0.13	0.12	0.09	0
110	15.93	0.15	0.13	0.12	0.11	0.08	0
111	16	0.15	0.14	0.12	0.11	0.08	0
112	16.2	0.14	0.12	0.11	0.1	0.07	0
113	16.4	0.13	0.12	0.11	0.1	0.07	0
114	16.6	0.13	0.11	0.1	0.09	0.07	0
115	16.8	0.12	0.11	0.1	0.09	0.06	0
116	17	0.11	0.1	0.09	0.08	0.06	0
117	17.2	0.11	0.1	0.09	0.08	0.06	0
118	17.4	0.1	0.09	0.08	0.08	0.06	0
119	17.6	0.1	0.09	0.08	0.07	0.05	0
120	17.8	0.09	0.09	0.08	0.07	0.05	0
121	18	0.09	0.08	0.07	0.07	0.05	0
122	18.2	0.09	0.08	0.07	0.06	0.05	0
123	18.4	0.08	0.07	0.07	0.06	0.04	0
124	18.6	0.08	0.07	0.06	0.06	0.04	0

Table 22

FIG. 46

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1982	0.1980	0.0002	0.0812
2	-4.8000	0.2063	0.2061	0.0002	0.0902
3	-4.6000	0.2147	0.2145	0.0002	0.1001
4	-4.4000	0.2234	0.2232	0.0002	0.1110
5	-4.2000	0.2324	0.2321	0.0003	0.1229
6	-4.0000	0.2417	0.2414	0.0003	0.1359
7	-3.8000	0.2514	0.2510	0.0004	0.1501
8	-3.6000	0.2613	0.2609	0.0004	0.1656
9	-3.4000	0.2715	0.2710	0.0005	0.1825
10	-3.2000	0.2821	0.2816	0.0006	0.2009
11	-3.0000	0.2930	0.2924	0.0006	0.2209
12	-2.8000	0.3043	0.3036	0.0007	0.2425
13	-2.6000	0.3159	0.3151	0.0008	0.2659
14	-2.4000	0.3279	0.3269	0.0010	0.2911
15	-2.2000	0.3402	0.3391	0.0011	0.3183
16	-2.0000	0.3529	0.3516	0.0012	0.3476
17	-1.8000	0.3659	0.3645	0.0014	0.3791
18	-1.6000	0.3793	0.3777	0.0016	0.4128
19	-1.4000	0.3930	0.3913	0.0018	0.4488
20	-1.2000	0.4072	0.4052	0.0020	0.4872
21	-1.0000	0.4217	0.4195	0.0022	0.5281
22	-0.8000	0.4366	0.4341	0.0025	0.5715
23	-0.6000	0.4519	0.4491	0.0028	0.6175
24	-0.4000	0.4675	0.4644	0.0031	0.6663
25	-0.2000	0.4835	0.4801	0.0034	0.7177
26	0.0000	0.4999	0.4961	0.0038	0.7719
27	0.0008	0.5000	0.4962	0.0038	0.7658
28	0.2000	0.5167	0.5125	0.0042	0.8289
29	0.4000	0.5339	0.5292	0.0047	0.8886
30	0.6000	0.5514	0.5462	0.0052	0.9511
31	0.8000	0.5694	0.5636	0.0057	1.0164
32	1.0000	0.5877	0.5814	0.0063	1.0844
33	1.2000	0.6063	0.5994	0.0069	1.1552
34	1.4000	0.6254	0.6178	0.0076	1.2283
35	1.6000	0.6448	0.6365	0.0083	1.3040
36	1.8000	0.6646	0.6555	0.0091	1.3820
37	2.0000	0.6847	0.6748	0.0099	1.4625
38	2.2000	0.7052	0.6945	0.0107	1.5451
39	2.4000	0.7260	0.7144	0.0116	1.6297
40	2.6000	0.7472	0.7346	0.0126	1.7162

Table 23

FIG. 47

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7688	0.7551	0.0136	1.8045
42	3.0000	0.7906	0.7759	0.0147	1.8944
43	3.2000	0.8128	0.7970	0.0158	1.9858
44	3.4000	0.8354	0.8184	0.0170	2.0783
45	3.6000	0.8582	0.8400	0.0182	2.1719
46	3.8000	0.8814	0.8619	0.0195	2.2663
47	4.0000	0.9049	0.8840	0.0209	2.3612
48	4.2000	0.9286	0.9064	0.0223	2.4564
49	4.4000	0.9527	0.9290	0.0237	2.5516
50	4.6000	0.9771	0.9519	0.0252	2.6465
51	4.7864	1.0000	0.9734	0.0266	2.7327
52	4.8000	1.0017	0.9750	0.0267	2.7407
53	5.0000	1.0266	0.9983	0.0283	2.8339
54	5.2000	1.0517	1.0218	0.0299	2.9256
55	5.4000	1.0771	1.0456	0.0315	3.0153
56	5.6000	1.1028	1.0696	0.0332	3.1025
57	5.8000	1.1286	1.0938	0.0349	3.1868
58	6.0000	1.1547	1.1182	0.0366	3.2699
59	6.2000	1.1810	1.1427	0.0383	3.3517
60	6.4000	1.2076	1.1675	0.0401	3.4318
61	6.6000	1.2343	1.1925	0.0419	3.5103
62	6.8000	1.2613	1.2176	0.0437	3.5868
63	7.0000	1.2885	1.2430	0.0455	3.6606
64	7.2000	1.3158	1.2685	0.0473	3.7311
65	7.4000	1.3433	1.2941	0.0491	3.7978
66	7.6000	1.3710	1.3200	0.0510	3.8609
67	7.8000	1.3988	1.3460	0.0528	3.9211
68	8.0000	1.4268	1.3722	0.0546	3.9785
69	8.2000	1.4549	1.3985	0.0564	4.0328
70	8.4000	1.4832	1.4250	0.0582	4.0837
71	8.5186	1.5000	1.4408	0.0592	4.1088
72	8.6000	1.5116	1.4516	0.0600	4.1311
73	8.8000	1.5401	1.4784	0.0617	4.1748
74	9.0000	1.5688	1.5053	0.0635	4.2162
75	9.2000	1.5976	1.5324	0.0652	4.2547
76	9.4000	1.6265	1.5596	0.0669	4.2906
77	9.6000	1.6555	1.5869	0.0686	4.3237
78	9.8000	1.6846	1.6143	0.0703	4.3538
79	10.0000	1.7138	1.6419	0.0719	4.3805
80	10.2000	1.7431	1.6696	0.0735	4.4034

Table 24

FIG. 48

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7725	1.6974	0.0751	4.4223
82	10.6000	1.8019	1.7254	0.0765	4.4366
83	10.8000	1.8314	1.7534	0.0780	4.4462
84	11.0000	1.8609	1.7816	0.0793	4.4514
85	11.2000	1.8905	1.8099	0.0806	4.4542
86	11.4000	1.9202	1.8383	0.0819	4.4549
87	11.6000	1.9499	1.8667	0.0831	4.4534
88	11.8000	1.9797	1.8953	0.0843	4.4497
89	11.9363	2.0000	1.9149	0.0851	4.4441
90	12.0000	2.0095	1.9240	0.0855	4.4438
91	12.2000	2.0394	1.9528	0.0866	4.4355
92	12.4000	2.0694	1.9817	0.0877	4.4249
93	12.6000	2.0994	2.0107	0.0887	4.4121
94	12.8000	2.1294	2.0397	0.0897	4.3969
95	13.0000	2.1595	2.0689	0.0906	4.3798
96	13.2000	2.1896	2.0981	0.0915	4.3610
97	13.4000	2.2197	2.1274	0.0923	4.3403
98	13.6000	2.2499	2.1568	0.0931	4.3178
99	13.8000	2.2801	2.1863	0.0939	4.2934
100	14.0000	2.3104	2.2158	0.0946	4.2671
101	14.2000	2.3406	2.2454	0.0952	4.2389
102	14.4000	2.3709	2.2751	0.0958	4.2085
103	14.6000	2.4012	2.3049	0.0963	4.1761
104	14.8000	2.4314	2.3347	0.0967	4.1414
105	15.0000	2.4617	2.3646	0.0971	4.1045
106	15.2000	2.4920	2.3946	0.0974	4.0654
107	15.2531	2.5000	2.4026	0.0974	4.0539
108	15.4000	2.5222	2.4247	0.0976	4.0241
109	15.6000	2.5525	2.4547	0.0977	3.9805
110	15.8000	2.5827	2.4849	0.0978	3.9347
111	16.0000	2.6129	2.5151	0.0978	3.8868
112	16.2000	2.6430	2.5454	0.0977	3.8366
113	16.4000	2.6732	2.5757	0.0975	3.7842
114	16.6000	2.7033	2.6061	0.0972	3.7296
115	16.8000	2.7333	2.6365	0.0968	3.6729
116	17.0000	2.7634	2.6670	0.0964	3.6139
117	17.2000	2.7933	2.6975	0.0958	3.5527
118	17.4000	2.8233	2.7281	0.0952	3.4893
119	17.6000	2.8531	2.7587	0.0944	3.4237
120	17.8000	2.8829	2.7893	0.0936	3.3558

Table 25

FIG. 49

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.9127	2.8200	0.0927	3.2856
122	18.2000	2.9424	2.8508	0.0916	3.2130
123	18.4000	2.9720	2.8815	0.0904	3.1382
124	18.5898	3.0000	2.9108	0.0892	3.0644
125	18.6000	3.0015	2.9123	0.0891	3.0610
126	18.8000	3.0309	2.9432	0.0877	2.9814
127	19.0000	3.0603	2.9740	0.0862	2.8995
128	19.2000	3.0895	3.0049	0.0846	2.8153
129	19.4000	3.1187	3.0358	0.0828	2.7288
130	19.6000	3.1477	3.0667	0.0810	2.6400
131	19.8000	3.1766	3.0977	0.0790	2.5491
132	20.0000	3.2054	3.1286	0.0768	2.4563
133	20.2000	3.2341	3.1594	0.0746	2.3616
134	20.4000	3.2626	3.1903	0.0723	2.2653
135	20.6000	3.2909	3.2211	0.0698	2.1677
136	20.8000	3.3191	3.2518	0.0673	2.0691
137	21.0000	3.3471	3.2824	0.0647	1.9700
138	21.2000	3.3749	3.3129	0.0620	1.8706
139	21.4000	3.4025	3.3433	0.0592	1.7714
140	21.6000	3.4298	3.3734	0.0564	1.6730
141	21.8000	3.4569	3.4033	0.0536	1.5758
142	22.0000	3.4837	3.4329	0.0508	1.4802
143	22.1229	3.5000	3.4510	0.0490	1.4199
144	22.2000	3.5102	3.4622	0.0480	1.3867
145	22.4000	3.5364	3.4911	0.0452	1.2956
146	22.6000	3.5621	3.5196	0.0425	1.2074
147	22.8000	3.5874	3.5476	0.0398	1.1222
148	23.0000	3.6123	3.5751	0.0372	1.0403
149	23.2000	3.6366	3.6019	0.0346	0.9619
150	23.4000	3.6603	3.6281	0.0322	0.8870
151	23.6000	3.6835	3.6537	0.0298	0.8158
152	23.8000	3.7059	3.6784	0.0275	0.7482
153	24.0000	3.7277	3.7023	0.0253	0.6842
154	24.2000	3.7487	3.7254	0.0232	0.6239
155	24.4000	3.7689	3.7476	0.0213	0.5672
156	24.6000	3.7882	3.7688	0.0194	0.5139
157	24.8000	3.8067	3.7891	0.0176	0.4641
158	25.0000	3.8242	3.8083	0.0159	0.4176
159	25.2000	3.8409	3.8266	0.0143	0.3744
160	25.4000	3.8566	3.8437	0.0129	0.3344

Table 26

FIG. 50

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
161	25.6000	3.8713	3.8598	0.0115	0.2974
162	25.8000	3.8851	3.8749	0.0102	0.2634
163	26.0000	3.8979	3.8889	0.0090	0.2323
164	26.2000	3.9097	3.9018	0.0079	0.2037
165	26.4000	3.9207	3.9137	0.0070	0.1779
166	26.6000	3.9306	3.9246	0.0061	0.1545
167	26.8000	3.9397	3.9345	0.0052	0.1334
168	27.0000	3.9479	3.9434	0.0045	0.1145
169	27.2000	3.9553	3.9515	0.0039	0.0977
170	27.4000	3.9619	3.9587	0.0033	0.0829
171	27.6000	3.9678	3.9650	0.0028	0.0698
172	27.8000	3.9729	3.9706	0.0023	0.0584
173	28.0000	3.9775	3.9755	0.0019	0.0485
174	28.2000	3.9814	3.9798	0.0016	0.0399
175	28.4000	3.9847	3.9834	0.0013	0.0326
176	28.6000	3.9876	3.9865	0.0011	0.0264
177	28.8000	3.9900	3.9892	0.0008	0.0212
178	29.0000	3.9920	3.9914	0.0007	0.0169
179	29.2000	3.9937	3.9932	0.0005	0.0134
180	29.4000	3.9951	3.9947	0.0004	0.0103

Table 27

FIG. 51

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-20.036	-6.369	-6.322	-6.315	-6.314	-6.302	-6.294	-6.240
2	-20.035	-6.309	-6.309	-6.309	-6.309	-6.309	-6.309	-6.309
3	-20.032	-6.337	-6.280	-6.271	-6.346	-6.289	-6.366	-6.282
4	-20.029	-6.320	-6.311	-6.311	-6.311	-6.311	-6.307	-6.307
5	-20.026	-6.323	-6.323	-6.323	-6.305	-6.305	-6.305	-6.305
6	-20.024	-6.335	-6.335	-6.335	-6.335	-6.335	-6.260	-6.260
7	-20.021	-6.315	-6.316	-6.316	-6.315	-6.317	-6.317	-6.313
8	-20.017	-6.320	-6.320	-6.320	-6.320	-6.320	-6.320	-6.300
9	-20.014	-6.341	-6.329	-6.329	-6.329	-6.300	-6.300	-6.300
10	-20.011	-6.308	-6.320	-6.316	-6.307	-6.343	-6.326	-6.318
11	-20.008	-6.363	-6.326	-6.318	-6.318	-6.318	-6.306	-6.300
12	-20.004	-6.333	-6.325	-6.325	-6.323	-6.323	-6.323	-6.310
13	-20.001	-6.336	-6.333	-6.320	-6.326	-6.317	-6.316	-6.323
14	-19.997	-6.323	-6.331	-6.323	-6.331	-6.327	-6.322	-6.325
15	-19.993	-6.328	-6.328	-6.328	-6.328	-6.328	-6.328	-6.328
16	-19.990	-6.332	-6.331	-6.331	-6.332	-6.324	-6.328	-6.326
17	-19.986	-6.331	-6.331	-6.331	-6.331	-6.331	-6.331	-6.331
18	-19.983	-6.332	-6.332	-6.332	-6.332	-6.332	-6.332	-6.332
19	-19.979	-6.339	-6.339	-6.336	-6.336	-6.330	-6.330	-6.330
20	-19.976	-6.332	-6.330	-6.336	-6.336	-6.346	-6.332	-6.339
21	-19.972	-6.362	-6.362	-6.362	-6.333	-6.315	-6.315	-6.312
22	-19.950	-6.582	-6.589	-6.596	-6.580	-6.574	-6.584	-4.681
23	-19.919	-6.734	-6.725	-6.725	-6.725	-6.724	-6.730	-3.421
24	-19.889	-6.822	-6.823	-6.821	-6.829	-6.826	-6.831	-2.221
25	-19.862	-6.899	-6.898	-6.901	-6.903	-6.894	-6.895	-0.058
26	-19.850	-6.922	-6.913	-6.913	-6.913	-6.911	-6.911	0.144
27	-19.840	-7.180	-7.160	-7.150	-7.140	-7.140	-5.430	-0.150
28	-19.823	-6.964	-6.962	-6.967	-6.955	-6.965	-6.961	0.939
29	-19.796	-7.238	-7.238	-7.238	-7.238	-7.233	-5.505	1.455
30	-19.756	-7.424	-7.426	-7.426	-7.426	-7.425	-4.126	1.219
31	-19.713	-7.576	-7.574	-7.574	-7.574	-7.574	-0.998	-0.906
32	-19.683	-7.631	-7.628	-7.630	-7.626	-7.627	-0.542	-0.543
33	-19.663	-7.660	-7.660	-7.660	-7.659	-7.659	0.000	0.000
34	-19.654	-7.668	-7.668	-7.668	-7.664	-7.655	-0.001	0.000
35	-19.597	-8.155	-8.155	-8.154	-8.154	-5.289	-0.161	-0.159
36	-19.601	-7.739	-7.739	-7.737	-7.737	-7.734	0.324	0.324
37	-19.507	-8.421	-8.421	-8.421	-8.421	-3.082	-0.540	-0.540
38	-19.551	-7.804	-7.804	-7.804	-7.804	-7.804	1.020	1.020
39	-19.445	-8.528	-8.528	-8.528	-8.528	-0.978	-0.977	-0.977
40	-19.418	-8.562	-8.562	-8.562	-8.562	-0.916	-0.916	-0.916

Table 28

FIG. 52

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	6.314	6.351	6.351	6.315	6.320	6.236	6.269	20.036
2	6.159	6.287	6.287	6.287	6.381	6.381	6.381	20.034
3	6.316	6.262	6.286	6.298	6.298	6.296	6.414	20.032
4	6.248	6.309	6.309	6.328	6.328	6.328	6.328	20.029
5	6.303	6.303	6.303	6.303	6.326	6.326	6.326	20.026
6	6.200	6.267	6.267	6.267	6.398	6.398	6.398	20.023
7	6.309	6.312	6.318	6.316	6.320	6.320	6.313	20.020
8	6.273	6.321	6.321	6.321	6.321	6.330	6.330	20.017
9	6.302	6.302	6.308	6.329	6.329	6.329	6.329	20.014
10	6.312	6.312	6.326	6.311	6.325	6.325	6.328	20.011
11	6.321	6.321	6.321	6.321	6.321	6.321	6.321	20.007
12	6.320	6.320	6.320	6.324	6.326	6.326	6.326	20.004
13	6.317	6.322	6.322	6.323	6.327	6.323	6.335	20.001
14	6.323	6.329	6.328	6.324	6.323	6.324	6.332	19.997
15	6.324	6.324	6.324	6.324	6.326	6.333	6.337	19.993
16	6.324	6.323	6.328	6.325	6.327	6.336	6.341	19.990
17	6.328	6.328	6.330	6.332	6.332	6.332	6.334	19.986
18	6.331	6.331	6.331	6.331	6.333	6.333	6.338	19.983
19	6.330	6.331	6.333	6.333	6.337	6.337	6.337	19.979
20	6.333	6.336	6.339	6.339	6.334	6.328	6.340	19.975
21	6.313	6.313	6.313	6.355	6.355	6.355	6.355	19.972
22	4.681	6.578	6.571	6.582	6.587	6.603	6.582	19.950
23	3.421	6.716	6.716	6.722	6.737	6.738	6.734	19.919
24	2.221	6.820	6.831	6.822	6.830	6.826	6.822	19.889
25	0.059	6.893	6.894	6.902	6.901	6.901	6.899	19.862
26	0.144	5.429	7.147	7.147	7.154	7.154	7.169	19.843
27	-0.140	6.900	6.900	6.900	6.910	6.920	6.950	19.850
28	0.895	2.960	7.406	7.403	7.400	7.399	7.399	19.796
29	1.467	1.467	7.466	7.466	7.466	7.466	7.466	19.770
30	1.219	1.225	7.521	7.521	7.521	7.521	7.521	19.741
31	-0.906	2.985	7.536	7.537	7.537	7.537	7.545	19.719
32	-0.546	1.662	7.624	7.621	7.621	7.624	7.620	19.685
33	0.000	0.000	7.659	7.660	7.660	7.660	7.660	19.663
34	0.000	0.000	7.663	7.663	7.663	7.665	7.668	19.654
35	-0.159	-0.159	7.703	7.703	7.703	7.703	7.703	19.627
36	0.324	0.327	4.201	8.309	8.309	8.309	8.309	19.551
37	-0.540	-0.540	7.771	7.771	7.771	7.771	7.771	19.576
38	1.020	1.020	1.021	8.501	8.501	8.501	8.501	19.468
39	-0.977	-0.977	6.441	8.121	8.121	8.121	8.121	19.517
40	-0.915	-0.915	5.622	8.286	8.286	8.286	8.287	19.476

Table 29

FIG. 53

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-19.438	-8.393	-8.393	-8.393	-8.393	-5.030	0.855	0.855
42	-19.365	-8.624	-8.624	-8.624	-8.624	-0.796	-0.796	-0.795
43	-19.369	-8.542	-8.542	-8.542	-8.542	-4.089	0.735	0.735
44	-19.315	-8.681	-8.681	-8.681	-8.681	-0.676	-0.676	-0.675
45	-19.290	-8.706	-8.707	-8.707	-8.707	-0.617	-0.617	-0.617
46	-19.267	-8.731	-8.731	-8.731	-8.731	-0.559	-0.559	-0.558
47	-19.252	-8.724	-8.724	-8.724	-8.724	-2.617	0.500	0.500
48	-19.221	-8.776	-8.776	-8.776	-8.776	-0.444	-0.444	-0.444
49	-19.199	-8.796	-8.796	-8.796	-8.795	-0.388	-0.388	-0.388
50	-19.181	-8.805	-8.805	-8.805	-8.805	-1.696	0.329	0.329
51	-19.160	-8.820	-8.820	-8.820	-8.820	-1.430	0.280	0.280
52	-19.158	-8.831	-8.831	-8.831	-8.831	-0.285	-0.278	-0.278
53	-19.138	-8.846	-8.847	-8.846	-8.846	-0.241	-0.227	-0.241
54	-19.119	-8.860	-8.860	-8.860	-8.860	-0.702	-0.066	-0.066
55	-19.100	-8.873	-8.873	-8.873	-8.873	-0.232	-0.232	-0.212
56	-19.081	-8.884	-8.884	-8.884	-8.884	-0.360	-0.342	0.100
57	-19.043	-9.266	-9.266	-9.265	-7.601	-1.129	-1.129	0.534
58	-18.995	-9.478	-9.478	-9.478	-6.660	-1.335	-1.335	-1.335
59	-18.952	-9.606	-9.606	-9.606	-5.932	-1.717	-1.717	-1.715
60	-18.914	-9.697	-9.697	-9.697	-5.244	-2.057	-2.057	-2.056
61	-18.879	-9.767	-9.767	-9.767	-4.469	-2.412	-2.412	-2.412
62	-18.848	-9.816	-9.816	-9.816	-2.987	-2.987	-2.987	-2.987
63	-18.824	-9.832	-9.832	-9.832	-2.985	-2.985	-2.985	-2.985
64	-18.800	-9.848	-9.848	-9.848	-2.986	-2.986	-2.986	-2.986
65	-18.775	-9.862	-9.862	-9.862	-2.988	-2.988	-2.988	-2.988
66	-18.742	-10.161	-10.161	-9.278	-3.006	-3.006	-3.006	-3.006
67	-18.705	-10.362	-10.362	-8.862	-3.036	-3.036	-3.036	-3.036
68	-18.671	-10.482	-10.482	-8.612	-3.061	-3.061	-3.061	-3.061
69	-18.638	-10.573	-10.573	-8.428	-3.083	-3.083	-3.083	-3.083
70	-18.608	-10.645	-10.645	-8.285	-3.102	-3.102	-3.102	-3.102
71	-18.600	-10.660	-10.660	-8.250	-3.110	-3.110	-3.110	-3.110
72	-18.579	-10.705	-10.705	-8.170	-3.118	-3.118	-3.118	-3.118
73	-18.545	-10.797	-10.797	-7.881	-3.559	-3.559	-3.559	-1.674
74	-18.512	-10.880	-10.880	-7.584	-3.788	-3.788	-3.788	0.000
75	-18.481	-10.932	-10.932	-7.425	-3.867	-3.867	-3.867	0.259
76	-18.450	-11.005	-11.005	-6.996	-4.647	-4.647	-2.290	0.000
77	-18.421	-11.052	-11.052	-6.725	-4.860	-4.861	-1.978	0.000
78	-18.394	-11.089	-11.089	-6.487	-5.027	-5.027	-1.764	0.000
79	-18.366	-11.120	-11.120	-6.261	-5.182	-5.181	-1.146	-1.075
80	-18.339	-11.146	-11.146	-5.849	-5.849	-4.951	-1.079	-1.078

Table 30

FIG. 54

	Label							
	8	9	10	11	12	13	14	15
Design #								
41	0.855	0.855	0.856	8.594	8.594	8.594	8.594	19.391
42	-0.795	-0.792	4.532	8.475	8.475	8.475	8.476	19.403
43	0.735	0.735	0.737	8.653	8.653	8.653	8.653	19.340
44	-0.675	-0.675	3.684	8.599	8.599	8.599	8.599	19.338
45	-0.617	-0.617	3.307	8.646	8.646	8.646	8.647	19.308
46	-0.558	-0.558	2.953	8.688	8.688	8.688	8.688	19.279
47	0.500	0.501	0.502	8.754	8.754	8.754	8.754	19.243
48	-0.443	-0.443	2.296	8.755	8.755	8.755	8.755	19.227
49	-0.387	-0.387	1.989	8.782	8.782	8.782	8.782	19.203
50	0.335	0.335	0.336	8.814	8.814	8.814	8.814	19.178
51	0.280	0.290	0.290	8.830	8.830	8.830	8.830	19.160
52	-0.278	-0.278	1.416	8.826	8.826	8.826	8.826	19.159
53	-0.208	-0.224	1.152	8.843	8.844	8.844	8.844	19.139
54	0.002	0.148	0.683	8.860	8.860	8.860	8.860	19.119
55	-0.212	0.445	0.445	8.873	8.873	8.873	8.873	19.100
56	0.181	0.181	0.241	8.884	8.884	8.885	8.885	19.081
57	0.534	0.543	0.619	7.675	9.250	9.250	9.250	19.044
58	1.335	1.335	1.335	6.660	9.478	9.478	9.478	18.995
59	1.716	1.716	1.717	5.932	9.606	9.606	9.606	18.952
60	2.056	2.056	2.056	5.244	9.697	9.697	9.697	18.914
61	2.411	2.412	2.412	4.469	9.767	9.767	9.767	18.879
62	2.987	2.987	2.987	2.987	9.816	9.816	9.816	18.848
63	2.985	2.985	2.985	2.985	9.832	9.832	9.832	18.824
64	2.986	2.986	2.986	2.986	9.848	9.848	9.848	18.800
65	2.988	2.988	2.988	2.988	9.862	9.862	9.862	18.775
66	3.006	3.006	3.006	3.006	9.278	10.161	10.161	18.742
67	3.036	3.036	3.036	3.036	8.862	10.362	10.362	18.705
68	3.061	3.061	3.061	3.061	8.612	10.482	10.482	18.671
69	3.083	3.083	3.083	3.083	8.428	10.573	10.573	18.638
70	3.102	3.102	3.102	3.102	8.285	10.645	10.645	18.608
71	3.110	3.110	3.110	3.110	8.250	10.660	10.660	18.600
72	3.118	3.118	3.118	3.118	8.170	10.705	10.705	18.579
73	1.674	3.559	3.559	3.559	7.881	10.797	10.797	18.545
74	0.000	3.788	3.788	3.788	7.584	10.880	10.880	18.512
75	0.259	2.208	4.527	4.527	7.200	10.956	10.956	18.481
76	0.000	2.290	4.647	4.647	6.997	11.005	11.005	18.450
77	0.001	1.977	4.861	4.861	6.724	11.052	11.052	18.421
78	0.000	1.764	5.027	5.027	6.487	11.089	11.089	18.394
79	1.111	1.111	5.182	5.182	6.261	11.120	11.120	18.366
80	1.078	1.078	4.951	5.848	5.849	11.146	11.146	18.339

Table 31

FIG. 55

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-18.310	-11.166	-11.166	-5.820	-5.820	-5.038	-1.059	-1.059
82	-18.281	-11.184	-11.184	-5.829	-5.829	-5.042	-1.064	-1.064
83	-18.250	-11.200	-11.200	-5.869	-5.869	-4.977	-1.088	-1.088
84	-18.216	-11.625	-10.769	-6.029	-6.028	-4.637	-1.196	-1.196
85	-18.183	-11.856	-10.518	-6.167	-6.167	-4.318	-1.321	-1.321
86	-18.151	-12.003	-10.359	-6.260	-6.260	-4.096	-1.419	-1.419
87	-18.121	-12.114	-10.244	-6.327	-6.327	-3.928	-1.497	-1.497
88	-18.092	-12.202	-10.156	-6.378	-6.378	-3.800	-1.559	-1.559
89	-18.070	-12.250	-10.110	-6.410	-6.410	-3.730	-1.590	-1.590
90	-18.064	-12.276	-10.088	-6.417	-6.417	-3.703	-1.607	-1.607
91	-18.037	-12.339	-10.034	-6.448	-6.448	-3.630	-1.644	-1.643
92	-18.011	-12.397	-9.985	-6.483	-6.483	-3.412	-2.368	-0.967
93	-17.985	-12.447	-9.947	-6.510	-6.510	-2.960	-2.960	-0.633
94	-17.960	-12.496	-9.905	-6.787	-6.251	-2.977	-2.977	-0.578
95	-17.935	-12.548	-9.844	-7.024	-6.017	-3.029	-3.029	-0.316
96	-17.911	-12.592	-9.800	-7.137	-5.915	-3.056	-3.056	0.000
97	-17.887	-12.632	-9.760	-7.242	-5.805	-3.365	-2.785	0.000
98	-17.863	-12.668	-9.730	-7.318	-5.727	-3.481	-2.701	0.000
99	-17.839	-12.702	-9.704	-7.391	-5.629	-3.688	-2.476	-0.486
100	-17.814	-12.732	-9.684	-7.450	-5.545	-3.844	-2.283	-0.732
101	-17.789	-12.759	-9.675	-7.487	-5.497	-3.924	-2.181	-0.851
102	-17.763	-12.784	-9.674	-7.511	-5.476	-3.962	-2.136	-0.902
103	-17.736	-12.807	-9.678	-7.526	-5.473	-3.975	-2.128	-0.914
104	-17.708	-12.829	-9.687	-7.535	-5.482	-3.975	-2.140	-0.903
105	-17.679	-12.849	-9.700	-7.541	-5.496	-3.966	-2.162	-0.882
106	-17.650	-12.869	-9.716	-7.544	-5.514	-3.955	-2.187	-0.858
107	-17.640	-12.870	-9.720	-7.550	-5.520	-3.950	-2.190	-0.850
108	-17.619	-12.888	-9.734	-7.547	-5.533	-3.944	-2.212	-0.835
109	-17.588	-12.906	-9.753	-7.550	-5.552	-3.934	-2.234	-0.815
110	-17.556	-12.925	-9.774	-7.554	-5.571	-3.926	-2.253	-0.799
111	-17.523	-12.942	-9.795	-7.558	-5.588	-3.921	-2.269	-0.787
112	-17.490	-12.960	-9.818	-7.563	-5.604	-3.918	-2.282	-0.777
113	-17.456	-12.977	-9.841	-7.570	-5.619	-3.917	-2.293	-0.771
114	-17.421	-12.993	-9.864	-7.578	-5.633	-3.919	-2.301	-0.767
115	-17.385	-13.009	-9.888	-7.587	-5.646	-3.922	-2.308	-0.765
116	-17.349	-13.025	-9.913	-7.598	-5.659	-3.927	-2.314	-0.765
117	-17.312	-13.040	-9.938	-7.611	-5.671	-3.933	-2.320	-0.765
118	-17.274	-13.054	-9.963	-7.625	-5.683	-3.940	-2.325	-0.766
119	-17.235	-13.067	-9.989	-7.641	-5.695	-3.948	-2.329	-0.768
120	-17.196	-13.080	-10.015	-7.658	-5.708	-3.956	-2.334	-0.770

Table 32

FIG. 56

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	1.059	1.059	5.038	5.820	5.820	11.166	11.166	18.310
82	1.064	1.064	5.042	5.829	5.829	11.184	11.184	18.281
83	1.088	1.088	4.977	5.869	5.869	11.200	11.200	18.250
84	1.196	1.196	4.637	6.028	6.028	10.769	11.625	18.216
85	1.321	1.321	4.318	6.167	6.167	10.518	11.856	18.183
86	1.419	1.419	4.096	6.260	6.260	10.359	12.003	18.151
87	1.497	1.497	3.928	6.327	6.327	10.244	12.114	18.121
88	1.559	1.559	3.800	6.378	6.378	10.156	12.202	18.092
89	1.590	1.590	3.730	6.410	6.410	10.110	12.250	18.070
90	1.607	1.607	3.703	6.417	6.417	10.088	12.276	18.064
91	1.644	1.644	3.630	6.448	6.448	10.034	12.339	18.037
92	0.967	2.369	3.412	6.483	6.483	9.985	12.397	18.011
93	0.633	2.960	2.960	6.510	6.510	9.947	12.447	17.985
94	0.578	2.977	2.977	6.251	6.787	9.905	12.496	17.960
95	0.316	3.029	3.029	6.017	7.024	9.844	12.548	17.935
96	0.000	3.056	3.056	5.915	7.137	9.800	12.592	17.911
97	0.000	2.785	3.365	5.805	7.242	9.760	12.632	17.887
98	0.000	2.701	3.481	5.727	7.318	9.730	12.668	17.863
99	0.486	2.476	3.688	5.629	7.391	9.704	12.702	17.839
100	0.732	2.283	3.844	5.545	7.450	9.684	12.732	17.814
101	0.851	2.181	3.924	5.497	7.487	9.675	12.759	17.789
102	0.902	2.136	3.962	5.476	7.511	9.674	12.784	17.763
103	0.914	2.128	3.975	5.473	7.526	9.678	12.807	17.736
104	0.903	2.140	3.975	5.482	7.535	9.687	12.829	17.708
105	0.882	2.162	3.966	5.496	7.541	9.700	12.849	17.679
106	0.858	2.187	3.955	5.514	7.544	9.716	12.869	17.650
107	0.850	2.190	3.950	5.520	7.550	9.720	12.870	17.640
108	0.835	2.212	3.944	5.533	7.547	9.734	12.888	17.619
109	0.815	2.234	3.934	5.552	7.550	9.753	12.906	17.588
110	0.799	2.253	3.926	5.571	7.554	9.774	12.925	17.556
111	0.787	2.269	3.921	5.588	7.558	9.795	12.942	17.523
112	0.777	2.282	3.918	5.604	7.563	9.818	12.960	17.490
113	0.771	2.293	3.917	5.619	7.570	9.841	12.977	17.456
114	0.767	2.301	3.919	5.633	7.578	9.864	12.993	17.421
115	0.765	2.308	3.922	5.646	7.587	9.888	13.009	17.385
116	0.765	2.314	3.927	5.659	7.598	9.913	13.025	17.349
117	0.765	2.320	3.933	5.671	7.611	9.938	13.040	17.312
118	0.766	2.325	3.940	5.683	7.625	9.963	13.054	17.274
119	0.768	2.329	3.948	5.695	7.641	9.989	13.067	17.235
120	0.770	2.334	3.956	5.708	7.658	10.015	13.080	17.196

Table 33

FIG. 57

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-17.156	-13.092	-10.041	-7.676	-5.720	-3.966	-2.339	-0.772
122	-17.115	-13.104	-10.068	-7.696	-5.734	-3.975	-2.344	-0.774
123	-17.074	-13.114	-10.095	-7.718	-5.748	-3.986	-2.349	-0.776
124	-17.030	-13.120	-10.120	-7.740	-5.760	-4.000	-2.350	-0.780
125	-17.031	-13.124	-10.122	-7.741	-5.762	-3.997	-2.355	-0.778
126	-16.988	-13.132	-10.150	-7.765	-5.778	-4.008	-2.362	-0.780
127	-16.944	-13.140	-10.177	-7.791	-5.795	-4.020	-2.368	-0.783
128	-16.900	-13.147	-10.205	-7.817	-5.814	-4.032	-2.376	-0.785
129	-16.854	-13.152	-10.232	-7.845	-5.833	-4.046	-2.383	-0.788
130	-16.809	-13.157	-10.259	-7.875	-5.855	-4.060	-2.392	-0.790
131	-16.762	-13.161	-10.286	-7.905	-5.877	-4.075	-2.401	-0.793
132	-16.715	-13.163	-10.313	-7.936	-5.901	-4.091	-2.410	-0.796
133	-16.667	-13.165	-10.339	-7.967	-5.927	-4.109	-2.420	-0.800
134	-16.619	-13.166	-10.365	-8.000	-5.955	-4.128	-2.431	-0.803
135	-16.570	-13.165	-10.390	-8.033	-5.983	-4.148	-2.443	-0.807
136	-16.521	-13.164	-10.415	-8.066	-6.014	-4.170	-2.456	-0.811
137	-16.471	-13.162	-10.439	-8.100	-6.045	-4.193	-2.470	-0.816
138	-16.422	-13.159	-10.462	-8.134	-6.077	-4.217	-2.484	-0.821
139	-16.372	-13.155	-10.484	-8.167	-6.111	-4.243	-2.500	-0.826
140	-16.323	-13.151	-10.506	-8.201	-6.144	-4.270	-2.517	-0.832
141	-16.273	-13.146	-10.527	-8.234	-6.179	-4.297	-2.534	-0.838
142	-16.225	-13.140	-10.547	-8.266	-6.213	-4.326	-2.552	-0.844
143	-16.190	-13.140	-10.560	-8.290	-6.230	-4.340	-2.560	-0.850
144	-16.176	-13.134	-10.566	-8.298	-6.247	-4.354	-2.571	-0.850
145	-16.129	-13.128	-10.585	-8.329	-6.281	-4.383	-2.589	-0.857
146	-16.083	-13.122	-10.603	-8.359	-6.313	-4.411	-2.608	-0.863
147	-16.038	-13.116	-10.620	-8.388	-6.345	-4.439	-2.627	-0.870
148	-15.994	-13.110	-10.636	-8.416	-6.376	-4.466	-2.645	-0.876
149	-15.952	-13.104	-10.652	-8.443	-6.406	-4.492	-2.662	-0.882
150	-15.911	-13.098	-10.667	-8.469	-6.435	-4.517	-2.679	-0.888
151	-15.872	-13.093	-10.681	-8.494	-6.462	-4.541	-2.695	-0.894
152	-15.834	-13.087	-10.695	-8.517	-6.488	-4.564	-2.710	-0.899
153	-15.798	-13.083	-10.708	-8.540	-6.513	-4.585	-2.725	-0.904
154	-15.763	-13.078	-10.721	-8.561	-6.536	-4.606	-2.739	-0.909
155	-15.730	-13.074	-10.733	-8.582	-6.558	-4.625	-2.752	-0.913
156	-15.698	-13.070	-10.745	-8.601	-6.579	-4.643	-2.764	-0.918
157	-15.668	-13.066	-10.756	-8.619	-6.599	-4.660	-2.776	-0.922
158	-15.639	-13.062	-10.767	-8.637	-6.618	-4.677	-2.787	-0.926
159	-15.611	-13.059	-10.777	-8.654	-6.636	-4.692	-2.797	-0.929
160	-15.584	-13.056	-10.787	-8.669	-6.653	-4.707	-2.807	-0.933

Table 34

FIG. 58

	Label							
	8	9	10	11	12	13	14	15
Design #								
121	0.772	2.339	3.966	5.720	7.676	10.041	13.092	17.156
122	0.774	2.344	3.975	5.734	7.696	10.068	13.104	17.115
123	0.776	2.349	3.986	5.748	7.718	10.095	13.114	17.074
124	0.780	2.350	4.000	5.760	7.740	10.120	13.120	17.030
125	0.778	2.355	3.997	5.762	7.741	10.122	13.124	17.031
126	0.780	2.362	4.008	5.778	7.765	10.150	13.132	16.988
127	0.783	2.368	4.020	5.795	7.791	10.177	13.140	16.944
128	0.785	2.376	4.032	5.814	7.817	10.205	13.147	16.900
129	0.788	2.383	4.046	5.833	7.845	10.232	13.152	16.854
130	0.790	2.392	4.060	5.855	7.875	10.259	13.157	16.809
131	0.793	2.401	4.075	5.877	7.905	10.286	13.161	16.762
132	0.796	2.410	4.091	5.901	7.936	10.313	13.163	16.715
133	0.800	2.420	4.109	5.927	7.967	10.339	13.165	16.667
134	0.803	2.431	4.128	5.955	8.000	10.365	13.166	16.619
135	0.807	2.443	4.148	5.983	8.033	10.390	13.165	16.570
136	0.811	2.456	4.170	6.014	8.066	10.415	13.164	16.521
137	0.816	2.470	4.193	6.045	8.100	10.439	13.162	16.471
138	0.821	2.484	4.217	6.077	8.134	10.462	13.159	16.422
139	0.826	2.500	4.243	6.111	8.167	10.484	13.155	16.372
140	0.832	2.517	4.270	6.144	8.201	10.506	13.151	16.323
141	0.838	2.534	4.297	6.179	8.234	10.527	13.146	16.273
142	0.844	2.552	4.326	6.213	8.266	10.547	13.140	16.225
143	0.850	2.560	4.340	6.230	8.290	10.560	13.140	16.190
144	0.850	2.571	4.354	6.247	8.298	10.566	13.134	16.176
145	0.857	2.589	4.383	6.281	8.329	10.585	13.128	16.129
146	0.863	2.608	4.411	6.313	8.359	10.603	13.122	16.083
147	0.870	2.627	4.439	6.345	8.388	10.620	13.116	16.038
148	0.876	2.645	4.466	6.376	8.416	10.636	13.110	15.994
149	0.882	2.662	4.492	6.406	8.443	10.652	13.104	15.952
150	0.888	2.679	4.517	6.435	8.469	10.667	13.098	15.911
151	0.894	2.695	4.541	6.462	8.494	10.681	13.093	15.872
152	0.899	2.710	4.564	6.488	8.517	10.695	13.087	15.834
153	0.904	2.725	4.585	6.513	8.540	10.708	13.083	15.798
154	0.909	2.739	4.606	6.536	8.561	10.721	13.078	15.763
155	0.913	2.752	4.625	6.558	8.582	10.733	13.074	15.730
156	0.918	2.764	4.643	6.579	8.601	10.745	13.070	15.698
157	0.922	2.776	4.660	6.599	8.619	10.756	13.066	15.668
158	0.926	2.787	4.677	6.618	8.637	10.767	13.062	15.639
159	0.929	2.797	4.692	6.636	8.654	10.777	13.059	15.611
160	0.933	2.807	4.707	6.653	8.669	10.787	13.056	15.584

Table 35

FIG. 59

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-15.559	-13.053	-10.796	-8.684	-6.670	-4.721	-2.816	-0.936
162	-15.534	-13.050	-10.805	-8.699	-6.685	-4.734	-2.825	-0.939
163	-15.512	-13.047	-10.813	-8.712	-6.699	-4.747	-2.833	-0.942
164	-15.489	-13.045	-10.822	-8.726	-6.714	-4.758	-2.841	-0.945
165	-15.468	-13.042	-10.829	-8.738	-6.727	-4.770	-2.848	-0.947
166	-15.447	-13.040	-10.837	-8.750	-6.739	-4.781	-2.856	-0.950
167	-15.428	-13.038	-10.844	-8.761	-6.751	-4.791	-2.862	-0.952
168	-15.409	-13.036	-10.851	-8.772	-6.763	-4.801	-2.869	-0.954
169	-15.392	-13.035	-10.857	-8.782	-6.773	-4.809	-2.874	-0.956
170	-15.374	-13.033	-10.864	-8.792	-6.784	-4.818	-2.880	-0.958
171	-15.359	-13.032	-10.870	-8.801	-6.793	-4.826	-2.886	-0.960
172	-15.344	-13.030	-10.875	-8.810	-6.802	-4.834	-2.891	-0.962
173	-15.328	-13.028	-10.881	-8.819	-6.812	-4.842	-2.896	-0.964
174	-15.312	-13.027	-10.887	-8.827	-6.821	-4.850	-2.901	-0.966
175	-15.300	-13.025	-10.891	-8.835	-6.829	-4.856	-2.906	-0.967
176	-15.287	-13.024	-10.896	-8.842	-6.836	-4.862	-2.910	-0.969
177	-15.275	-13.023	-10.900	-8.849	-6.843	-4.868	-2.913	-0.970
178	-15.261	-13.022	-10.905	-8.856	-6.851	-4.875	-2.918	-0.971
179	-15.252	-13.022	-10.909	-8.861	-6.856	-4.879	-2.921	-0.973
180	-15.240	-13.020	-10.913	-8.869	-6.865	-4.887	-2.926	-0.974

Table 36

FIG. 60

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	0.936	2.816	4.721	6.670	8.684	10.796	13.053	15.559
162	0.939	2.825	4.734	6.685	8.699	10.805	13.050	15.534
163	0.942	2.833	4.747	6.699	8.712	10.813	13.047	15.512
164	0.945	2.841	4.758	6.714	8.726	10.822	13.045	15.489
165	0.947	2.848	4.770	6.727	8.738	10.829	13.042	15.468
166	0.950	2.856	4.781	6.739	8.750	10.837	13.040	15.447
167	0.952	2.862	4.791	6.751	8.761	10.844	13.038	15.428
168	0.954	2.869	4.801	6.763	8.772	10.851	13.036	15.409
169	0.956	2.874	4.809	6.773	8.782	10.857	13.035	15.392
170	0.958	2.880	4.818	6.784	8.792	10.864	13.033	15.374
171	0.960	2.886	4.826	6.793	8.801	10.870	13.032	15.359
172	0.962	2.891	4.834	6.802	8.810	10.875	13.030	15.344
173	0.964	2.896	4.842	6.812	8.819	10.881	13.028	15.328
174	0.966	2.901	4.850	6.821	8.827	10.887	13.027	15.312
175	0.967	2.906	4.856	6.829	8.835	10.891	13.025	15.300
176	0.969	2.909	4.862	6.836	8.842	10.896	13.024	15.287
177	0.970	2.913	4.868	6.843	8.849	10.900	13.023	15.275
178	0.971	2.918	4.875	6.851	8.856	10.905	13.022	15.261
179	0.973	2.921	4.879	6.856	8.861	10.909	13.022	15.252
180	0.974	2.926	4.887	6.865	8.869	10.913	13.019	15.240

Table 37

FIG. 61

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.27	0.23	0.2	0.17	0.14	0
2	-4.8	0.31	0.26	0.22	0.19	0.16	0
3	-4.6	0.38	0.32	0.27	0.23	0.18	0
4	-4.4	0.34	0.29	0.25	0.21	0.16	0
5	-4.2	0.34	0.29	0.25	0.21	0.18	0
6	-4	0.38	0.32	0.27	0.23	0.2	0
7	-3.8	0.38	0.32	0.27	0.23	0.2	0
8	-3.6	0.42	0.36	0.3	0.26	0.22	0
9	-3.4	0.38	0.32	0.27	0.23	0.2	0
10	-3.2	0.47	0.4	0.34	0.29	0.24	0
11	-3	0.52	0.4	0.34	0.26	0.22	0
12	-2.8	0.52	0.44	0.38	0.32	0.27	0
13	-2.6	0.47	0.4	0.34	0.29	0.24	0
14	-2.4	0.52	0.44	0.38	0.32	0.27	0
15	-2.2	0.58	0.49	0.42	0.36	0.3	0
16	-2	0.58	0.49	0.42	0.36	0.27	0
17	-1.8	0.65	0.55	0.47	0.4	0.34	0
18	-1.6	0.65	0.55	0.47	0.4	0.34	0
19	-1.4	0.72	0.61	0.52	0.44	0.37	0
20	-1.2	0.65	0.55	0.47	0.4	0.34	0
21	-1	0.71	0.61	0.52	0.44	0.37	0
22	-0.8	0.8	0.68	0.58	0.49	0.42	0
23	-0.6	0.74	0.63	0.53	0.45	0.38	0
24	-0.4	0.83	0.7	0.6	0.51	0.43	0
25	-0.2	0.93	0.79	0.67	0.57	0.44	0
26	0	0.93	0.79	0.67	0.57	0.49	0
27	0	0.93	0.79	0.67	0.57	0.44	0
28	0.2	1.04	0.89	0.75	0.64	0.49	0
29	0.4	0.97	0.82	0.7	0.59	0.45	0
30	0.6	0.97	0.82	0.7	0.59	0.5	0
31	0.8	0.97	0.82	0.7	0.6	0.51	0
32	1	0.97	0.83	0.7	0.6	0.51	0
33	1.2	0.98	0.83	0.71	0.6	0.51	0
34	1.4	1.09	0.92	0.79	0.67	0.57	0
35	1.6	1.09	0.93	0.79	0.67	0.57	0
36	1.8	1.22	1.04	0.88	0.75	0.64	0
37	2	1.23	1.04	0.89	0.75	0.64	0
38	2.2	1.23	1.05	0.89	0.76	0.64	0
39	2.4	1.26	1.07	0.91	0.77	0.66	0

Table 38

FIG. 62

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
40	2.6	1.25	1.06	0.9	0.77	0.65	0
41	2.8	1.25	1.06	0.9	0.77	0.66	0
42	3	1.38	1.18	1	0.85	0.65	0
43	3.2	1.38	1.18	1	0.85	0.65	0
44	3.4	1.38	1.18	1	0.85	0.72	0
45	3.6	1.34	1.14	0.97	0.83	0.7	0
46	3.8	1.2	1.02	0.87	0.74	0.63	0
47	4	1.46	1.24	1.05	0.9	0.69	0
48	4.2	1.42	1.21	1.03	0.87	0.74	0
49	4.4	1.52	1.29	1.1	0.93	0.71	0
50	4.6	1.44	1.23	1.04	0.89	0.68	0
51	4.79	1.22	1.04	0.88	0.75	0.64	0
52	4.8	1.2	1.02	0.87	0.74	0.63	0
53	5	0.98	0.83	0.71	0.6	0.51	0
54	5.2	0.6	0.51	0.43	0.37	0.31	0
55	5.4	0.38	0.33	0.28	0.24	0.2	0
56	5.6	0.37	0.32	0.27	0.23	0.19	0
57	5.8	1.09	0.93	0.79	0.67	0.57	0
58	6	1.45	1.23	1.05	0.89	0.76	0
59	6.2	1.46	1.24	1.06	0.9	0.69	0
60	6.4	1.4	1.19	1.01	0.86	0.73	0
61	6.6	1.41	1.2	1.02	0.86	0.74	0
62	6.8	1.42	1.2	1.02	0.87	0.67	0
63	7	1.42	1.21	1.02	0.87	0.67	0
64	7.2	1.42	1.21	1.03	0.87	0.67	0
65	7.4	1.28	1.09	0.92	0.79	0.67	0
66	7.6	1.42	1.21	1.02	0.87	0.67	0
67	7.8	1.29	1.1	0.93	0.79	0.67	0
68	8	1.3	1.11	0.94	0.8	0.68	0
69	8.2	1.31	1.11	0.95	0.8	0.68	0
70	8.4	1.32	1.12	0.95	0.81	0.69	0
71	8.52	1.32	1.12	0.95	0.81	0.62	0
72	8.6	1.32	1.13	0.96	0.81	0.69	0
73	8.8	1.29	1.09	0.93	0.79	0.67	0
74	9	1.35	1.15	0.97	0.83	0.7	0
75	9.2	1.38	1.17	0.99	0.85	0.65	0
76	9.4	1.24	1.05	0.89	0.76	0.65	0
77	9.6	1.24	1.06	0.89	0.76	0.65	0
78	9.8	1.25	1.06	0.89	0.76	0.64	0
79	10	1.25	1.06	0.89	0.76	0.64	0
80	10.2	1.18	1	0.85	0.72	0.61	0

Table 39

FIG. 63

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	1.2	1.02	0.86	0.73	0.62	0
82	10.6	1.2	1.02	0.87	0.74	0.63	0
83	10.8	1.19	1.01	0.86	0.73	0.56	0
84	11	1.08	0.92	0.6	0.51	0.43	0
85	11.2	0.92	0.78	0.44	0.37	0.32	0
86	11.4	0.82	0.7	0.36	0.31	0.26	0
87	11.6	0.78	0.67	0.37	0.31	0.27	0
88	11.8	0.79	0.67	0.38	0.33	0.28	0
89	11.94	0.79	0.67	0.4	0.34	0.29	0
90	12	0.79	0.67	0.39	0.33	0.28	0
91	12.2	0.8	0.68	0.41	0.35	0.3	0
92	12.4	0.8	0.68	0.58	0.49	0.42	0
93	12.6	0.81	0.68	0.58	0.49	0.42	0
94	12.8	0.68	0.58	0.49	0.42	0.36	0
95	13	0.66	0.56	0.48	0.41	0.35	0
96	13.2	0.67	0.57	0.48	0.41	0.35	0
97	13.4	0.61	0.52	0.44	0.38	0.32	0
98	13.6	0.61	0.52	0.44	0.38	0.32	0
99	13.8	0.22	0.18	0.16	0.13	0.11	0
100	14	0.06	0.05	0.05	0.04	0.03	0
101	14.2	0.09	0.07	0.06	0.05	0.04	0
102	14.4	0.13	0.11	0.09	0.08	0.07	0
103	14.6	0.14	0.12	0.1	0.08	0.07	0
104	14.8	0.15	0.13	0.11	0.09	0.08	0
105	15	0.17	0.14	0.12	0.1	0.09	0
106	15.2	0.19	0.16	0.14	0.12	0.1	0
107	15.25	0.19	0.17	0.14	0.12	0.1	0
108	15.4	0.21	0.18	0.15	0.13	0.11	0
109	15.6	0.23	0.2	0.17	0.14	0.12	0
110	15.8	0.25	0.21	0.18	0.15	0.13	0
111	16	0.27	0.23	0.2	0.17	0.14	0
112	16.2	0.29	0.25	0.21	0.18	0.15	0
113	16.4	0.31	0.26	0.22	0.19	0.16	0
114	16.6	0.33	0.28	0.24	0.2	0.17	0
115	16.8	0.35	0.3	0.25	0.22	0.18	0
116	17	0.37	0.32	0.27	0.23	0.19	0
117	17.2	0.39	0.33	0.28	0.24	0.21	0
118	17.4	0.41	0.35	0.3	0.25	0.22	0
119	17.6	0.43	0.37	0.31	0.27	0.23	0
120	17.8	0.45	0.39	0.33	0.28	0.24	0

Table 40

FIG. 64

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.47	0.4	0.34	0.29	0.25	0
122	18.2	0.49	0.42	0.35	0.3	0.25	0
123	18.4	0.46	0.39	0.33	0.28	0.24	0
124	18.59	0.48	0.41	0.31	0.27	0.23	0
125	18.6	0.48	0.41	0.31	0.27	0.23	0
126	18.8	0.45	0.38	0.3	0.25	0.21	0
127	19	0.46	0.39	0.28	0.24	0.21	0
128	19.2	0.43	0.37	0.28	0.24	0.21	0
129	19.4	0.4	0.34	0.28	0.24	0.2	0
130	19.6	0.41	0.35	0.28	0.24	0.2	0
131	19.8	0.35	0.3	0.25	0.22	0.18	0
132	20	0.33	0.28	0.24	0.2	0.17	0
133	20.2	0.34	0.29	0.25	0.21	0.18	0
134	20.4	0.32	0.27	0.23	0.2	0.17	0
135	20.6	0.3	0.26	0.22	0.18	0.16	0
136	20.8	0.31	0.26	0.22	0.19	0.15	0
137	21	0.26	0.22	0.19	0.16	0.14	0
138	21.2	0.27	0.23	0.19	0.17	0.14	0
139	21.4	0.25	0.21	0.18	0.15	0.13	0
140	21.6	0.24	0.2	0.17	0.15	0.11	0
141	21.8	0.21	0.18	0.15	0.13	0.11	0
142	22	0.2	0.17	0.14	0.12	0.1	0
143	22.12	0.22	0.18	0.16	0.13	0.1	0
144	22.2	0.19	0.16	0.14	0.12	0.1	0
145	22.4	0.18	0.16	0.13	0.11	0.1	0
146	22.6	0.18	0.15	0.13	0.11	0.09	0
147	22.8	0.17	0.14	0.12	0.1	0.09	0
148	23	0.14	0.12	0.1	0.09	0.08	0
149	23.2	0.15	0.13	0.11	0.09	0.07	0
150	23.4	0.13	0.11	0.09	0.08	0.07	0
151	23.6	0.14	0.12	0.1	0.09	0.07	0
152	23.8	0.13	0.11	0.1	0.08	0.07	0
153	24	0.13	0.11	0.09	0.08	0.06	0
154	24.2	0.11	0.09	0.08	0.07	0.06	0
155	24.4	0.1	0.09	0.08	0.06	0.05	0
156	24.6	0.1	0.08	0.07	0.06	0.05	0
157	24.8	0.09	0.08	0.07	0.06	0.05	0
158	25	0.09	0.08	0.07	0.06	0.05	0
159	25.2	0.09	0.07	0.06	0.05	0.05	0
160	25.4	0.08	0.07	0.06	0.05	0.04	0

Table 41

FIG. 65

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
161	25.6	0.08	0.07	0.06	0.05	0.04	0
162	25.8	0.08	0.06	0.05	0.05	0.04	0
163	26	0.07	0.06	0.05	0.04	0.04	0
164	26.2	0.07	0.06	0.05	0.04	0.04	0
165	26.4	0.07	0.06	0.05	0.04	0.03	0
166	26.6	0.06	0.05	0.05	0.04	0.03	0
167	26.8	0.05	0.05	0.04	0.03	0.03	0
168	27	0.06	0.05	0.04	0.04	0.03	0
169	27.2	0.06	0.05	0.04	0.03	0.03	0
170	27.4	0.05	0.04	0.04	0.03	0.03	0
171	27.6	0.05	0.04	0.04	0.03	0.03	0
172	27.8	0.05	0.04	0.03	0.03	0.02	0
173	28	0.04	0.04	0.03	0.03	0.02	0
174	28.2	0.04	0.04	0.03	0.03	0.02	0
175	28.4	0.04	0.04	0.03	0.03	0.02	0
176	28.6	0.04	0.03	0.03	0.02	0.02	0
177	28.8	0.03	0.03	0.03	0.02	0.02	0
178	29	0.04	0.03	0.03	0.02	0.02	0
179	29.2	0.04	0.03	0.03	0.02	0.02	0
180	29.4	0.03	0.03	0.02	0.02	0.02	0

Table 42

FIG. 66

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1977	0.1576	0.0401	25.4629
2	-4.8000	0.2058	0.1643	0.0415	25.2261
3	-4.6000	0.2141	0.1713	0.0428	24.9841
4	-4.4000	0.2227	0.1785	0.0442	24.7360
5	-4.2000	0.2316	0.1860	0.0455	24.4818
6	-4.0000	0.2407	0.1938	0.0469	24.2216
7	-3.8000	0.2502	0.2019	0.0484	23.9548
8	-3.6000	0.2600	0.2102	0.0498	23.6816
9	-3.4000	0.2700	0.2188	0.0512	23.4016
10	-3.2000	0.2804	0.2277	0.0526	23.1144
11	-3.0000	0.2910	0.2370	0.0541	22.8200
12	-2.8000	0.3020	0.2465	0.0555	22.5180
13	-2.6000	0.3133	0.2563	0.0569	22.2080
14	-2.4000	0.3248	0.2665	0.0583	21.8898
15	-2.2000	0.3367	0.2770	0.0597	21.5628
16	-2.0000	0.3489	0.2878	0.0611	21.2267
17	-1.8000	0.3613	0.2989	0.0624	20.8808
18	-1.6000	0.3741	0.3104	0.0637	20.5252
19	-1.4000	0.3872	0.3222	0.0650	20.1587
20	-1.2000	0.4005	0.3344	0.0661	19.7810
21	-1.0000	0.4141	0.3469	0.0673	19.3912
22	-0.8000	0.4280	0.3597	0.0683	18.9893
23	-0.6000	0.4421	0.3729	0.0693	18.5739
24	-0.4000	0.4565	0.3864	0.0701	18.1444
25	-0.2000	0.4711	0.4003	0.0708	17.7001
26	0.0000	0.4859	0.4145	0.0715	17.2401
27	0.2000	0.5010	0.4291	0.0719	16.7635
28	0.4000	0.5162	0.4440	0.0722	16.2693
29	0.6000	0.5316	0.4592	0.0724	15.7567
30	0.8000	0.5471	0.4748	0.0723	15.2244
31	1.0000	0.5628	0.4908	0.0720	14.6717
32	1.2000	0.5786	0.5071	0.0715	14.0974
33	1.4000	0.5944	0.5237	0.0707	13.5004
34	1.6000	0.6104	0.5407	0.0698	12.9055
35	1.8000	0.6270	0.5580	0.0691	12.3775
36	2.0000	0.6442	0.5756	0.0686	11.9162
37	2.2000	0.6619	0.5936	0.0684	11.5177
38	2.4000	0.6802	0.6118	0.0684	11.1782
39	2.6000	0.6991	0.6305	0.0687	10.8936
40	2.8000	0.7186	0.6494	0.0692	10.6597

Table 43

FIG. 67

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	3.0000	0.7386	0.6686	0.0700	10.4722
42	3.2000	0.7593	0.6882	0.0711	10.3265
43	3.4000	0.7804	0.7081	0.0724	10.2182
44	3.6000	0.8021	0.7282	0.0739	10.1425
45	3.8000	0.8243	0.7487	0.0756	10.0945
46	4.0000	0.8470	0.7695	0.0775	10.0696
47	4.2000	0.8702	0.7906	0.0796	10.0628
48	4.4000	0.8938	0.8120	0.0818	10.0689
49	4.6000	0.9178	0.8337	0.0841	10.0835
50	4.8000	0.9421	0.8557	0.0864	10.1016
51	5.0000	0.9668	0.8780	0.0888	10.1186
52	5.2000	0.9917	0.9005	0.0912	10.1305
53	5.4000	1.0169	0.9234	0.0936	10.1329
54	5.6000	1.0423	0.9465	0.0958	10.1224
55	5.8000	1.0678	0.9699	0.0979	10.0957
56	6.0000	1.0934	0.9936	0.0999	10.0497
57	6.2000	1.1191	1.0175	0.1016	9.9824
58	6.4000	1.1448	1.0417	0.1030	9.8921
59	6.6000	1.1704	1.0662	0.1042	9.7773
60	6.8000	1.1960	1.0909	0.1051	9.6380
61	7.0000	1.2216	1.1159	0.1057	9.4739
62	7.2000	1.2470	1.1411	0.1060	9.2859
63	7.4000	1.2725	1.1665	0.1060	9.0866
64	7.6000	1.2988	1.1922	0.1067	8.9466
65	7.8000	1.3259	1.2180	0.1079	8.8548
66	8.0000	1.3535	1.2441	0.1094	8.7928
67	8.2000	1.3814	1.2704	0.1110	8.7338
68	8.4000	1.4094	1.2969	0.1125	8.6745
69	8.6000	1.4376	1.3236	0.1140	8.6134
70	8.8000	1.4696	1.3505	0.1191	8.8172
71	9.0000	1.4999	1.3776	0.1223	8.8779
72	9.0008	1.5000	1.3777	0.1223	8.8771
73	9.2000	1.5300	1.4048	0.1252	8.9121
74	9.4000	1.5600	1.4323	0.1278	8.9202
75	9.6000	1.5899	1.4599	0.1300	8.9029
76	9.8000	1.6195	1.4877	0.1318	8.8608
77	10.0000	1.6490	1.5157	0.1333	8.7947
78	10.2000	1.6784	1.5439	0.1345	8.7124
79	10.4000	1.7079	1.5722	0.1357	8.6296
80	10.6000	1.7376	1.6008	0.1368	8.5474

Table 44

FIG. 68

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.8000	1.7719	1.6296	0.1424	8.7355
82	11.0000	1.8033	1.6586	0.1447	8.7246
83	11.2000	1.8346	1.6877	0.1468	8.6985
84	11.4000	1.8659	1.7172	0.1487	8.6597
85	11.6000	1.8972	1.7468	0.1504	8.6109
86	11.8000	1.9287	1.7767	0.1520	8.5555
87	12.0000	1.9603	1.8068	0.1535	8.4959
88	12.2000	1.9920	1.8371	0.1549	8.4335
89	12.2496	2.0000	1.8447	0.1553	8.4187
90	12.4000	2.0240	1.8677	0.1563	8.3688
91	12.6000	2.0561	1.8985	0.1576	8.2996
92	12.8000	2.0883	1.9296	0.1587	8.2232
93	13.0000	2.1205	1.9609	0.1596	8.1376
94	13.2000	2.1527	1.9925	0.1602	8.0402
95	13.4000	2.1848	2.0242	0.1605	7.9297
96	13.6000	2.2167	2.0563	0.1605	7.8050
97	13.8000	2.2486	2.0885	0.1601	7.6656
98	14.0000	2.2803	2.1209	0.1593	7.5115
99	14.2000	2.3117	2.1536	0.1581	7.3434
100	14.4000	2.3430	2.1864	0.1566	7.1620
101	14.6000	2.3741	2.2194	0.1547	6.9689
102	14.8000	2.4050	2.2526	0.1524	6.7660
103	15.0000	2.4358	2.2859	0.1499	6.5558
104	15.2000	2.4664	2.3194	0.1471	6.3410
105	15.4000	2.4971	2.3529	0.1441	6.1245
106	15.4192	2.5000	2.3562	0.1438	6.1030
107	15.6000	2.5276	2.3866	0.1410	5.9087
108	15.8000	2.5582	2.4204	0.1379	5.6954
109	16.0000	2.5888	2.4542	0.1346	5.4854
110	16.2000	2.6194	2.4881	0.1313	5.2791
111	16.4000	2.6500	2.5220	0.1280	5.0762
112	16.6000	2.6806	2.5560	0.1246	4.8762
113	16.8000	2.7111	2.5899	0.1212	4.6790
114	17.0000	2.7415	2.6238	0.1177	4.4842
115	17.2000	2.7718	2.6577	0.1141	4.2915
116	17.4000	2.8020	2.6916	0.1104	4.1009
117	17.6000	2.8321	2.7254	0.1066	3.9130
118	17.8000	2.8620	2.7591	0.1029	3.7285
119	18.0000	2.8919	2.7928	0.0991	3.5488
120	18.2000	2.9218	2.8263	0.0954	3.3762

Table 45

FIG. 69

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
121	18.4000	2.9517	2.8598	0.0919	3.2131
122	18.6000	2.9817	2.8931	0.0886	3.0622
123	18.7214	3.0000	2.9133	0.0867	2.9760
124	18.8000	3.0119	2.9263	0.0856	2.9259
125	19.0000	3.0423	2.9593	0.0830	2.8046
126	19.2000	3.0729	2.9922	0.0807	2.6964
127	19.4000	3.1035	3.0249	0.0786	2.5978
128	19.6000	3.1341	3.0575	0.0766	2.5058
129	19.8000	3.1646	3.0899	0.0747	2.4178
130	20.0000	3.1949	3.1221	0.0728	2.3321
131	20.2000	3.2250	3.1541	0.0709	2.2472
132	20.4000	3.2548	3.1859	0.0689	2.1625
133	20.6000	3.2844	3.2176	0.0668	2.0772
134	20.8000	3.3137	3.2490	0.0647	1.9911
135	21.0000	3.3427	3.2802	0.0625	1.9039
136	21.2000	3.3714	3.3112	0.0601	1.8158
137	21.4000	3.3997	3.3420	0.0577	1.7270
138	21.6000	3.4276	3.3724	0.0552	1.6377
139	21.8000	3.4553	3.4026	0.0527	1.5483
140	22.0000	3.4825	3.4324	0.0501	1.4593
141	22.1304	3.5000	3.4516	0.0484	1.4022
142	22.2000	3.5093	3.4618	0.0475	1.3712
143	22.4000	3.5357	3.4909	0.0448	1.2844
144	22.6000	3.5616	3.5194	0.0422	1.1994
145	22.8000	3.5871	3.5475	0.0396	1.1167
146	23.0000	3.6120	3.5750	0.0371	1.0366
147	23.2000	3.6364	3.6019	0.0346	0.9594
148	23.4000	3.6602	3.6281	0.0321	0.8854
149	23.6000	3.6834	3.6536	0.0298	0.8148
150	23.8000	3.7059	3.6784	0.0275	0.7476
151	24.0000	3.7277	3.7023	0.0253	0.6839
152	24.2000	3.7487	3.7254	0.0232	0.6237
153	24.4000	3.7689	3.7476	0.0213	0.5670
154	24.6000	3.7882	3.7688	0.0194	0.5139
155	24.8000	3.8067	3.7891	0.0176	0.4641
156	25.0000	3.8242	3.8083	0.0159	0.4176
157	25.2000	3.8409	3.8266	0.0143	0.3744
158	25.4000	3.8566	3.8437	0.0129	0.3344
159	25.6000	3.8713	3.8598	0.0115	0.2974
160	25.8000	3.8851	3.8749	0.0102	0.2634

Table 46

FIG. 70

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	26.0000	3.8979	3.8889	0.0090	0.2323
162	26.2000	3.9097	3.9018	0.0079	0.2037
163	26.4000	3.9207	3.9137	0.0070	0.1779
164	26.6000	3.9306	3.9246	0.0061	0.1545
165	26.8000	3.9397	3.9345	0.0052	0.1334
166	27.0000	3.9479	3.9434	0.0045	0.1145
167	27.2000	3.9553	3.9515	0.0039	0.0977
168	27.4000	3.9619	3.9587	0.0033	0.0829
169	27.6000	3.9678	3.9650	0.0028	0.0698
170	27.8000	3.9729	3.9706	0.0023	0.0584
171	28.0000	3.9775	3.9755	0.0019	0.0485
172	28.2000	3.9814	3.9798	0.0016	0.0399
173	28.4000	3.9847	3.9834	0.0013	0.0326
174	28.6000	3.9876	3.9865	0.0011	0.0264
175	28.8000	3.9900	3.9892	0.0008	0.0212
176	29.0000	3.9920	3.9914	0.0007	0.0169
177	29.2000	3.9937	3.9932	0.0005	0.0134
178	29.4000	3.9951	3.9947	0.0004	0.0103

Table 47

FIG. 71

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-9.221	-9.221	-9.221	-9.221	-9.216	-9.216	-9.221	-9.221
2	-9.253	-9.252	-9.200	-9.252	-9.200	-9.200	-9.200	-9.200
3	-9.232	-9.222	-9.216	-9.222	-9.216	-9.216	-9.216	-9.216
4	-9.224	-9.220	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219
5	-9.226	-9.226	-9.226	-9.226	-9.195	-9.219	-9.220	-9.220
6	-9.230	-9.225	-9.218	-9.218	-9.217	-9.217	-9.217	-9.217
7	-9.229	-9.224	-9.224	-9.224	-9.214	-9.214	-9.214	-9.214
8	-9.225	-9.225	-9.225	-9.225	-9.192	-9.219	-9.225	-9.219
9	-9.225	-9.224	-9.219	-9.219	-9.215	-9.218	-9.219	-9.218
10	-9.227	-9.227	-9.227	-9.227	-9.193	-9.200	-9.227	-9.227
11	-9.221	-9.221	-9.221	-9.221	-9.214	-9.218	-9.221	-9.220
12	-9.220	-9.220	-9.220	-9.220	-9.217	-9.217	-9.220	-9.220
13	-9.228	-9.228	-9.215	-9.228	-9.215	-9.215	-9.215	-9.215
14	-9.221	-9.221	-9.221	-9.221	-9.216	-9.216	-9.221	-9.220
15	-9.220	-9.220	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
16	-9.220	-9.220	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
17	-9.232	-9.232	-9.232	-9.232	-9.189	-9.189	-9.231	-9.219
18	-9.224	-9.224	-9.224	-9.224	-9.211	-9.212	-9.224	-9.212
19	-9.222	-9.220	-9.220	-9.220	-9.219	-9.219	-9.220	-9.219
20	-9.223	-9.220	-9.219	-9.220	-9.217	-9.219	-9.219	-9.219
21	-9.220	-9.220	-9.220	-9.220	-9.219	-9.219	-9.220	-9.220
22	-9.222	-9.222	-9.220	-9.220	-9.218	-9.219	-9.219	-9.219
23	-9.229	-9.223	-9.219	-9.219	-9.217	-9.217	-9.217	-9.217
24	-9.225	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219
25	-9.224	-9.222	-9.220	-9.221	-9.217	-9.217	-9.217	-9.217
26	-9.223	-9.223	-9.223	-9.223	-9.210	-9.212	-9.222	-9.222
27	-9.222	-9.221	-9.221	-9.221	-9.217	-9.217	-9.220	-9.218
28	-9.235	-9.235	-9.230	-9.235	-9.203	-9.206	-9.207	-9.207
29	-9.226	-9.220	-9.219	-9.219	-9.218	-9.218	-9.218	-9.218
30	-9.236	-9.236	-9.211	-9.235	-9.207	-9.210	-9.210	-9.210
31	-9.225	-9.225	-9.216	-9.225	-9.216	-9.216	-9.216	-9.216
32	-9.226	-9.226	-9.224	-9.225	-9.214	-9.214	-9.214	-9.214
33	-9.221	-9.221	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
34	-10.280	-10.279	-10.279	-10.279	-8.021	-8.021	-8.021	-8.021
35	-10.773	-10.773	-10.772	-10.772	-7.346	-7.346	-7.346	-7.346
36	-11.094	-11.094	-11.094	-11.094	-6.850	-6.850	-6.850	-6.850
37	-11.336	-11.336	-11.336	-11.336	-6.442	-6.442	-6.442	-6.442
38	-11.527	-11.527	-11.527	-11.527	-6.094	-6.094	-6.094	-6.094
39	-11.684	-11.684	-11.684	-11.684	-5.786	-5.786	-5.786	-5.786
40	-11.817	-11.816	-11.816	-11.816	-5.512	-5.512	-5.512	-5.512

Table 48

FIG. 72

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	9.220	9.220	9.220	9.220	9.220	9.220	9.220	9.220
2	9.220	9.220	9.220	9.220	9.218	9.218	9.220	9.220
3	9.232	9.233	9.232	9.232	9.205	9.205	9.210	9.207
4	9.223	9.223	9.223	9.223	9.193	9.223	9.223	9.223
5	9.226	9.219	9.219	9.219	9.219	9.219	9.219	9.219
6	9.220	9.220	9.220	9.220	9.218	9.220	9.220	9.220
7	9.225	9.225	9.225	9.225	9.197	9.208	9.225	9.225
8	9.220	9.220	9.219	9.219	9.219	9.219	9.219	9.219
9	9.221	9.221	9.219	9.221	9.218	9.218	9.219	9.219
10	9.227	9.221	9.221	9.221	9.214	9.215	9.221	9.217
11	9.229	9.229	9.214	9.229	9.214	9.214	9.214	9.214
12	9.224	9.224	9.217	9.224	9.217	9.217	9.217	9.217
13	9.228	9.221	9.217	9.221	9.217	9.217	9.217	9.217
14	9.221	9.219	9.219	9.219	9.219	9.219	9.219	9.219
15	9.222	9.220	9.220	9.220	9.219	9.219	9.220	9.219
16	9.233	9.218	9.218	9.218	9.218	9.218	9.218	9.218
17	9.232	9.225	9.223	9.223	9.213	9.213	9.214	9.214
18	9.226	9.224	9.224	9.224	9.214	9.214	9.218	9.214
19	9.222	9.222	9.221	9.222	9.217	9.218	9.218	9.218
20	9.223	9.219	9.219	9.219	9.219	9.219	9.219	9.219
21	9.246	9.246	9.247	9.246	9.193	9.193	9.193	9.193
22	9.222	9.220	9.220	9.220	9.219	9.219	9.219	9.219
23	9.228	9.228	9.217	9.217	9.217	9.217	9.217	9.217
24	9.223	9.223	9.219	9.219	9.218	9.218	9.218	9.218
25	9.224	9.224	9.220	9.224	9.214	9.214	9.220	9.214
26	9.224	9.224	9.224	9.224	9.199	9.217	9.223	9.224
27	9.222	9.219	9.219	9.219	9.219	9.219	9.219	9.219
28	9.235	9.228	9.226	9.228	9.210	9.210	9.210	9.210
29	9.220	9.220	9.220	9.220	9.219	9.219	9.219	9.219
30	9.227	9.225	9.224	9.225	9.214	9.214	9.214	9.214
31	9.222	9.222	9.221	9.221	9.218	9.218	9.218	9.218
32	9.229	9.229	9.214	9.229	9.214	9.214	9.214	9.214
33	9.222	9.222	9.222	9.222	9.217	9.217	9.218	9.217
34	10.279	10.279	10.279	10.279	8.021	8.021	8.022	8.021
35	10.773	10.773	10.773	10.773	7.345	7.345	7.345	7.345
36	11.096	11.096	11.089	11.096	6.850	6.850	6.850	6.850
37	11.336	11.334	11.334	11.334	6.443	6.443	6.443	6.443
38	11.527	11.527	11.527	11.527	6.094	6.094	6.094	6.094
39	11.684	11.684	11.684	11.684	5.786	5.786	5.786	5.786
40	11.817	11.816	11.815	11.815	5.512	5.512	5.512	5.512

Table 49

FIG. 73

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-11.928	-11.928	-11.928	-11.928	-5.265	-5.265	-5.265	-5.265
42	-12.025	-12.025	-12.023	-12.025	-5.042	-5.042	-5.042	-5.042
43	-12.108	-12.108	-12.107	-12.108	-4.838	-4.838	-4.838	-4.838
44	-12.181	-12.180	-12.180	-12.180	-4.652	-4.652	-4.652	-4.652
45	-12.244	-12.244	-12.244	-12.244	-4.482	-4.482	-4.482	-4.482
46	-12.300	-12.300	-12.300	-12.300	-4.327	-4.327	-4.327	-4.327
47	-12.348	-12.348	-12.348	-12.348	-4.187	-4.187	-4.187	-4.187
48	-12.391	-12.391	-12.390	-12.390	-4.058	-4.058	-4.058	-4.058
49	-12.429	-12.429	-12.428	-12.428	-3.942	-3.942	-3.942	-3.942
50	-12.461	-12.461	-12.461	-12.461	-3.837	-3.837	-3.837	-3.837
51	-12.491	-12.490	-12.489	-12.489	-3.743	-3.743	-3.743	-3.743
52	-12.515	-12.515	-12.514	-12.514	-3.660	-3.660	-3.660	-3.660
53	-12.541	-12.541	-12.517	-12.522	-3.592	-3.592	-3.594	-3.594
54	-12.556	-12.552	-12.552	-12.552	-3.524	-3.524	-3.524	-3.524
55	-12.568	-12.568	-12.568	-12.568	-3.471	-3.471	-3.471	-3.471
56	-12.586	-12.586	-12.569	-12.582	-3.426	-3.426	-3.427	-3.426
57	-12.595	-12.595	-12.582	-12.587	-3.391	-3.391	-3.393	-3.392
58	-12.605	-12.605	-12.575	-12.602	-3.366	-3.366	-3.368	-3.366
59	-12.605	-12.605	-12.595	-12.602	-3.349	-3.349	-3.350	-3.349
60	-12.606	-12.606	-12.593	-12.605	-3.340	-3.341	-3.343	-3.342
61	-12.615	-12.605	-12.596	-12.596	-3.339	-3.340	-3.340	-3.340
62	-12.669	-12.669	-12.531	-12.538	-3.334	-3.334	-3.357	-3.356
63	-16.220	-12.525	-10.823	-10.823	-2.815	-3.060	-3.520	-3.520
64	-12.374	-12.374	-12.374	-12.374	-3.419	-3.419	-3.419	-3.419
65	-16.429	-12.067	-10.510	-10.510	-2.915	-3.151	-3.545	-3.545
66	-16.646	-11.953	-10.412	-10.412	-2.868	-3.118	-3.554	-3.554
67	-16.808	-11.873	-10.335	-10.335	-2.828	-3.086	-3.564	-3.564
68	-16.934	-11.816	-10.270	-10.270	-2.793	-3.054	-3.575	-3.575
69	-17.032	-11.780	-10.213	-10.213	-2.763	-3.021	-3.588	-3.588
70	-15.874	-15.874	-7.431	-7.431	15.874	15.874	-5.725	-5.725
71	-15.852	-15.845	-7.463	-7.464	15.852	15.845	-5.752	-5.752
72	-15.850	-15.850	-7.460	-7.460	15.850	15.850	-5.750	-5.750
73	-15.822	-15.820	-7.500	-7.500	15.822	15.822	-5.777	-5.777
74	-15.793	-15.792	-7.542	-7.542	15.794	15.791	-5.800	-5.800
75	-15.765	-15.761	-7.587	-7.588	15.765	15.759	-5.821	-5.821
76	-15.758	-15.702	-7.633	-7.648	15.757	15.703	-5.837	-5.837
77	-15.738	-15.654	-7.688	-7.710	15.738	15.654	-5.850	-5.850
78	-16.426	-14.932	-7.512	-7.920	16.426	14.932	-5.820	-5.820
79	-16.777	-14.559	-7.409	-8.036	16.778	14.559	-5.775	-5.775
80	-17.021	-14.296	-7.335	-8.129	17.021	14.296	-5.726	-5.726

Table 50

FIG. 74

	Label							
	8	9	10	11	12	13	14	15
Design #								
41	11.928	11.928	11.928	11.928	5.265	5.265	5.265	5.265
42	12.025	12.025	12.023	12.023	5.042	5.042	5.042	5.042
43	12.108	12.108	12.108	12.108	4.838	4.838	4.838	4.838
44	12.181	12.181	12.181	12.181	4.652	4.652	4.652	4.652
45	12.244	12.244	12.244	12.244	4.482	4.482	4.482	4.482
46	12.299	12.299	12.299	12.299	4.328	4.328	4.328	4.328
47	12.348	12.348	12.348	12.348	4.187	4.187	4.187	4.187
48	12.391	12.392	12.390	12.391	4.058	4.058	4.058	4.058
49	12.429	12.429	12.429	12.429	3.941	3.941	3.941	3.941
50	12.461	12.461	12.461	12.461	3.837	3.837	3.837	3.837
51	12.491	12.491	12.489	12.489	3.743	3.743	3.743	3.743
52	12.515	12.515	12.514	12.514	3.660	3.660	3.660	3.660
53	12.540	12.540	12.540	12.540	3.583	3.583	3.585	3.583
54	12.556	12.556	12.547	12.555	3.523	3.523	3.524	3.523
55	12.569	12.569	12.568	12.569	3.470	3.470	3.471	3.471
56	12.581	12.581	12.579	12.579	3.427	3.427	3.427	3.427
57	12.595	12.588	12.587	12.587	3.392	3.393	3.393	3.393
58	12.599	12.596	12.595	12.595	3.366	3.367	3.367	3.367
59	12.600	12.600	12.600	12.600	3.351	3.351	3.351	3.351
60	12.604	12.604	12.603	12.604	3.341	3.341	3.341	3.341
61	12.607	12.607	12.597	12.607	3.338	3.338	3.339	3.338
62	12.602	12.602	12.602	12.602	3.345	3.345	3.345	3.345
63	12.424	12.424	12.424	12.424	3.403	3.403	3.403	3.403
64	16.734	12.329	10.663	10.663	2.735	3.014	3.518	3.518
65	16.429	12.066	10.511	10.511	2.915	3.151	3.545	3.545
66	16.646	11.953	10.412	10.412	2.868	3.118	3.554	3.554
67	16.808	11.873	10.335	10.335	2.828	3.086	3.564	3.564
68	16.933	11.816	10.270	10.270	2.793	3.054	3.575	3.575
69	17.033	11.780	10.213	10.213	2.762	3.022	3.588	3.588
70	5.725	5.725	0.168	0.168	7.431	7.431	-0.168	-0.168
71	5.752	5.752	0.200	0.200	7.464	7.465	-0.200	-0.200
72	5.750	5.750	0.200	0.200	7.460	7.460	-0.200	-0.200
73	5.777	5.777	0.231	0.231	7.501	7.501	-0.232	-0.232
74	5.800	5.800	0.264	0.264	7.542	7.542	-0.264	-0.264
75	5.821	5.821	0.297	0.297	7.587	7.589	-0.297	-0.297
76	5.837	5.837	0.333	0.333	7.633	7.648	-0.333	-0.333
77	5.850	5.850	0.372	0.372	7.687	7.710	-0.371	-0.371
78	5.820	5.820	0.403	0.403	7.512	7.920	-0.403	-0.403
79	5.775	5.775	0.435	0.435	7.409	8.036	-0.435	-0.435
80	5.726	5.726	0.469	0.469	7.335	8.129	-0.469	-0.469

Table 51

FIG. 75

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-18.783	-12.062	18.197	-10.830	-3.578	-5.451	-3.578	-5.645
82	-18.750	-12.045	18.134	-10.833	-3.574	-5.494	-3.574	-5.700
83	-18.705	-12.026	18.079	-10.832	-3.567	-5.547	-3.567	-5.764
84	-18.646	-12.005	18.034	-10.825	-3.556	-5.611	-3.556	-5.837
85	-18.574	-11.980	17.998	-10.814	-3.540	-5.687	-3.540	-5.920
86	-18.490	-11.954	17.970	-10.798	-3.520	-5.774	-3.520	-6.011
87	-18.398	-11.926	17.948	-10.779	-3.496	-5.869	-3.496	-6.108
88	-18.302	-11.901	17.929	-10.761	-3.470	-5.966	-3.470	-6.205
89	-18.280	-11.900	17.920	-10.760	-3.460	-5.990	-3.460	-6.230
90	-18.207	-11.880	17.909	-10.744	-3.444	-6.061	-3.444	-6.299
91	-18.120	-11.867	17.886	-10.731	-3.418	-6.149	-3.418	-6.386
92	-18.040	-11.861	17.859	-10.721	-3.393	-6.227	-3.393	-6.465
93	-17.966	-11.862	17.829	-10.715	-3.371	-6.294	-3.371	-6.537
94	-17.899	-11.872	17.796	-10.711	-3.351	-6.352	-3.351	-6.600
95	-17.837	-11.888	17.761	-10.709	-3.333	-6.400	-3.333	-6.658
96	-17.780	-11.911	17.724	-10.708	-3.317	-6.440	-3.317	-6.709
97	-17.724	-11.941	17.688	-10.705	-3.304	-6.472	-3.304	-6.756
98	-17.680	-11.982	17.639	-10.701	-3.291	-6.494	-3.291	-6.798
99	-17.625	-12.022	17.609	-10.691	-3.282	-6.514	-3.281	-6.839
100	-17.580	-12.076	17.569	-10.677	-3.277	-6.522	-3.265	-6.877
101	-17.536	-12.137	17.532	-10.655	-3.274	-6.524	-3.250	-6.913
102	-17.500	-12.211	17.494	-10.623	-3.270	-6.513	-3.233	-6.947
103	-17.466	-12.295	17.462	-10.581	-3.267	-6.491	-3.214	-6.979
104	-17.441	-12.388	17.434	-10.529	-3.263	-6.456	-3.192	-7.006
105	-17.421	-12.484	17.414	-10.470	-3.259	-6.413	-3.168	-7.028
106	-17.420	-12.490	17.410	-10.460	-3.260	-6.410	-3.170	-7.030
107	-17.407	-12.577	17.398	-10.411	-3.255	-6.363	-3.142	-7.046
108	-17.395	-12.663	17.388	-10.354	-3.252	-6.313	-3.115	-7.061
109	-17.385	-12.740	17.378	-10.304	-3.251	-6.266	-3.089	-7.074
110	-17.374	-12.807	17.370	-10.261	-3.252	-6.224	-3.063	-7.087
111	-17.363	-12.865	17.359	-10.225	-3.256	-6.186	-3.039	-7.101
112	-17.346	-12.915	17.346	-10.197	-3.264	-6.156	-3.017	-7.119
113	-17.335	-12.960	17.333	-10.172	-3.273	-6.124	-2.993	-7.137
114	-17.317	-12.998	17.316	-10.153	-3.287	-6.098	-2.970	-7.162
115	-17.296	-13.031	17.295	-10.140	-3.305	-6.074	-2.945	-7.193
116	-17.272	-13.058	17.272	-10.131	-3.328	-6.051	-2.919	-7.232
117	-17.244	-13.082	17.244	-10.127	-3.356	-6.029	-2.888	-7.279
118	-17.211	-13.101	17.211	-10.128	-3.393	-6.006	-2.852	-7.338
119	-17.172	-13.115	17.172	-10.134	-3.438	-5.983	-2.811	-7.406
120	-17.126	-13.125	17.126	-10.146	-3.496	-5.964	-2.765	-7.483

Table 52

FIG. 76

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	8.945	7.590	10.894	7.590	1.248	2.166	1.131	2.166
82	9.040	7.532	11.014	7.532	1.186	2.227	1.081	2.227
83	9.154	7.464	11.131	7.464	1.111	2.291	1.022	2.291
84	9.291	7.388	11.242	7.388	1.022	2.360	0.950	2.360
85	9.452	7.304	11.344	7.304	0.918	2.436	0.865	2.436
86	9.629	7.215	11.435	7.215	0.803	2.516	0.768	2.516
87	9.809	7.127	11.512	7.127	0.683	2.601	0.664	2.601
88	9.978	7.055	11.580	7.036	0.566	2.685	0.560	2.685
89	10.020	7.040	11.600	7.010	0.540	2.710	0.540	2.710
90	10.127	7.007	11.643	6.939	0.461	2.766	0.461	2.766
91	10.247	6.969	11.696	6.864	0.372	2.841	0.372	2.841
92	10.344	6.939	11.742	6.805	0.297	2.909	0.297	2.909
93	10.423	6.918	11.784	6.758	0.234	2.968	0.234	2.968
94	10.486	6.904	11.823	6.722	0.183	3.019	0.183	3.019
95	10.536	6.897	11.861	6.694	0.141	3.063	0.141	3.063
96	10.575	6.897	11.898	6.673	0.108	3.099	0.108	3.099
97	10.604	6.901	11.939	6.655	0.082	3.128	0.082	3.128
98	10.626	6.911	11.977	6.646	0.065	3.153	0.065	3.153
99	10.633	6.924	12.029	6.627	0.045	3.166	0.045	3.173
100	10.633	6.941	12.082	6.613	0.034	3.175	0.034	3.193
101	10.623	6.961	12.144	6.595	0.025	3.178	0.025	3.207
102	10.600	6.983	12.213	6.572	0.019	3.178	0.019	3.220
103	10.565	7.004	12.293	6.541	0.015	3.172	0.015	3.228
104	10.518	7.024	12.381	6.500	0.012	3.160	0.012	3.234
105	10.464	7.041	12.474	6.451	0.009	3.143	0.009	3.237
106	10.460	7.040	12.480	6.450	0.010	3.140	0.010	3.240
107	10.407	7.055	12.566	6.397	0.008	3.123	0.008	3.238
108	10.352	7.067	12.653	6.342	0.010	3.101	0.002	3.239
109	10.303	7.079	12.731	6.291	0.015	3.077	-0.006	3.241
110	10.260	7.090	12.800	6.244	0.021	3.054	-0.015	3.244
111	10.225	7.103	12.860	6.202	0.029	3.031	-0.025	3.250
112	10.197	7.121	12.913	6.167	0.038	3.010	-0.037	3.259
113	10.172	7.139	12.958	6.134	0.050	2.987	-0.048	3.269
114	10.153	7.163	12.997	6.105	0.063	2.965	-0.063	3.284
115	10.140	7.194	13.030	6.080	0.079	2.942	-0.079	3.303
116	10.131	7.232	13.058	6.056	0.098	2.916	-0.099	3.326
117	10.127	7.280	13.082	6.032	0.122	2.885	-0.122	3.355
118	10.128	7.338	13.101	6.008	0.151	2.850	-0.152	3.392
119	10.135	7.407	13.115	5.985	0.188	2.809	-0.189	3.438
120	10.147	7.484	13.125	5.965	0.235	2.763	-0.236	3.496

Table 53

FIG. 77

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-17.073	-13.130	17.073	-10.163	-3.570	-5.950	-2.716	-7.565
122	-17.011	-13.129	17.012	-10.183	-3.661	-5.947	-2.667	-7.649
123	-16.970	-13.130	16.970	-10.200	-3.720	-5.950	-2.640	-7.700
124	-16.945	-13.125	16.945	-10.205	-3.765	-5.954	-2.624	-7.729
125	-16.879	-13.119	16.879	-10.228	-3.867	-5.968	-2.591	-7.798
126	-16.817	-13.115	16.817	-10.249	-3.954	-5.984	-2.568	-7.856
127	-16.763	-13.114	16.763	-10.271	-4.022	-5.999	-2.551	-7.903
128	-16.713	-13.115	16.713	-10.292	-4.074	-6.013	-2.539	-7.942
129	-16.667	-13.117	16.667	-10.313	-4.116	-6.026	-2.531	-7.975
130	-16.624	-13.120	16.624	-10.334	-4.149	-6.039	-2.526	-8.005
131	-16.582	-13.123	16.582	-10.356	-4.176	-6.053	-2.524	-8.033
132	-16.540	-13.127	16.541	-10.378	-4.199	-6.067	-2.524	-8.060
133	-16.499	-13.129	16.500	-10.400	-4.220	-6.083	-2.525	-8.087
134	-16.459	-13.132	16.458	-10.422	-4.238	-6.100	-2.528	-8.113
135	-16.418	-13.134	16.417	-10.444	-4.256	-6.119	-2.532	-8.140
136	-16.375	-13.134	16.376	-10.466	-4.275	-6.139	-2.539	-8.167
137	-16.335	-13.135	16.332	-10.487	-4.293	-6.162	-2.545	-8.195
138	-16.291	-13.134	16.290	-10.508	-4.312	-6.186	-2.555	-8.223
139	-16.249	-13.132	16.247	-10.528	-4.332	-6.212	-2.565	-8.251
140	-16.204	-13.129	16.204	-10.548	-4.354	-6.239	-2.577	-8.280
141	-16.180	-13.130	16.180	-10.560	-4.370	-6.260	-2.590	-8.300
142	-16.160	-13.126	16.161	-10.567	-4.377	-6.267	-2.590	-8.309
143	-16.117	-13.122	16.117	-10.585	-4.400	-6.296	-2.604	-8.337
144	-16.074	-13.117	16.074	-10.603	-4.424	-6.324	-2.619	-8.365
145	-16.031	-13.112	16.032	-10.620	-4.448	-6.353	-2.635	-8.393
146	-15.989	-13.107	15.989	-10.636	-4.472	-6.382	-2.650	-8.419
147	-15.948	-13.102	15.949	-10.652	-4.497	-6.410	-2.666	-8.445
148	-15.910	-13.098	15.908	-10.667	-4.519	-6.437	-2.681	-8.470
149	-15.872	-13.092	15.869	-10.682	-4.542	-6.463	-2.696	-8.495
150	-15.833	-13.087	15.833	-10.695	-4.565	-6.489	-2.712	-8.518
151	-15.797	-13.082	15.797	-10.708	-4.586	-6.513	-2.726	-8.540
152	-15.763	-13.078	15.763	-10.721	-4.606	-6.536	-2.739	-8.562
153	-15.730	-13.073	15.730	-10.733	-4.625	-6.559	-2.752	-8.582
154	-15.698	-13.069	15.698	-10.745	-4.643	-6.580	-2.764	-8.601
155	-15.668	-13.066	15.668	-10.756	-4.660	-6.599	-2.776	-8.619
156	-15.639	-13.062	15.639	-10.767	-4.677	-6.618	-2.787	-8.637
157	-15.610	-13.059	15.610	-10.777	-4.692	-6.637	-2.797	-8.654
158	-15.584	-13.056	15.584	-10.787	-4.707	-6.654	-2.807	-8.670
159	-15.559	-13.053	15.559	-10.796	-4.721	-6.670	-2.816	-8.685
160	-15.535	-13.050	15.535	-10.805	-4.734	-6.685	-2.825	-8.699

Table 54

FIG. 78

	Label							
	8	9	10	11	12	13	14	15
Design #								
121	10.163	7.566	13.130	5.951	0.294	2.713	-0.295	3.570
122	10.184	7.650	13.130	5.947	0.368	2.666	-0.369	3.661
123	10.200	7.700	13.130	5.950	0.420	2.640	-0.420	3.720
124	10.206	7.729	13.125	5.954	0.452	2.623	-0.452	3.765
125	10.228	7.799	13.120	5.969	0.534	2.590	-0.534	3.867
126	10.250	7.856	13.115	5.985	0.604	2.567	-0.604	3.954
127	10.271	7.903	13.114	6.000	0.658	2.550	-0.659	4.022
128	10.292	7.942	13.115	6.013	0.700	2.539	-0.700	4.074
129	10.313	7.975	13.117	6.026	0.732	2.531	-0.732	4.116
130	10.334	8.005	13.120	6.039	0.757	2.526	-0.757	4.149
131	10.356	8.033	13.123	6.053	0.776	2.524	-0.776	4.176
132	10.378	8.060	13.127	6.067	0.791	2.523	-0.791	4.199
133	10.400	8.087	13.130	6.083	0.803	2.525	-0.803	4.220
134	10.422	8.113	13.132	6.100	0.813	2.528	-0.813	4.238
135	10.444	8.140	13.133	6.119	0.822	2.533	-0.821	4.257
136	10.466	8.167	13.135	6.139	0.828	2.538	-0.828	4.274
137	10.487	8.195	13.134	6.163	0.835	2.547	-0.834	4.294
138	10.508	8.223	13.134	6.186	0.840	2.555	-0.840	4.312
139	10.528	8.251	13.131	6.212	0.846	2.566	-0.845	4.333
140	10.548	8.280	13.129	6.239	0.851	2.577	-0.850	4.354
141	10.560	8.300	13.130	6.260	0.850	2.590	-0.850	4.370
142	10.567	8.309	13.126	6.267	0.856	2.590	-0.856	4.376
143	10.585	8.337	13.122	6.296	0.861	2.604	-0.861	4.400
144	10.603	8.365	13.117	6.324	0.867	2.619	-0.866	4.424
145	10.620	8.392	13.113	6.353	0.872	2.634	-0.872	4.448
146	10.636	8.419	13.107	6.382	0.878	2.650	-0.878	4.472
147	10.652	8.446	13.102	6.410	0.883	2.666	-0.884	4.496
148	10.667	8.471	13.096	6.437	0.890	2.682	-0.888	4.520
149	10.681	8.495	13.091	6.464	0.895	2.697	-0.894	4.543
150	10.695	8.518	13.087	6.489	0.899	2.711	-0.900	4.565
151	10.709	8.540	13.082	6.513	0.904	2.726	-0.904	4.586
152	10.721	8.561	13.078	6.537	0.909	2.739	-0.909	4.606
153	10.733	8.582	13.073	6.559	0.914	2.752	-0.914	4.625
154	10.745	8.601	13.070	6.580	0.918	2.764	-0.918	4.643
155	10.756	8.619	13.066	6.599	0.922	2.776	-0.922	4.661
156	10.767	8.637	13.062	6.618	0.926	2.787	-0.926	4.677
157	10.777	8.654	13.059	6.637	0.929	2.797	-0.929	4.692
158	10.787	8.670	13.056	6.654	0.933	2.807	-0.933	4.707
159	10.796	8.685	13.053	6.670	0.936	2.816	-0.936	4.721
160	10.805	8.699	13.050	6.685	0.939	2.825	-0.939	4.734

Table 55

FIG. 79

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-15.512	-13.048	15.512	-10.814	-4.746	-6.700	-2.833	-8.713
162	-15.489	-13.045	15.489	-10.822	-4.758	-6.713	-2.841	-8.726
163	-15.468	-13.043	15.468	-10.829	-4.770	-6.727	-2.848	-8.738
164	-15.447	-13.040	15.447	-10.837	-4.781	-6.739	-2.856	-8.750
165	-15.428	-13.038	15.428	-10.844	-4.790	-6.751	-2.862	-8.761
166	-15.408	-13.036	15.409	-10.851	-4.801	-6.763	-2.869	-8.773
167	-15.393	-13.034	15.392	-10.857	-4.809	-6.773	-2.875	-8.782
168	-15.375	-13.033	15.374	-10.864	-4.818	-6.784	-2.880	-8.792
169	-15.358	-13.032	15.358	-10.870	-4.826	-6.793	-2.886	-8.801
170	-15.344	-13.030	15.344	-10.875	-4.834	-6.802	-2.891	-8.810
171	-15.328	-13.028	15.328	-10.881	-4.842	-6.811	-2.896	-8.819
172	-15.312	-13.026	15.313	-10.886	-4.850	-6.821	-2.902	-8.827
173	-15.299	-13.025	15.299	-10.891	-4.856	-6.829	-2.906	-8.835
174	-15.288	-13.024	15.288	-10.895	-4.862	-6.836	-2.910	-8.841
175	-15.276	-13.024	15.276	-10.901	-4.867	-6.843	-2.912	-8.849
176	-15.263	-13.023	15.261	-10.905	-4.874	-6.850	-2.917	-8.856
177	-15.252	-13.022	15.251	-10.909	-4.879	-6.856	-2.921	-8.862
178	-15.242	-13.021	15.241	-10.913	-4.885	-6.863	-2.923	-8.869

Table 56

FIG. 80

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	10.813	8.712	13.047	6.699	0.942	2.833	-0.942	4.747
162	10.822	8.726	13.045	6.713	0.945	2.841	-0.945	4.758
163	10.829	8.738	13.042	6.727	0.947	2.848	-0.947	4.770
164	10.837	8.750	13.040	6.740	0.950	2.856	-0.950	4.781
165	10.844	8.761	13.038	6.751	0.952	2.862	-0.952	4.790
166	10.852	8.773	13.037	6.763	0.954	2.869	-0.955	4.801
167	10.857	8.782	13.035	6.773	0.956	2.874	-0.957	4.809
168	10.864	8.792	13.033	6.784	0.959	2.881	-0.958	4.818
169	10.870	8.801	13.032	6.793	0.960	2.886	-0.960	4.826
170	10.875	8.810	13.030	6.802	0.962	2.891	-0.962	4.834
171	10.881	8.819	13.029	6.811	0.964	2.896	-0.964	4.842
172	10.887	8.828	13.027	6.821	0.965	2.901	-0.967	4.849
173	10.891	8.835	13.025	6.829	0.967	2.906	-0.967	4.856
174	10.896	8.842	13.025	6.836	0.968	2.909	-0.969	4.862
175	10.899	8.848	13.023	6.843	0.971	2.914	-0.969	4.868
176	10.905	8.856	13.022	6.851	0.972	2.918	-0.971	4.875
177	10.909	8.861	13.021	6.857	0.973	2.921	-0.972	4.880
178	10.911	8.867	13.019	6.863	0.975	2.926	-0.972	4.886

Table 57

FIG. 81

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
63	7.4	1.36	1.23	1.1	0.99	0.65	0
64	7.6	1.35	1.22	1.1	0.99	0.65	0
65	7.8	1.24	1.11	1	0.9	0.59	0
66	8	1.17	1.06	0.95	0.85	0.56	0
67	8.2	1.14	1.02	0.92	0.83	0.54	0
68	8.4	1.12	1.01	0.91	0.82	0.59	0
69	8.6	1.23	1.1	0.99	0.8	0.53	0
70	8.8	1.17	0.65	0.58	0.53	0.47	0
71	9	1.17	0.64	0.57	0.52	0.47	0
72	9	1.17	0.64	0.57	0.52	0.47	0
73	9.2	1.17	0.63	0.56	0.51	0.46	0
74	9.4	1.16	0.61	0.55	0.49	0.45	0
75	9.6	1.16	0.6	0.54	0.48	0.44	0
76	9.8	1.15	0.59	0.53	0.48	0.43	0
77	10	1.14	0.58	0.52	0.47	0.42	0
78	10.2	1.18	0.79	0.71	0.64	0.57	0
79	10.4	1.1	0.91	0.82	0.73	0.54	0
80	10.6	1.08	0.97	0.87	0.79	0.57	0
81	10.8	1.04	0.93	0.84	0.68	0.5	0
82	11	1	0.9	0.81	0.73	0.53	0
83	11.2	1.07	0.96	0.87	0.7	0.51	0
84	11.4	1.03	0.92	0.83	0.75	0.55	0
85	11.6	1.09	0.98	0.88	0.71	0.52	0
86	11.8	1.03	0.92	0.83	0.75	0.49	0
87	12	0.97	0.87	0.78	0.7	0.51	0
88	12.2	1.01	0.9	0.81	0.73	0.48	0
89	12.25	0.99	0.89	0.8	0.72	0.52	0
90	12.4	0.94	0.84	0.76	0.68	0.5	0
91	12.6	0.97	0.88	0.79	0.71	0.47	0
92	12.8	0.93	0.83	0.75	0.67	0.49	0
93	13	0.89	0.8	0.72	0.65	0.47	0
94	13.2	0.86	0.77	0.7	0.63	0.46	0
95	13.4	0.93	0.83	0.75	0.68	0.44	0
96	13.6	0.9	0.81	0.73	0.66	0.43	0
97	13.8	0.88	0.79	0.71	0.64	0.42	0
98	14	0.87	0.78	0.69	0.62	0.45	0
99	14.2	0.77	0.69	0.62	0.56	0.45	0
100	14.4	0.83	0.75	0.63	0.57	0.41	0
101	14.6	0.73	0.66	0.59	0.53	0.39	0
102	14.8	0.72	0.65	0.55	0.5	0.4	0
103	15	0.71	0.64	0.52	0.47	0.38	0

Table 58

FIG. 82

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
104	15.2	0.7	0.63	0.49	0.44	0.36	0
105	15.4	0.68	0.61	0.47	0.43	0.34	0
106	15.42	0.68	0.61	0.47	0.42	0.38	0
107	15.6	0.67	0.6	0.47	0.42	0.34	0
108	15.8	0.72	0.65	0.46	0.41	0.33	0
109	16	0.63	0.57	0.45	0.41	0.33	0
110	16.2	0.61	0.55	0.44	0.4	0.32	0
111	16.4	0.6	0.54	0.43	0.39	0.32	0
112	16.6	0.58	0.53	0.42	0.38	0.31	0
113	16.8	0.57	0.52	0.41	0.37	0.3	0
114	17	0.56	0.5	0.4	0.36	0.26	0
115	17.2	0.49	0.44	0.38	0.35	0.28	0
116	17.4	0.53	0.47	0.37	0.33	0.27	0
117	17.6	0.51	0.46	0.35	0.31	0.25	0
118	17.8	0.48	0.43	0.32	0.29	0.23	0
119	18	0.46	0.41	0.34	0.3	0.25	0
120	18.2	0.43	0.39	0.35	0.31	0.23	0
121	18.4	0.4	0.36	0.33	0.29	0.21	0
122	18.6	0.42	0.38	0.34	0.31	0.2	0
123	18.72	0.39	0.35	0.31	0.28	0.21	0
124	18.8	0.39	0.35	0.32	0.29	0.21	0
125	19	0.37	0.33	0.3	0.27	0.2	0
126	19.2	0.39	0.35	0.3	0.27	0.2	0
127	19.4	0.37	0.33	0.28	0.25	0.18	0
128	19.6	0.34	0.31	0.26	0.24	0.19	0
129	19.8	0.36	0.32	0.25	0.23	0.18	0
130	20	0.33	0.3	0.27	0.24	0.18	0
131	20.2	0.34	0.31	0.28	0.23	0.16	0
132	20.4	0.32	0.29	0.26	0.23	0.17	0
133	20.6	0.3	0.27	0.24	0.22	0.16	0
134	20.8	0.27	0.25	0.22	0.2	0.15	0
135	21	0.28	0.25	0.23	0.21	0.14	0
136	21.2	0.26	0.24	0.21	0.19	0.14	0
137	21.4	0.26	0.23	0.21	0.19	0.14	0
138	21.6	0.25	0.22	0.2	0.18	0.12	0
139	21.8	0.24	0.21	0.19	0.17	0.13	0
140	22	0.23	0.21	0.19	0.17	0.11	0
141	22.13	0.2	0.18	0.16	0.15	0.11	0
142	22.2	0.2	0.18	0.16	0.14	0.11	0
143	22.4	0.19	0.17	0.15	0.14	0.1	0
144	22.6	0.18	0.17	0.15	0.13	0.1	0

Table 59

FIG. 83

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
145	22.8	0.18	0.16	0.14	0.13	0.09	0
146	23	0.17	0.15	0.14	0.12	0.09	0
147	23.2	0.16	0.14	0.13	0.12	0.08	0
148	23.4	0.15	0.14	0.12	0.11	0.08	0
149	23.6	0.13	0.12	0.11	0.1	0.07	0
150	23.8	0.14	0.13	0.11	0.1	0.07	0
151	24	0.12	0.11	0.1	0.09	0.06	0
152	24.2	0.12	0.1	0.09	0.08	0.06	0
153	24.4	0.11	0.1	0.09	0.08	0.06	0
154	24.6	0.11	0.09	0.09	0.08	0.06	0
155	24.8	0.1	0.09	0.08	0.07	0.05	0
156	25	0.1	0.09	0.08	0.07	0.05	0
157	25.2	0.09	0.08	0.07	0.07	0.05	0
158	25.4	0.09	0.08	0.07	0.06	0.05	0
159	25.6	0.08	0.08	0.07	0.06	0.04	0
160	25.8	0.08	0.07	0.06	0.06	0.04	0
161	26	0.08	0.07	0.06	0.06	0.04	0
162	26.2	0.07	0.07	0.06	0.05	0.04	0
163	26.4	0.07	0.06	0.06	0.05	0.04	0
164	26.6	0.07	0.06	0.05	0.05	0.04	0
165	26.8	0.06	0.06	0.05	0.05	0.03	0
166	27	0.06	0.05	0.05	0.04	0.03	0
167	27.2	0.05	0.05	0.04	0.04	0.03	0
168	27.4	0.06	0.05	0.05	0.04	0.03	0
169	27.6	0.05	0.05	0.04	0.04	0.03	0
170	27.8	0.05	0.04	0.04	0.03	0.02	0
171	28	0.05	0.04	0.04	0.04	0.03	0
172	28.2	0.05	0.04	0.04	0.03	0.02	0
173	28.4	0.04	0.04	0.03	0.03	0.02	0
174	28.6	0.04	0.04	0.03	0.03	0.02	0
175	28.8	0.04	0.03	0.03	0.03	0.02	0
176	29	0.03	0.03	0.03	0.03	0.02	0
177	29.2	0.03	0.03	0.03	0.02	0.02	0
178	29.4	0.03	0.03	0.03	0.02	0.02	0

Table 60

FIG. 84

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
1	-5.000	0.1982	0.1980	0.0002	0.0804
2	-4.800	0.2063	0.2061	0.0002	0.0894
3	-4.600	0.2147	0.2145	0.0002	0.0992
4	-4.400	0.2234	0.2232	0.0002	0.1099
5	-4.200	0.2324	0.2322	0.0003	0.1217
6	-4.000	0.2417	0.2414	0.0003	0.1346
7	-3.800	0.2514	0.2510	0.0004	0.1488
8	-3.600	0.2613	0.2609	0.0004	0.1642
9	-3.400	0.2715	0.2711	0.0005	0.1809
10	-3.200	0.2821	0.2816	0.0006	0.1992
11	-3.000	0.2931	0.2924	0.0006	0.2190
12	-2.800	0.3043	0.3036	0.0007	0.2405
13	-2.600	0.3159	0.3151	0.0008	0.2637
14	-2.400	0.3279	0.3269	0.0009	0.2888
15	-2.200	0.3402	0.3391	0.0011	0.3159
16	-2.000	0.3529	0.3516	0.0012	0.3451
17	-1.800	0.3659	0.3645	0.0014	0.3764
18	-1.600	0.3793	0.3777	0.0015	0.4100
19	-1.400	0.3931	0.3913	0.0017	0.4460
20	-1.200	0.4072	0.4052	0.0020	0.4845
21	-1.000	0.4217	0.4195	0.0022	0.5255
22	-0.800	0.4366	0.4341	0.0025	0.5692
23	-0.600	0.4519	0.4491	0.0028	0.6155
24	-0.400	0.4675	0.4644	0.0031	0.6647
25	-0.200	0.4836	0.4801	0.0034	0.7167
26	0.000	0.5000	0.4962	0.0038	0.7716
27	0.200	0.5168	0.5125	0.0043	0.8294
28	0.400	0.5340	0.5292	0.0047	0.8902
29	0.600	0.5515	0.5463	0.0052	0.9540
30	0.800	0.5695	0.5637	0.0058	1.0207
31	1.000	0.5878	0.5814	0.0063	1.0903
32	1.200	0.6065	0.5995	0.0070	1.1629
33	1.400	0.6255	0.6179	0.0077	1.2383
34	1.600	0.6450	0.6366	0.0084	1.3165
35	1.800	0.6648	0.6556	0.0092	1.3973
36	2.000	0.6850	0.6750	0.0100	1.4807
37	2.200	0.7055	0.6946	0.0109	1.5665
38	2.400	0.7264	0.7146	0.0118	1.6545
39	2.600	0.7476	0.7348	0.0128	1.7448
40	2.800	0.7692	0.7553	0.0139	1.8370

Table 61

FIG. 85

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
41	3.000	0.7912	0.7762	0.0150	1.9309
42	3.200	0.8134	0.7973	0.0162	2.0265
43	3.400	0.8360	0.8186	0.0174	2.1237
44	3.600	0.8589	0.8403	0.0187	2.2223
45	3.800	0.8822	0.8622	0.0200	2.3216
46	4.000	0.9057	0.8843	0.0214	2.4217
47	4.200	0.9296	0.9067	0.0229	2.5225
48	4.400	0.9538	0.9294	0.0244	2.6237
49	4.600	0.9782	0.9523	0.0260	2.7253
50	4.776	1.0000	0.9726	0.0274	2.8172
51	4.800	1.0030	0.9754	0.0276	2.8268
52	5.000	1.0280	0.9988	0.0292	2.9281
53	5.200	1.0533	1.0223	0.0310	3.0290
54	5.400	1.0789	1.0461	0.0327	3.1291
55	5.600	1.1047	1.0702	0.0345	3.2281
56	5.800	1.1308	1.0944	0.0364	3.3258
57	6.000	1.1571	1.1188	0.0383	3.4217
58	6.200	1.1836	1.1434	0.0402	3.5157
59	6.400	1.2104	1.1682	0.0422	3.6083
60	6.600	1.2374	1.1932	0.0441	3.6994
61	6.800	1.2646	1.2184	0.0462	3.7888
62	7.000	1.2920	1.2438	0.0482	3.8764
63	7.200	1.3197	1.2694	0.0503	3.9615
64	7.400	1.3475	1.2951	0.0524	4.0439
65	7.600	1.3755	1.3210	0.0545	4.1236
66	7.800	1.4037	1.3471	0.0566	4.2007
67	8.000	1.4320	1.3733	0.0587	4.2754
68	8.200	1.4605	1.3997	0.0609	4.3475
69	8.400	1.4892	1.4262	0.0630	4.4169
70	8.475	1.5000	1.4362	0.0638	4.4423
71	8.600	1.5180	1.4529	0.0651	4.4836
72	8.800	1.5470	1.4797	0.0673	4.5478
73	9.000	1.5762	1.5067	0.0695	4.6094
74	9.200	1.6054	1.5338	0.0716	4.6679
75	9.400	1.6348	1.5611	0.0737	4.7231
76	9.600	1.6643	1.5885	0.0758	4.7749
77	9.800	1.6940	1.6160	0.0779	4.8235
78	10.000	1.7237	1.6437	0.0800	4.8693
79	10.200	1.7536	1.6715	0.0821	4.9121
80	10.400	1.7835	1.6994	0.0842	4.9523

Table 62

FIG. 86

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.600	1.8136	1.7274	0.0862	4.9903
82	10.800	1.8437	1.7555	0.0882	5.0256
83	11.000	1.8740	1.7838	0.0902	5.0582
84	11.200	1.9043	1.8121	0.0922	5.0883
85	11.400	1.9348	1.8406	0.0942	5.1158
86	11.600	1.9653	1.8692	0.0961	5.1407
87	11.800	1.9959	1.8979	0.0980	5.1630
88	11.827	2.0000	1.9018	0.0982	5.1635
89	12.000	2.0265	1.9267	0.0998	5.1824
90	12.200	2.0572	1.9555	0.1017	5.1988
91	12.400	2.0880	1.9845	0.1034	5.2121
92	12.600	2.1188	2.0136	0.1052	5.2225
93	12.800	2.1496	2.0428	0.1069	5.2307
94	13.000	2.1805	2.0720	0.1085	5.2365
95	13.200	2.2115	2.1014	0.1101	5.2400
96	13.400	2.2425	2.1308	0.1117	5.2417
97	13.600	2.2736	2.1603	0.1132	5.2416
98	13.800	2.3047	2.1899	0.1147	5.2398
99	14.000	2.3358	2.2196	0.1162	5.2362
100	14.200	2.3670	2.2494	0.1177	5.2310
101	14.400	2.3983	2.2792	0.1191	5.2241
102	14.600	2.4296	2.3091	0.1204	5.2155
103	14.800	2.4609	2.3391	0.1218	5.2052
104	15.000	2.4922	2.3692	0.1230	5.1934
105	15.050	2.5000	2.3766	0.1234	5.1923
106	15.200	2.5236	2.3993	0.1243	5.1802
107	15.400	2.5550	2.4295	0.1255	5.1654
108	15.600	2.5864	2.4598	0.1267	5.1492
109	15.800	2.6179	2.4901	0.1278	5.1316
110	16.000	2.6493	2.5205	0.1289	5.1124
111	16.200	2.6808	2.5509	0.1299	5.0918
112	16.400	2.7123	2.5814	0.1309	5.0696
113	16.600	2.7438	2.6120	0.1318	5.0459
114	16.800	2.7753	2.6426	0.1327	5.0207
115	17.000	2.8068	2.6733	0.1335	4.9940
116	17.200	2.8383	2.7041	0.1343	4.9658
117	17.400	2.8698	2.7348	0.1350	4.9361
118	17.600	2.9013	2.7657	0.1357	4.9051
119	17.800	2.9329	2.7966	0.1363	4.8727
120	18.000	2.9643	2.8275	0.1368	4.8389

Table 63

FIG. 87

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.200	2.9958	2.8585	0.1373	4.8038
122	18.226	3.0000	2.8626	0.1374	4.7998
123	18.400	3.0273	2.8896	0.1378	4.7674
124	18.600	3.0588	2.9206	0.1381	4.7297
125	18.800	3.0902	2.9518	0.1385	4.6907
126	19.000	3.1217	2.9830	0.1387	4.6505
127	19.200	3.1531	3.0142	0.1389	4.6090
128	19.400	3.1845	3.0454	0.1391	4.5663
129	19.600	3.2159	3.0767	0.1391	4.5223
130	19.800	3.2472	3.1081	0.1392	4.4772
131	20.000	3.2786	3.1395	0.1391	4.4307
132	20.200	3.3099	3.1709	0.1390	4.3831
133	20.400	3.3411	3.2023	0.1388	4.3342
134	20.600	3.3723	3.2338	0.1385	4.2841
135	20.800	3.4035	3.2653	0.1382	4.2328
136	21.000	3.4347	3.2969	0.1378	4.1802
137	21.200	3.4658	3.3285	0.1373	4.1265
138	21.400	3.4969	3.3601	0.1368	4.0714
139	21.420	3.5000	3.3633	0.1367	4.0645
140	21.600	3.5279	3.3917	0.1362	4.0151
141	21.800	3.5589	3.4234	0.1355	3.9576
142	22.000	3.5898	3.4551	0.1347	3.8988
143	22.200	3.6207	3.4869	0.1338	3.8386
144	22.400	3.6515	3.5186	0.1329	3.7772
145	22.600	3.6823	3.5504	0.1319	3.7144
146	22.800	3.7130	3.5823	0.1308	3.6503
147	23.000	3.7437	3.6141	0.1296	3.5848
148	23.200	3.7742	3.6460	0.1283	3.5179
149	23.400	3.8047	3.6779	0.1269	3.4495
150	23.600	3.8351	3.7098	0.1254	3.3797
151	23.800	3.8655	3.7417	0.1238	3.3084
152	24.000	3.8958	3.7737	0.1221	3.2356
153	24.200	3.9259	3.8056	0.1203	3.1611
154	24.400	3.9560	3.8376	0.1184	3.0851
155	24.600	3.9860	3.8696	0.1164	3.0074
156	24.694	4.0000	3.8846	0.1154	2.9707
157	24.800	4.0159	3.9016	0.1142	2.9280
158	25.000	4.0457	3.9337	0.1120	2.8470
159	25.200	4.0753	3.9657	0.1096	2.7642
160	25.400	4.1049	3.9978	0.1071	2.6798

Table 64

FIG. 88

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	25.600	4.1343	4.0298	0.1045	2.5936
162	25.800	4.1636	4.0618	0.1018	2.5059
163	26.000	4.1928	4.0938	0.0989	2.4166
164	26.200	4.2218	4.1258	0.0960	2.3259
165	26.400	4.2507	4.1578	0.0929	2.2339
166	26.600	4.2793	4.1896	0.0897	2.1408
167	26.800	4.3079	4.2214	0.0864	2.0470
168	27.000	4.3362	4.2531	0.0830	1.9525
169	27.200	4.3643	4.2847	0.0796	1.8579
170	27.400	4.3922	4.3161	0.0761	1.7634
171	27.600	4.4199	4.3473	0.0726	1.6694
172	27.800	4.4473	4.3782	0.0690	1.5763
173	28.000	4.4744	4.4089	0.0655	1.4846
174	28.192	4.5000	4.4380	0.0620	1.3970
175	28.200	4.5011	4.4392	0.0619	1.3945
176	28.400	4.5276	4.4692	0.0584	1.3065
177	28.600	4.5536	4.4987	0.0549	1.2209
178	28.800	4.5793	4.5277	0.0515	1.1379
179	29.000	4.6044	4.5562	0.0482	1.0577
180	29.200	4.6290	4.5841	0.0450	0.9806
181	29.400	4.6531	4.6113	0.0418	0.9067
182	29.600	4.6765	4.6378	0.0388	0.8361
183	29.800	4.6993	4.6635	0.0359	0.7687
184	30.000	4.7214	4.6884	0.0330	0.7048
185	30.200	4.7427	4.7124	0.0304	0.6442
186	30.400	4.7632	4.7354	0.0278	0.5870
187	30.600	4.7829	4.7575	0.0254	0.5332
188	30.800	4.8017	4.7786	0.0231	0.4826
189	31.000	4.8196	4.7987	0.0209	0.4352
190	31.200	4.8366	4.8177	0.0188	0.3910
191	31.400	4.8526	4.8356	0.0169	0.3500
192	31.600	4.8676	4.8525	0.0151	0.3119
193	31.800	4.8817	4.8682	0.0135	0.2768
194	32.000	4.8948	4.8828	0.0119	0.2445
195	32.200	4.9069	4.8964	0.0105	0.2149
196	32.400	4.9181	4.9088	0.0092	0.1880
197	32.600	4.9283	4.9203	0.0080	0.1636
198	32.800	4.9376	4.9307	0.0070	0.1415
199	33.000	4.9461	4.9401	0.0060	0.1217
200	33.200	4.9537	4.9485	0.0051	0.1041

Table 65

FIG. 89

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
201	33.400	4.9605	4.9561	0.0044	0.0884
202	33.600	4.9665	4.9628	0.0037	0.0746
203	33.800	4.9718	4.9687	0.0031	0.0625
204	34.000	4.9765	4.9739	0.0026	0.0520
205	34.200	4.9805	4.9784	0.0021	0.0429
206	34.400	4.9840	4.9823	0.0017	0.0351
207	34.600	4.9870	4.9856	0.0014	0.0285
208	34.800	4.9895	4.9884	0.0011	0.0229
209	35.000	4.9916	4.9907	0.0009	0.0183
210	35.200	4.9934	4.9927	0.0007	0.0144
211	35.400	4.9948	4.9943	0.0006	0.0112
212	35.600	4.9960	4.9956	0.0004	0.0087

Table 66

FIG. 90

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-45.343	-24.394	-22.729	-22.548	-19.985	-18.269	-17.663	-14.325
2	-45.283	-24.883	-23.866	-21.906	-18.916	-18.279	-17.226	-15.598
3	-45.204	-23.374	-22.109	-19.418	-20.610	-22.694	-19.681	-19.323
4	-45.358	-20.100	-20.409	-20.348	-20.339	-20.192	-20.138	-20.147
5	-45.157	-29.835	-16.217	-16.186	-16.191	-16.174	-16.180	-16.171
6	-45.052	-27.227	-26.705	-14.338	-14.246	-14.277	-14.357	-14.108
7	-45.255	-18.839	-23.089	-23.230	-19.868	-21.696	-21.424	-13.250
8	-24.362	-45.114	-22.442	-22.568	-19.567	-15.644	3.330	-1.541
9	-45.303	-21.093	-20.986	-19.939	-20.641	-20.205	-20.253	-19.419
10	-45.073	-21.391	-21.412	-21.316	-21.414	-21.401	-21.433	-21.295
11	-45.215	-21.016	-21.354	-21.801	-21.401	-21.841	-20.198	-16.773
12	-45.219	-19.542	-19.569	-20.712	-21.194	-20.510	-20.220	-21.190
13	-17.864	-45.095	-24.540	-22.942	-23.784	-18.452	-13.887	-17.940
14	-45.158	-20.872	-20.650	-20.889	-20.103	-20.425	-20.291	-20.430
15	-45.118	-20.368	-20.347	-20.452	-20.577	-20.637	-20.511	-20.575
16	-45.172	-20.834	-19.751	-20.049	-20.844	-19.309	-20.258	-21.056
17	-45.108	-20.531	-20.445	-20.446	-20.492	-20.508	-20.508	-20.518
18	-45.094	-22.132	-22.129	-21.433	-21.433	-21.450	-16.562	-16.560
19	-45.032	-20.649	-20.648	-20.648	-20.650	-20.558	-20.552	-20.553
20	-45.043	-20.498	-20.551	-20.740	-20.638	-20.664	-20.223	-20.505
21	-44.984	-20.591	-20.757	-20.602	-20.753	-20.530	-20.561	-20.568
22	-44.957	-20.684	-20.684	-20.657	-20.646	-20.650	-20.652	-20.652
23	-44.931	-20.682	-20.682	-20.682	-20.682	-20.682	-20.682	-20.672
24	-44.914	-20.681	-20.681	-20.681	-20.681	-20.681	-20.681	-20.681
25	-44.884	-20.700	-20.710	-20.704	-20.706	-20.699	-20.706	-20.712
26	-44.850	-20.750	-20.740	-20.740	-20.740	-20.740	-20.740	-20.740
27	-44.829	-20.756	-20.756	-20.756	-20.756	-20.756	-20.756	-20.756
28	-44.817	-20.760	-20.760	-20.760	-20.762	-20.762	-20.762	-20.752
29	-44.805	-20.765	-20.764	-20.764	-20.764	-20.762	-20.761	-20.764
30	-44.786	-20.777	-20.781	-20.775	-20.772	-20.794	-20.766	-20.790
31	-44.736	-21.134	-21.019	-21.139	-21.139	-21.111	-21.114	-21.123
32	-44.702	-21.199	-21.183	-21.183	-21.183	-21.183	-21.183	-21.183
33	-44.668	-21.250	-21.250	-21.247	-21.249	-21.253	-21.255	-21.248
34	-44.635	-21.306	-21.306	-21.306	-21.306	-21.306	-21.301	-21.300
35	-44.604	-21.353	-21.353	-21.353	-21.353	-21.351	-21.351	-21.351
36	-44.573	-21.391	-21.391	-21.391	-21.391	-21.395	-21.395	-21.395
37	-44.543	-21.431	-21.430	-21.430	-21.430	-21.430	-21.428	-21.428
38	-44.486	-21.617	-21.657	-21.541	-21.590	-21.608	-21.546	-21.516
39	-44.404	-21.764	-21.767	-21.777	-21.785	-21.778	-21.774	-21.770
40	-44.380	-21.767	-21.794	-21.780	-21.780	-21.788	-21.789	-21.789

Table 67

FIG. 91

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	-13.635	-11.934	-8.649	-6.824	-5.114	-4.635	-3.002	-1.841
2	-12.831	-10.336	-9.442	-8.114	-5.012	-3.798	-3.272	-2.251
3	-4.779	-3.027	-3.096	-2.272	-0.919	-1.470	-2.114	-0.429
4	-20.325	-7.492	-2.105	-1.838	-1.625	-1.116	-1.116	-1.127
5	-16.199	-16.203	-13.514	-11.005	-8.551	-7.350	-4.689	-3.100
6	-14.280	-14.036	-14.051	-14.399	-12.854	-1.255	-0.433	1.263
7	-11.272	-14.648	-4.877	-6.864	-0.261	-3.154	-4.705	-2.791
8	2.680	-0.839	-22.596	-17.342	-9.498	-9.135	-3.810	-5.056
9	-19.872	-5.248	-1.888	-1.112	-1.283	-1.245	-1.367	-0.633
10	-4.409	-4.056	-3.087	-3.109	-3.056	-2.997	-2.974	-2.994
11	-15.579	-10.382	-7.645	-2.175	-1.624	-0.887	-1.247	-1.441
12	-19.896	-8.370	-2.717	-2.138	-0.283	-0.277	-0.303	-1.010
13	-15.407	-15.505	-3.399	-2.573	-3.270	-3.907	2.693	0.560
14	-20.141	-4.751	-2.071	-1.303	-0.418	-0.959	-1.139	-0.930
15	-20.770	-2.574	-0.053	-1.009	-1.099	-0.986	-1.226	-0.812
16	-20.343	-12.201	0.377	1.140	-0.046	1.530	0.760	0.683
17	-20.495	-7.614	-1.738	-1.162	-0.583	-0.540	-0.526	-0.545
18	-16.560	-16.560	-0.513	0.198	0.226	0.659	1.095	1.087
19	-20.552	-1.210	-0.755	-0.754	-0.754	-0.756	-0.756	-0.756
20	-20.469	-8.598	1.013	1.057	0.913	0.724	0.947	0.744
21	-20.712	-1.009	-0.885	-1.016	-0.934	-0.885	-0.996	-0.906
22	-20.652	-0.569	-0.569	-0.569	-0.569	-0.569	-0.569	-0.532
23	-20.673	-0.459	-0.460	-0.459	-0.459	-0.459	-0.459	-0.459
24	-20.681	-5.138	0.198	0.198	0.198	0.198	0.198	0.198
25	-20.742	-4.073	0.302	0.292	0.297	0.291	0.302	0.293
26	-20.730	-0.420	-0.380	-0.360	-0.320	-0.300	-0.280	-0.230
27	-20.756	-0.082	0.006	0.006	0.006	0.006	0.006	0.006
28	-20.752	-0.007	-0.007	-0.007	-0.010	0.001	-0.010	0.000
29	-20.754	-0.001	-0.002	-0.001	-0.002	-0.003	-0.004	-0.002
30	-20.715	0.011	0.013	0.011	0.014	0.012	0.011	0.010
31	-18.293	0.011	-0.004	0.004	-0.005	0.008	-0.020	-0.001
32	-17.778	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	0.003
33	-17.315	-0.003	0.000	-0.004	0.001	0.000	-0.002	0.003
34	-16.932	-0.005	-0.005	0.001	0.001	0.001	0.001	0.001
35	-16.587	-0.013	-0.010	-0.008	-0.009	0.004	0.004	0.004
36	-16.301	-0.017	-0.011	-0.011	0.004	0.004	0.004	0.004
37	-16.041	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
38	-13.874	-3.119	-3.130	-2.370	0.046	0.224	0.372	-0.010
39	-4.540	-4.609	-4.579	-4.575	-4.642	-4.538	-4.465	-4.463
40	-4.516	-4.521	-4.550	-4.546	-4.540	-4.557	-4.537	-4.536

Table 68

FIG. 92

	Label							
	16	17	18	19	20	21	22	23
Design #								
1	0.559	0.878	1.628	7.378	8.322	13.022	13.872	13.904
2	-1.704	3.042	3.725	7.812	9.088	9.252	9.824	13.966
3	-3.795	-5.863	-7.934	-6.868	18.715	18.996	20.161	15.141
4	-1.369	-0.980	-0.902	-0.750	-0.105	11.883	15.567	15.460
5	6.651	7.690	12.424	12.446	12.429	12.433	12.427	12.454
6	1.620	3.323	3.412	4.223	6.954	8.333	13.210	16.400
7	0.423	-0.188	1.768	1.979	5.146	12.533	19.507	17.546
8	-5.326	-5.776	-6.925	8.076	-0.329	7.836	21.080	19.904
9	-1.159	-2.404	-1.143	-1.834	18.015	-1.591	16.255	16.368
10	-3.042	-3.091	-3.066	-2.633	4.068	9.019	19.530	19.493
11	1.066	1.431	1.863	2.221	3.093	5.760	8.956	15.215
12	-0.833	-1.102	1.022	0.334	1.654	5.346	8.910	19.145
13	-1.453	-0.557	-1.031	4.251	-1.363	19.205	23.735	17.787
14	-0.890	-0.800	-0.582	0.426	0.057	2.811	14.025	15.501
15	-0.834	-0.404	-0.640	-0.676	-0.608	0.390	12.322	20.810
16	1.356	1.543	0.606	0.849	0.368	0.937	0.269	20.383
17	-0.104	-0.019	0.225	0.802	2.289	3.473	5.851	20.505
18	1.097	1.095	1.102	1.095	1.095	1.095	1.090	19.720
19	-0.756	-0.755	-0.494	-0.414	-0.413	-0.174	9.614	20.386
20	0.904	0.792	0.627	0.791	0.617	-0.641	-0.520	20.637
21	-1.020	-0.917	-0.884	-1.015	5.030	5.165	20.457	20.734
22	-0.569	-0.532	-0.545	-0.557	-0.542	-0.072	7.122	20.594
23	-0.459	-0.459	-0.460	-0.459	-0.459	-0.459	6.213	20.636
24	0.238	0.249	0.653	0.668	0.668	0.668	0.668	20.696
25	0.303	0.298	0.313	0.318	0.321	0.324	0.351	20.731
26	-0.160	-0.070	-0.030	-0.030	-0.010	0.010	2.590	20.730
27	0.006	0.006	0.006	0.006	0.006	0.006	0.006	20.734
28	-0.008	0.005	0.010	0.006	0.006	0.011	0.009	20.762
29	-0.003	-0.003	-0.003	-0.003	-0.003	0.014	0.016	20.734
30	0.009	0.006	0.008	0.014	0.012	0.010	0.016	18.707
31	-0.001	0.017	-0.021	0.023	-0.011	21.115	21.080	-0.032
32	0.003	0.003	0.003	0.003	0.003	0.003	0.003	17.791
33	-0.001	0.000	0.001	0.000	0.001	0.001	0.003	17.320
34	0.001	0.001	0.001	0.001	0.001	0.001	0.002	16.920
35	0.004	0.004	0.004	0.004	0.004	0.004	0.004	16.590
36	0.004	0.003	0.003	0.003	0.003	0.003	0.003	16.298
37	-0.004	-0.004	-0.004	-0.004	0.015	0.015	0.015	16.041
38	-0.001	-0.158	-0.092	0.249	2.273	2.631	3.034	13.943
39	4.450	4.538	4.602	4.675	4.557	4.555	4.550	4.485
40	4.533	4.547	4.538	4.524	4.524	4.538	4.568	4.532

Table 69

FIG. 93

	Label							
	24	25	26	27	28	29	30	31
Design #								
1	16.228	16.944	17.064	17.457	18.036	23.620	26.682	45.294
2	17.928	17.982	18.192	19.553	20.125	20.783	26.101	45.344
3	19.782	17.858	12.125	20.371	5.939	19.767	20.586	45.539
4	18.190	18.596	18.715	18.637	18.711	22.562	24.189	45.372
5	12.433	12.435	12.444	12.434	12.433	28.408	28.406	44.777
6	18.299	18.306	18.313	18.364	18.214	18.234	27.926	45.223
7	16.584	18.303	17.321	14.171	18.208	19.779	26.886	45.258
8	14.532	23.670	18.114	16.877	22.057	16.800	17.548	45.369
9	17.625	16.310	16.764	16.787	17.047	21.555	26.663	45.231
10	19.360	19.413	19.361	19.445	19.349	19.391	19.391	45.431
11	19.561	21.712	21.119	21.344	20.929	21.212	19.875	45.222
12	21.093	20.712	20.654	19.848	20.639	19.954	20.537	45.233
13	25.021	18.153	20.805	16.034	13.962	13.485	12.121	45.159
14	19.945	21.365	21.592	20.221	20.547	20.507	20.601	45.201
15	20.279	20.327	20.400	20.168	20.126	20.118	20.137	45.198
16	20.621	20.677	20.467	20.558	20.568	20.490	20.580	45.102
17	20.519	20.515	20.448	20.613	20.475	20.513	20.556	45.098
18	20.462	20.637	20.639	20.641	20.665	20.633	21.019	45.074
19	20.448	20.510	20.513	20.511	20.509	20.514	20.507	45.077
20	20.696	20.484	20.699	20.605	20.653	20.514	20.663	45.008
21	20.547	20.590	20.655	20.626	20.600	0.532	20.590	45.000
22	20.596	20.604	20.603	20.607	20.627	20.628	20.636	44.979
23	20.637	20.636	20.636	20.651	20.661	20.661	20.661	44.947
24	20.696	20.697	20.697	20.697	20.697	20.707	20.707	44.905
25	20.710	20.714	20.719	20.714	20.717	20.743	20.708	44.879
26	20.730	20.724	20.736	20.742	20.742	20.738	20.746	44.852
27	20.747	20.758	20.758	20.758	20.758	20.762	20.768	44.829
28	20.757	20.757	20.759	20.758	20.760	20.762	20.757	44.817
29	20.733	20.760	20.760	20.760	20.761	20.760	20.830	44.805
30	21.012	21.041	21.063	21.047	21.040	21.063	21.058	44.767
31	18.509	21.040	21.141	21.071	20.994	-0.005	21.154	44.740
32	21.184	21.184	21.184	21.184	21.184	21.184	21.184	44.702
33	21.240	21.248	21.251	21.252	21.254	21.250	21.252	44.668
34	21.305	21.305	21.305	21.305	21.305	21.305	21.309	44.635
35	21.350	21.352	21.352	21.352	21.353	21.352	21.352	44.604
36	21.379	21.387	21.395	21.399	21.400	21.398	21.394	44.573
37	21.427	21.427	21.427	21.432	21.432	21.432	21.432	44.543
38	21.535	21.638	21.561	21.528	21.640	21.580	21.575	44.487
39	21.768	21.771	21.771	21.773	21.761	21.796	21.773	44.404
40	21.783	21.784	21.779	21.783	21.786	21.784	21.787	44.380

Table 70

FIG. 94

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-44.331	-22.050	-22.075	-22.065	-22.069	-22.039	-22.053	-20.292
42	-44.215	-22.590	-22.587	-22.587	-22.587	-22.587	-22.586	-15.296
43	-44.114	-22.822	-22.823	-22.822	-22.822	-22.821	-22.821	-7.777
44	-44.082	-22.838	-22.838	-22.838	-22.838	-22.838	-22.838	-6.530
45	-44.052	-22.851	-22.851	-22.851	-22.851	-22.849	-22.847	-6.521
46	-43.976	-23.394	-23.394	-23.392	-23.391	-23.391	-19.819	-6.612
47	-43.890	-23.657	-23.657	-23.657	-23.657	-23.655	-17.939	-6.749
48	-43.769	-23.993	-23.993	-23.992	-23.992	-23.992	-12.510	-8.124
49	-43.747	-23.932	-23.932	-23.932	-23.932	-23.932	-15.409	-7.000
50	-43.690	-24.010	-24.010	-24.010	-24.010	-24.010	-14.480	-7.110
51	-43.685	-24.022	-24.021	-24.021	-24.021	-24.021	-14.359	-7.122
52	-43.628	-24.091	-24.091	-24.091	-24.091	-24.091	-13.389	-7.240
53	-43.575	-24.146	-24.148	-24.147	-24.146	-24.147	-12.479	-7.354
54	-43.526	-24.191	-24.191	-24.191	-24.191	-24.191	-11.630	-7.457
55	-43.480	-24.223	-24.225	-24.227	-24.223	-24.227	-7.537	-7.562
56	-43.435	-24.253	-24.251	-24.252	-24.253	-24.252	-8.209	-8.208
57	-43.393	-24.270	-24.269	-24.269	-24.269	-24.269	-8.182	-8.170
58	-43.323	-24.795	-24.795	-24.795	-24.795	-21.912	-8.434	-8.432
59	-43.224	-25.171	-25.171	-25.170	-25.170	-19.536	-9.211	-9.200
60	-43.143	-25.366	-25.366	-25.366	-25.366	-17.669	-10.066	-10.066
61	-43.058	-25.545	-25.545	-25.545	-25.545	-11.874	-11.891	-11.892
62	-43.009	-25.568	-25.568	-25.568	-25.569	-11.884	-11.875	-11.875
63	-42.959	-25.586	-25.586	-25.586	-25.586	-11.885	-11.885	-11.885
64	-42.907	-25.604	-25.604	-25.604	-25.604	-11.890	-11.889	-11.890
65	-42.832	-26.175	-26.175	-26.175	-23.810	-11.947	-11.947	-11.947
66	-42.761	-26.407	-26.407	-26.407	-23.008	-12.221	-12.221	-12.221
67	-42.685	-26.623	-26.623	-26.623	-22.127	-12.457	-12.457	-12.457
68	-42.617	-26.762	-26.762	-26.762	-21.528	-12.600	-12.600	-12.600
69	-42.547	-26.909	-26.909	-26.909	-20.691	-12.903	-12.905	-12.910
70	-42.530	-26.930	-26.930	-26.930	-20.620	-12.920	-12.920	-12.920
71	-42.477	-27.050	-27.050	-27.050	-19.462	-13.823	-13.823	-13.822
72	-42.415	-27.121	-27.121	-27.121	-18.901	-14.000	-14.000	-13.989
73	-42.351	-27.211	-27.212	-27.211	-15.640	-15.640	-15.640	-15.640
74	-42.293	-27.236	-27.236	-27.236	-15.640	-15.639	-15.639	-15.639
75	-42.232	-27.259	-27.259	-27.259	-15.648	-15.648	-15.648	-15.648
76	-42.169	-27.280	-27.279	-27.279	-15.663	-15.663	-15.663	-15.663
77	-42.094	-27.964	-27.964	-25.850	-15.745	-15.745	-15.745	-15.745
78	-42.024	-28.222	-28.222	-25.276	-15.825	-15.826	-15.826	-15.825
79	-41.959	-28.366	-28.366	-24.979	-15.860	-15.858	-15.860	-15.860
80	-41.887	-28.620	-28.620	-24.173	-16.615	-16.615	-16.616	-16.615

Table 71

FIG. 95

	Label							
	8	9	10	11	12	13	14	15
Design #								
41	-3.639	-3.628	-3.647	-3.629	-3.641	-3.634	-3.627	-3.653
42	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609
43	-6.370	-6.373	-6.369	-6.369	-6.369	-6.370	-6.368	-6.370
44	-6.527	-6.527	-6.527	-6.527	-6.527	-6.527	-6.524	-6.524
45	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521
46	-6.612	-6.612	-6.612	-6.612	-6.612	-6.610	-6.610	-6.610
47	-6.749	-6.749	-6.749	-6.748	-6.748	-6.747	-6.746	-6.744
48	-8.124	-8.124	-8.124	-8.124	-8.121	-8.121	-8.121	6.031
49	-7.000	-7.000	-7.000	-7.000	-7.000	-7.000	-7.000	-6.998
50	-7.110	-7.110	-7.110	-7.110	-7.100	-7.100	-7.100	-7.100
51	-7.122	-7.122	-7.122	-7.122	-7.122	-7.122	-7.116	-7.114
52	-7.240	-7.239	-7.239	-7.239	-7.239	-7.234	-7.234	-7.233
53	-7.312	-7.406	-7.344	-7.380	-7.341	-7.346	-7.335	-7.327
54	-7.457	-7.457	-7.457	-7.457	-7.457	-7.457	-7.450	-7.451
55	-7.675	-7.559	-7.013	-7.420	-10.845	-7.658	-7.783	-7.752
56	-8.209	-8.208	-8.209	-8.209	-8.206	-8.172	-8.158	-4.855
57	-8.166	-8.166	-8.166	-8.166	-8.166	-8.166	-8.166	-5.244
58	-8.432	-8.432	-8.432	-8.432	-8.432	-8.432	-8.432	0.000
59	-9.200	-9.200	-9.200	-9.200	-9.200	-9.200	0.506	0.519
60	-10.066	-10.066	-10.063	-10.057	-10.057	-0.011	-0.010	0.005
61	-11.893	-11.892	-11.892	-11.892	-0.308	-0.308	-0.308	-0.310
62	-11.874	-11.874	-11.874	-11.870	-2.509	0.345	0.345	0.345
63	-11.884	-11.885	-11.885	-11.885	-0.377	-0.376	-0.376	-0.371
64	-11.890	-11.890	-11.890	-11.890	-0.365	-0.365	-0.365	-0.365
65	-11.947	-11.947	-11.947	-11.946	-0.003	0.000	0.000	0.000
66	-12.221	-12.221	-12.221	-10.474	-0.012	-0.012	-0.012	-0.012
67	-12.456	-12.456	-12.456	-9.335	0.000	0.000	0.000	0.000
68	-12.600	-12.600	-12.600	-8.667	-0.002	-0.001	-0.001	0.000
69	-12.901	-12.897	-12.902	-3.081	-3.046	-3.034	-3.074	-3.000
70	-12.910	-12.910	-12.910	-3.110	-3.080	-3.020	-3.000	-2.950
71	-13.822	-13.823	-5.103	-5.055	-5.055	-5.054	-5.054	2.768
72	-13.987	-13.987	-4.360	-4.360	-4.360	-4.360	-4.360	-4.360
73	-15.640	-9.495	-4.617	-4.616	-4.616	-4.617	-4.616	-4.614
74	-15.640	-9.565	-4.609	-4.609	-4.609	-4.609	-4.609	-4.608
75	-15.648	-9.562	-4.613	-4.613	-4.613	-4.613	-4.613	-4.613
76	-15.663	-9.504	-4.627	-4.626	-4.626	-4.626	-4.626	-4.626
77	-15.745	-8.854	-4.726	-4.724	-4.724	-4.724	-4.724	-4.724
78	-15.825	-6.846	-5.638	-5.642	-5.644	-5.641	-5.659	0.068
79	-15.859	-5.840	-5.840	-5.840	-5.840	-5.840	-5.839	-0.179
80	-12.278	-6.699	-6.692	-6.691	-6.694	-6.690	0.001	0.002

Table 72

FIG. 96

	Label							
	16	17	18	19	20	21	22	23
Design #								
41	-3.686	-3.668	6.587	6.464	6.471	6.465	6.464	6.507
42	-4.609	6.228	6.228	6.229	6.229	6.229	6.246	6.269
43	5.753	5.801	6.614	6.773	6.758	6.762	6.762	6.750
44	6.524	6.524	6.524	6.524	6.528	6.528	6.528	6.528
45	6.519	6.519	6.519	6.519	6.523	6.523	6.523	6.523
46	6.611	6.611	6.611	6.611	6.611	6.611	6.611	6.611
47	6.741	6.746	6.746	6.749	6.749	6.749	6.749	6.749
48	6.031	6.031	6.031	6.032	6.032	6.032	6.033	6.035
49	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
50	7.100	7.100	7.100	7.100	7.110	7.110	7.110	7.110
51	7.120	7.120	7.120	7.121	7.121	7.121	7.121	7.121
52	7.237	7.237	7.237	7.237	7.237	7.238	7.238	7.238
53	7.399	7.365	7.267	7.327	7.392	7.354	7.264	7.414
54	7.444	7.457	7.457	7.457	7.457	7.457	7.457	7.457
55	7.469	7.260	7.080	8.013	10.246	9.165	7.233	8.013
56	4.856	8.207	8.190	8.190	8.202	8.210	8.186	8.196
57	5.244	8.168	8.168	8.168	8.168	8.168	8.168	8.168
58	0.000	8.432	8.432	8.432	8.432	8.432	8.432	8.432
59	0.524	0.524	3.854	9.711	9.711	9.711	9.711	9.711
60	0.005	0.005	0.005	10.061	10.061	10.061	10.064	10.064
61	-0.310	-0.311	-0.310	2.201	11.885	11.885	11.886	11.885
62	0.345	0.345	0.345	0.384	11.884	11.884	11.885	11.885
63	-0.353	-0.353	-0.354	2.624	11.874	11.874	11.874	11.874
64	-0.365	-0.365	-0.340	2.595	11.879	11.879	11.879	11.879
65	0.000	0.000	0.000	0.002	11.947	11.947	11.947	11.947
66	-0.012	-0.012	-0.012	-0.012	11.972	11.973	11.975	11.975
67	-0.001	0.000	0.000	0.002	9.338	12.455	12.456	12.456
68	-0.001	-0.001	0.002	0.004	8.664	12.601	12.601	12.601
69	2.963	3.151	3.052	3.030	3.041	12.913	12.904	12.896
70	2.900	2.900	3.040	3.090	3.240	12.910	12.910	12.910
71	2.768	2.768	2.769	2.767	2.768	9.507	13.611	13.611
72	4.357	4.357	4.362	4.362	4.362	4.362	13.979	13.996
73	4.613	4.617	4.617	4.617	4.616	4.617	9.495	15.639
74	4.607	4.609	4.610	4.608	4.608	4.609	9.565	15.639
75	4.613	4.613	4.613	4.613	4.613	4.614	9.562	15.647
76	4.626	4.626	4.626	4.626	4.626	4.626	9.504	15.663
77	4.723	4.724	4.724	4.724	4.725	4.725	8.854	15.745
78	0.039	4.532	5.978	6.085	6.071	6.159	6.139	15.825
79	0.179	5.839	5.840	5.839	5.840	5.841	5.841	15.859
80	0.002	-0.004	6.686	6.685	6.689	6.695	6.709	12.278

Table 73

FIG. 97

	Label							
	24	25	26	27	28	29	30	31
Design #								
41	15.111	22.519	22.506	22.519	22.521	22.511	22.503	44.275
42	12.543	22.685	22.685	22.686	22.686	22.688	22.688	44.193
43	6.754	22.824	22.822	22.823	22.823	22.824	22.823	44.113
44	6.535	22.837	22.838	22.838	22.838	22.838	22.838	44.082
45	6.523	22.848	22.849	22.849	22.851	22.851	22.851	44.051
46	6.611	19.820	23.392	23.392	23.392	23.392	23.392	43.976
47	6.749	17.938	23.656	23.656	23.656	23.657	23.657	43.890
48	6.044	19.120	23.583	23.583	23.583	23.583	23.583	43.859
49	7.000	15.409	23.932	23.932	23.932	23.932	23.932	43.747
50	7.120	14.480	24.010	24.010	24.010	24.010	24.010	43.690
51	7.121	14.359	24.022	24.021	24.021	24.021	24.021	43.685
52	7.238	13.389	24.091	24.091	24.091	24.091	24.091	43.628
53	12.477	7.364	24.146	24.146	24.147	24.147	24.148	43.575
54	7.457	11.630	24.191	24.191	24.191	24.191	24.191	43.526
55	7.057	7.253	24.230	24.230	24.227	24.226	24.227	43.479
56	8.190	8.214	24.253	24.252	24.252	24.252	24.252	43.435
57	8.168	8.168	24.269	24.269	24.269	24.270	24.270	43.393
58	8.432	8.433	21.911	24.795	24.795	24.795	24.795	43.323
59	9.711	9.711	19.089	25.210	25.210	25.210	25.210	43.219
60	10.064	10.064	17.669	25.366	25.366	25.366	25.366	43.143
61	11.883	11.882	11.878	25.546	25.546	25.546	25.546	43.058
62	11.885	11.885	11.885	25.567	25.567	25.566	25.566	43.010
63	11.874	11.874	11.874	25.588	25.588	25.588	25.588	42.959
64	11.879	11.879	11.879	25.606	25.606	25.606	25.606	42.907
65	11.947	11.947	11.947	23.810	26.175	26.175	26.175	42.832
66	11.975	11.975	11.977	23.203	26.366	26.366	26.366	42.764
67	12.456	12.456	12.456	22.128	26.622	26.622	26.622	42.685
68	12.600	12.601	12.600	21.527	26.762	26.762	26.762	42.617
69	12.895	12.899	12.910	20.692	26.909	26.910	26.909	42.547
70	12.920	12.920	12.920	20.620	26.930	26.930	26.930	42.530
71	13.612	13.613	13.612	19.786	27.028	27.028	27.028	42.479
72	13.996	13.996	13.996	18.901	27.121	27.121	27.121	42.415
73	15.640	15.640	15.640	15.640	27.211	27.211	27.212	42.351
74	15.639	15.641	15.639	15.640	27.236	27.236	27.236	42.293
75	15.647	15.648	15.648	15.648	27.259	27.259	27.259	42.232
76	15.663	15.663	15.663	15.663	27.279	27.279	27.279	42.169
77	15.745	15.745	15.745	15.745	25.850	27.964	27.964	42.094
78	15.825	15.824	15.825	15.825	25.278	28.221	28.221	42.024
79	15.859	15.859	15.859	15.860	24.979	28.366	28.366	41.959
80	16.616	16.615	16.615	16.615	24.173	28.620	28.620	41.887

Table 74

FIG. 98

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-41.821	-28.769	-28.769	-23.684	-16.845	-16.844	-16.842	-16.844
82	-41.760	-28.848	-28.848	-23.491	-16.906	-16.906	-16.905	-16.904
83	-41.699	-28.961	-28.960	-23.010	-17.629	-17.630	-17.631	-14.794
84	-41.639	-29.050	-29.050	-22.578	-17.973	-17.973	-17.974	-13.458
85	-41.579	-29.119	-29.119	-22.235	-18.205	-18.206	-18.207	-10.709
86	-41.521	-29.202	-29.202	-19.869	-20.914	-19.870	-15.504	-11.049
87	-41.459	-29.236	-29.236	-20.250	-20.250	-20.250	-11.081	-15.369
88	-41.470	-29.230	-29.230	-20.250	-20.250	-20.250	-15.360	-11.090
89	-41.393	-29.263	-29.263	-20.260	-20.261	-20.262	-15.380	-11.085
90	-41.326	-29.287	-29.287	-20.278	-20.278	-20.277	-15.368	-11.099
91	-41.255	-29.310	-29.310	-20.298	-20.298	-20.299	-15.342	-11.114
92	-41.186	-29.946	-28.676	-20.382	-20.382	-20.382	-14.423	-12.070
93	-41.122	-30.267	-28.358	-20.437	-20.437	-20.437	-11.520	-13.125
94	-41.060	-30.466	-28.172	-20.467	-20.467	-20.467	-13.048	-13.053
95	-41.000	-30.689	-27.922	-20.987	-20.987	-19.413	-13.248	-13.248
96	-40.942	-30.884	-27.690	-21.251	-21.251	-18.827	-13.445	-13.445
97	-40.887	-31.034	-27.518	-21.408	-21.408	-18.464	-13.580	-13.581
98	-40.834	-31.159	-27.377	-21.533	-21.533	-18.136	-13.737	-13.737
99	-40.781	-31.262	-27.270	-21.623	-21.623	-17.864	-14.270	-14.266
100	-40.731	-31.355	-27.173	-21.712	-21.712	-17.499	-14.653	-14.653
101	-40.680	-31.436	-27.096	-21.782	-21.782	-17.099	-15.016	-15.016
102	-40.629	-31.502	-27.043	-21.827	-21.827	-16.360	-16.383	-14.324
103	-40.578	-31.565	-26.989	-22.249	-21.434	-16.400	-16.400	-14.293
104	-40.531	-31.636	-26.895	-22.684	-20.972	-16.569	-16.575	-13.874
105	-40.520	-31.650	-26.880	-22.740	-20.910	-16.600	-16.600	-13.820
106	-40.482	-31.696	-26.823	-22.912	-20.724	-16.706	-16.696	-13.471
107	-40.433	-31.749	-26.769	-23.062	-20.561	-16.800	-16.805	-12.532
108	-40.382	-31.796	-26.724	-23.204	-20.335	-17.642	-15.980	-12.703
109	-40.330	-31.839	-26.687	-23.329	-20.047	-18.214	-14.661	-14.680
110	-40.275	-31.875	-26.675	-23.385	-19.954	-18.338	-14.629	-14.608
111	-40.217	-31.906	-26.677	-23.414	-19.937	-18.371	-14.621	-14.621
112	-40.157	-31.935	-26.689	-23.430	-19.948	-18.379	-14.631	-14.631
113	-40.096	-31.960	-26.708	-23.437	-19.986	-18.357	-14.674	-14.638
114	-40.032	-31.984	-26.733	-23.437	-20.035	-18.325	-14.686	-14.686
115	-39.966	-32.007	-26.761	-23.435	-20.089	-18.291	-14.717	-14.717
116	-39.899	-32.028	-26.791	-23.436	-20.126	-18.282	-14.727	-14.734
117	-39.830	-32.048	-26.825	-23.430	-20.191	-18.225	-15.093	-14.397
118	-39.760	-32.067	-26.861	-23.423	-20.255	-18.167	-15.279	-14.232
119	-39.689	-32.086	-26.898	-23.419	-20.312	-18.119	-15.410	-14.123
120	-39.616	-32.103	-26.934	-23.418	-20.360	-18.084	-15.506	-14.041

Table 75

FIG. 99

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	-10.064	-4.016	-7.749	-7.790	-7.745	-7.783	0.710	0.600
82	-8.314	-8.314	-8.314	-8.314	-8.314	-2.668	0.000	0.000
83	-8.424	-8.424	-8.425	-8.424	-8.425	-2.035	-0.010	-0.010
84	-9.139	-9.131	-9.148	-9.133	-5.714	-1.273	-1.253	-1.255
85	-10.708	-10.711	-10.709	-7.012	-2.943	-2.947	-2.946	-2.946
86	-10.979	-11.029	-11.133	-2.350	-3.921	-3.938	-4.265	-4.066
87	-11.077	-11.070	-11.079	-4.614	-3.298	-3.771	-3.624	-3.316
88	-11.080	-11.070	-11.070	-4.470	-3.850	-3.740	-3.630	-2.900
89	-11.087	-11.083	-11.081	-4.419	-4.105	-3.440	-3.137	-3.531
90	-11.105	-11.095	-11.095	-3.757	-3.409	-3.572	-3.471	-4.449
91	-11.115	-11.118	-11.122	-3.938	-3.922	-3.779	-3.779	-3.260
92	-12.114	-12.134	-7.676	-5.315	-5.314	-5.318	-0.787	-0.790
93	-13.127	-13.129	-5.948	-5.949	-5.948	-5.948	-0.345	-0.320
94	-13.052	-11.799	-5.914	-5.916	-5.916	-5.916	-1.319	0.426
95	-13.248	-11.225	-5.983	-5.983	-5.982	-5.983	-0.233	-0.148
96	-13.446	-10.590	-6.363	-6.363	-6.363	-5.062	-0.001	-0.001
97	-13.580	-6.575	-10.100	-6.574	-4.630	-6.577	0.002	0.000
98	-13.731	-8.188	-8.183	-8.185	-5.489	-2.226	-2.219	-2.231
99	-12.752	-8.255	-8.253	-8.253	-5.304	-2.266	-2.266	-2.266
100	-12.010	-8.425	-8.425	-8.425	-4.723	-2.483	-2.393	-2.393
101	-10.050	-10.047	-10.050	-5.349	-5.160	-5.018	-1.627	-0.546
102	-10.084	-10.073	-10.052	-4.892	-5.291	-5.290	-1.843	-0.211
103	-10.075	-10.081	-10.079	-5.181	-5.085	-5.214	-1.847	-0.022
104	-10.625	-10.633	-9.069	-5.233	-5.235	-5.234	-1.098	-1.100
105	-10.670	-10.640	-9.040	-5.270	-5.230	-5.180	-1.690	-0.020
106	-10.976	-10.978	-8.219	-6.210	-5.838	-3.678	-1.657	-1.383
107	-12.532	-10.313	-7.356	-7.377	-5.724	-2.519	-2.613	-1.587
108	-12.703	-9.852	-7.796	-7.617	-4.400	-4.277	-2.310	-0.288
109	-11.046	-10.598	-8.374	-6.228	-5.911	-2.886	-2.891	-0.761
110	-11.630	-9.544	-9.075	-6.528	-4.747	-4.441	-1.922	2.042
111	-11.639	-9.258	-9.417	-6.176	-5.514	-3.923	-1.628	-1.313
112	-11.636	-9.367	-9.337	-5.875	-5.869	-3.813	-1.898	-1.040
113	-11.521	-9.916	-8.843	-5.892	-5.967	-3.727	-1.210	-1.836
114	-11.349	-10.251	-8.607	-5.991	-5.990	-3.645	-1.592	-1.552
115	-10.977	-10.743	-8.400	-6.068	-6.061	-3.492	-1.877	-1.380
116	-10.870	-10.866	-8.385	-6.096	-6.093	-2.911	-2.905	-0.561
117	-10.896	-10.898	-8.337	-6.306	-5.924	-2.887	-2.924	-0.568
118	-10.944	-10.946	-8.124	-6.860	-5.436	-3.346	-2.594	-0.489
119	-10.991	-10.989	-7.699	-7.441	-5.086	-3.647	-2.493	-0.229
120	-11.297	-10.718	-7.592	-7.592	-5.023	-3.732	-2.449	-0.229

Table 76

FIG. 100

	Label							
	16	17	18	19	20	21	22	23
Design #								
81	0.645	0.697	0.666	8.351	8.351	8.355	8.360	8.348
82	0.000	0.001	2.667	8.314	8.314	8.314	8.314	8.313
83	-0.009	-0.010	2.080	8.423	8.423	8.423	8.423	8.422
84	-1.110	2.734	2.911	4.726	9.212	9.223	9.217	9.219
85	2.943	2.949	2.944	2.948	7.012	10.711	10.708	10.709
86	3.314	3.315	3.187	3.641	5.117	11.020	11.009	11.046
87	4.672	3.541	3.560	3.537	3.314	11.074	11.083	11.073
88	2.870	3.480	3.890	4.110	4.250	11.070	11.070	11.080
89	3.414	3.445	3.365	3.791	4.620	11.084	11.081	11.085
90	3.381	3.458	3.883	4.403	3.534	11.099	11.096	11.098
91	3.241	3.862	3.858	3.858	3.859	11.118	11.118	11.119
92	-0.794	2.871	5.049	5.048	5.062	8.046	12.065	12.058
93	-0.342	1.023	5.944	5.943	5.943	5.941	11.537	13.123
94	0.432	0.426	5.927	5.927	5.928	5.928	11.755	13.064
95	-0.134	0.518	5.981	5.982	5.982	5.982	11.228	13.247
96	0.001	0.001	5.059	6.361	6.366	6.366	10.589	13.446
97	-0.002	0.000	4.630	6.569	6.570	6.586	10.100	13.580
98	2.256	2.211	2.210	5.488	8.196	8.184	8.177	13.741
99	2.266	2.266	2.266	5.305	8.254	8.254	8.254	12.753
100	2.412	2.423	2.433	4.725	8.424	8.424	8.427	12.010
101	0.292	1.989	4.607	5.235	5.585	10.018	10.058	10.061
102	0.199	1.862	4.857	5.294	5.316	10.059	10.067	10.082
103	0.027	1.840	5.158	5.161	5.163	10.081	10.077	10.076
104	0.507	1.769	5.187	5.185	5.194	9.676	9.674	11.026
105	0.020	1.690	5.210	5.230	5.240	9.040	10.650	10.660
106	1.491	1.542	3.681	6.008	6.031	8.239	11.040	10.907
107	1.618	2.523	2.564	5.792	7.592	7.091	10.313	12.546
108	0.281	2.326	4.252	4.414	7.640	7.776	9.850	12.681
109	1.329	1.578	3.802	5.883	5.951	10.149	11.333	8.689
110	-0.655	0.583	4.770	4.332	6.606	9.687	8.910	11.611
111	1.225	1.746	3.838	5.773	5.960	9.394	9.297	11.633
112	1.917	1.027	3.804	5.827	5.920	9.356	9.346	11.637
113	1.236	1.801	3.741	5.929	5.922	8.854	9.904	11.524
114	1.564	1.580	3.645	6.016	5.964	8.608	10.249	11.350
115	1.608	1.607	3.559	5.819	6.299	8.379	10.867	10.860
116	0.588	2.629	3.173	6.054	6.117	8.394	10.870	10.863
117	0.566	2.894	2.920	5.893	6.340	8.330	10.898	10.898
118	0.521	2.446	3.520	5.319	6.968	8.070	10.908	10.993
119	0.235	2.468	3.679	5.067	7.472	7.673	10.990	10.989
120	0.226	2.460	3.717	5.033	7.568	7.612	10.729	11.285

Table 77

FIG. 101

	Label							
	24	25	26	27	28	29	30	31
Design #								
81	16.864	16.864	16.864	16.864	23.655	28.775	28.775	41.821
82	16.904	16.904	16.906	16.906	23.491	28.848	28.848	41.760
83	14.802	17.628	17.628	17.628	23.011	28.960	28.960	41.699
84	13.339	17.988	17.988	17.988	22.564	29.051	29.051	41.639
85	10.707	18.208	18.205	18.205	22.235	29.119	29.119	41.579
86	11.056	15.550	20.962	19.857	19.813	29.201	29.201	41.521
87	11.074	15.370	20.252	20.248	20.251	29.236	29.236	41.459
88	11.090	15.360	20.250	20.250	20.250	29.230	29.230	41.470
89	11.082	15.382	20.261	20.261	20.260	29.263	29.263	41.393
90	11.101	15.368	20.277	20.278	20.278	29.287	29.287	41.326
91	11.113	15.342	20.298	20.298	20.299	29.310	29.310	41.255
92	12.054	14.489	20.380	20.379	20.380	28.682	29.941	41.186
93	13.123	13.122	20.437	20.437	20.437	28.357	30.267	41.122
94	13.060	13.065	20.466	20.466	20.467	28.173	30.465	41.060
95	13.247	13.247	19.415	20.987	20.986	27.922	30.688	41.000
96	13.445	13.446	18.826	21.251	21.251	27.690	30.884	40.942
97	13.580	13.580	18.464	21.408	21.408	27.518	31.034	40.887
98	13.732	13.732	18.136	21.534	21.533	27.377	31.159	40.834
99	14.267	14.269	17.864	21.623	21.623	27.270	31.262	40.781
100	14.653	14.653	17.499	21.712	21.712	27.173	31.355	40.731
101	15.019	15.019	17.095	21.783	21.782	27.095	31.436	40.680
102	14.325	16.399	16.342	21.827	21.827	27.043	31.502	40.629
103	14.293	16.400	16.400	21.434	22.249	26.989	31.565	40.578
104	13.899	16.562	16.562	20.987	22.674	26.897	31.635	40.530
105	13.820	16.600	16.600	20.910	22.740	26.880	31.650	40.520
106	13.473	16.699	16.702	20.724	22.912	26.823	31.696	40.482
107	12.517	16.811	16.794	20.560	23.062	26.769	31.749	40.433
108	12.727	15.977	17.644	20.334	23.205	26.724	31.796	40.382
109	14.629	14.688	18.227	20.039	23.331	26.686	31.839	40.330
110	14.610	14.629	18.338	19.954	23.385	26.675	31.875	40.275
111	14.628	14.617	18.369	19.939	23.413	26.678	31.906	40.217
112	14.629	14.632	18.379	19.947	23.430	26.689	31.935	40.158
113	14.658	14.654	18.357	19.986	23.437	26.708	31.960	40.096
114	14.686	14.686	18.325	20.035	23.437	26.733	31.984	40.032
115	14.716	14.719	18.290	20.089	23.435	26.761	32.007	39.966
116	14.734	14.728	18.280	20.126	23.436	26.791	32.028	39.899
117	14.392	15.097	18.224	20.191	23.429	26.825	32.048	39.830
118	14.220	15.290	18.164	20.256	23.423	26.861	32.067	39.760
119	14.124	15.410	18.119	20.312	23.419	26.898	32.086	39.689
120	14.045	15.504	18.085	20.360	23.418	26.934	32.103	39.616

Table 78

FIG. 102

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-39.542	-32.120	-26.971	-23.419	-20.406	-18.051	-15.604	-13.944
122	-39.530	-32.120	-26.980	-23.420	-20.410	-18.050	-15.620	-13.930
123	-39.467	-32.136	-27.007	-23.423	-20.446	-18.027	-15.684	-13.860
124	-39.390	-32.151	-27.044	-23.431	-20.479	-18.015	-15.738	-13.811
125	-39.312	-32.165	-27.080	-23.442	-20.509	-18.011	-15.782	-13.773
126	-39.233	-32.177	-27.115	-23.457	-20.535	-18.015	-15.810	-13.761
127	-39.152	-32.188	-27.151	-23.475	-20.558	-18.026	-15.831	-13.764
128	-39.070	-32.198	-27.186	-23.496	-20.579	-18.041	-15.846	-13.776
129	-38.986	-32.206	-27.221	-23.519	-20.600	-18.059	-15.859	-13.792
130	-38.901	-32.212	-27.256	-23.544	-20.620	-18.079	-15.871	-13.810
131	-38.814	-32.217	-27.292	-23.572	-20.641	-18.100	-15.884	-13.828
132	-38.725	-32.219	-27.326	-23.602	-20.662	-18.122	-15.897	-13.847
133	-38.635	-32.220	-27.361	-23.633	-20.684	-18.145	-15.912	-13.865
134	-38.544	-32.219	-27.396	-23.666	-20.706	-18.168	-15.928	-13.883
135	-38.451	-32.216	-27.430	-23.700	-20.731	-18.192	-15.945	-13.900
136	-38.357	-32.212	-27.464	-23.736	-20.756	-18.216	-15.964	-13.918
137	-38.261	-32.205	-27.497	-23.773	-20.783	-18.241	-15.984	-13.936
138	-38.164	-32.196	-27.529	-23.812	-20.812	-18.266	-16.005	-13.955
139	-38.150	-32.190	-27.530	-23.820	-20.820	-18.270	-16.010	-13.960
140	-38.065	-32.185	-27.561	-23.851	-20.843	-18.292	-16.028	-13.974
141	-37.965	-32.172	-27.592	-23.892	-20.876	-18.319	-16.051	-13.993
142	-37.863	-32.157	-27.621	-23.933	-20.910	-18.348	-16.076	-14.014
143	-37.760	-32.140	-27.650	-23.975	-20.946	-18.377	-16.102	-14.036
144	-37.656	-32.120	-27.677	-24.017	-20.984	-18.408	-16.129	-14.058
145	-37.550	-32.099	-27.702	-24.060	-21.024	-18.441	-16.157	-14.082
146	-37.443	-32.075	-27.727	-24.104	-21.066	-18.475	-16.186	-14.107
147	-37.334	-32.049	-27.749	-24.147	-21.110	-18.511	-16.217	-14.133
148	-37.224	-32.021	-27.770	-24.190	-21.155	-18.549	-16.249	-14.161
149	-37.112	-31.990	-27.789	-24.233	-21.201	-18.589	-16.283	-14.190
150	-37.000	-31.957	-27.806	-24.276	-21.249	-18.631	-16.318	-14.220
151	-36.886	-31.923	-27.822	-24.318	-21.298	-18.676	-16.356	-14.252
152	-36.770	-31.885	-27.835	-24.359	-21.348	-18.722	-16.395	-14.286
153	-36.654	-31.846	-27.846	-24.399	-21.399	-18.771	-16.437	-14.321
154	-36.536	-31.805	-27.855	-24.438	-21.450	-18.821	-16.481	-14.359
155	-36.417	-31.761	-27.862	-24.476	-21.502	-18.873	-16.527	-14.398
156	-36.360	-31.740	-27.860	-24.490	-21.530	-18.900	-16.550	-14.420
157	-36.296	-31.715	-27.867	-24.513	-21.554	-18.927	-16.576	-14.440
158	-36.175	-31.667	-27.869	-24.548	-21.605	-18.983	-16.627	-14.484
159	-36.052	-31.617	-27.869	-24.581	-21.657	-19.039	-16.681	-14.531
160	-35.928	-31.565	-27.867	-24.613	-21.708	-19.097	-16.737	-14.581

Table 79

FIG. 103

	Label							
	8	9	10	11	12	13	14	15
Design #								
121	-11.514	-10.528	-7.664	-7.612	-4.476	-4.476	-1.815	-1.123
122	-11.550	-10.490	-7.830	-7.450	-4.490	-4.480	-1.730	-1.220
123	-11.692	-10.348	-8.097	-7.207	-4.776	-4.237	-1.668	-1.313
124	-11.793	-10.248	-8.253	-7.066	-4.948	-4.085	-1.837	-1.141
125	-11.881	-10.145	-8.428	-6.865	-5.234	-3.784	-2.185	-0.797
126	-11.924	-10.105	-8.499	-6.786	-5.342	-3.663	-2.325	-0.650
127	-11.945	-10.098	-8.526	-6.766	-5.376	-3.631	-2.364	-0.611
128	-11.955	-10.110	-8.527	-6.781	-5.367	-3.652	-2.346	-0.634
129	-11.960	-10.127	-8.523	-6.804	-5.351	-3.681	-2.320	-0.665
130	-11.964	-10.147	-8.517	-6.828	-5.335	-3.709	-2.295	-0.694
131	-11.970	-10.165	-8.515	-6.848	-5.324	-3.731	-2.277	-0.716
132	-11.977	-10.182	-8.515	-6.865	-5.318	-3.748	-2.265	-0.731
133	-11.985	-10.198	-8.518	-6.879	-5.316	-3.760	-2.259	-0.741
134	-11.996	-10.212	-8.524	-6.890	-5.317	-3.768	-2.257	-0.746
135	-12.008	-10.225	-8.531	-6.900	-5.321	-3.775	-2.257	-0.750
136	-12.021	-10.238	-8.540	-6.909	-5.326	-3.780	-2.258	-0.752
137	-12.036	-10.251	-8.550	-6.917	-5.332	-3.785	-2.261	-0.753
138	-12.051	-10.264	-8.561	-6.926	-5.339	-3.790	-2.263	-0.754
139	-12.050	-10.270	-8.560	-6.930	-5.340	-3.790	-2.260	-0.750
140	-12.067	-10.278	-8.572	-6.935	-5.346	-3.795	-2.266	-0.754
141	-12.085	-10.292	-8.584	-6.944	-5.353	-3.799	-2.270	-0.755
142	-12.103	-10.307	-8.597	-6.954	-5.361	-3.805	-2.273	-0.756
143	-12.121	-10.322	-8.610	-6.964	-5.369	-3.810	-2.276	-0.757
144	-12.141	-10.338	-8.623	-6.975	-5.377	-3.816	-2.280	-0.758
145	-12.161	-10.355	-8.637	-6.986	-5.386	-3.822	-2.284	-0.760
146	-12.182	-10.373	-8.652	-6.998	-5.395	-3.829	-2.287	-0.761
147	-12.205	-10.392	-8.668	-7.011	-5.404	-3.835	-2.291	-0.762
148	-12.228	-10.411	-8.684	-7.024	-5.414	-3.842	-2.296	-0.764
149	-12.252	-10.432	-8.701	-7.037	-5.425	-3.850	-2.300	-0.765
150	-12.278	-10.454	-8.719	-7.052	-5.436	-3.858	-2.305	-0.767
151	-12.305	-10.477	-8.738	-7.067	-5.447	-3.866	-2.310	-0.768
152	-12.334	-10.501	-8.757	-7.083	-5.460	-3.874	-2.315	-0.770
153	-12.364	-10.526	-8.778	-7.099	-5.472	-3.883	-2.320	-0.772
154	-12.396	-10.553	-8.800	-7.117	-5.486	-3.893	-2.326	-0.774
155	-12.429	-10.581	-8.823	-7.136	-5.500	-3.903	-2.332	-0.776
156	-12.450	-10.590	-8.830	-7.140	-5.510	-3.910	-2.330	-0.780
157	-12.465	-10.611	-8.848	-7.155	-5.515	-3.914	-2.338	-0.778
158	-12.503	-10.642	-8.874	-7.176	-5.531	-3.925	-2.345	-0.780
159	-12.542	-10.676	-8.902	-7.199	-5.548	-3.937	-2.352	-0.782
160	-12.585	-10.711	-8.931	-7.222	-5.567	-3.950	-2.360	-0.785

Table 80

FIG. 104

	Label							
	16	17	18	19	20	21	22	23
Design #								
121	1.123	1.815	4.476	4.476	7.632	7.643	10.529	11.513
122	1.210	1.740	4.470	4.490	7.490	7.790	10.500	11.540
123	1.486	1.475	4.503	4.505	7.296	8.017	10.382	11.668
124	1.115	1.879	4.030	5.005	7.029	8.282	10.233	11.802
125	0.797	2.183	3.787	5.231	6.868	8.426	10.146	11.880
126	0.650	2.325	3.663	5.342	6.786	8.500	10.105	11.924
127	0.611	2.364	3.631	5.376	6.766	8.526	10.098	11.945
128	0.634	2.346	3.652	5.367	6.781	8.528	10.110	11.955
129	0.665	2.320	3.681	5.351	6.804	8.523	10.127	11.960
130	0.694	2.295	3.709	5.335	6.828	8.517	10.147	11.964
131	0.716	2.277	3.731	5.324	6.848	8.515	10.165	11.970
132	0.731	2.265	3.748	5.318	6.865	8.515	10.182	11.977
133	0.741	2.259	3.760	5.316	6.879	8.518	10.198	11.985
134	0.746	2.257	3.768	5.317	6.890	8.524	10.212	11.996
135	0.750	2.257	3.775	5.321	6.900	8.531	10.225	12.008
136	0.752	2.258	3.780	5.326	6.909	8.540	10.238	12.021
137	0.753	2.261	3.785	5.332	6.917	8.550	10.251	12.036
138	0.754	2.263	3.790	5.339	6.926	8.561	10.264	12.051
139	0.750	2.260	3.790	5.340	6.930	8.560	10.270	12.050
140	0.754	2.266	3.795	5.346	6.935	8.572	10.278	12.067
141	0.755	2.270	3.799	5.353	6.944	8.584	10.292	12.085
142	0.756	2.273	3.805	5.361	6.954	8.597	10.307	12.103
143	0.757	2.276	3.810	5.369	6.964	8.610	10.322	12.121
144	0.758	2.280	3.816	5.377	6.975	8.623	10.338	12.141
145	0.760	2.284	3.822	5.386	6.986	8.637	10.355	12.161
146	0.761	2.287	3.829	5.395	6.998	8.652	10.373	12.182
147	0.762	2.291	3.835	5.404	7.011	8.668	10.392	12.205
148	0.764	2.296	3.842	5.414	7.024	8.684	10.411	12.228
149	0.765	2.300	3.850	5.425	7.037	8.701	10.432	12.252
150	0.767	2.305	3.858	5.436	7.052	8.719	10.454	12.278
151	0.768	2.310	3.866	5.447	7.067	8.738	10.477	12.305
152	0.770	2.315	3.874	5.460	7.083	8.757	10.501	12.334
153	0.772	2.320	3.883	5.472	7.099	8.778	10.526	12.364
154	0.774	2.326	3.893	5.486	7.117	8.800	10.553	12.396
155	0.776	2.332	3.903	5.500	7.136	8.823	10.581	12.429
156	0.780	2.330	3.910	5.510	7.140	8.830	10.590	12.450
157	0.778	2.338	3.914	5.515	7.155	8.848	10.611	12.465
158	0.780	2.345	3.925	5.531	7.176	8.874	10.642	12.503
159	0.782	2.352	3.937	5.548	7.199	8.902	10.676	12.542
160	0.785	2.360	3.950	5.567	7.222	8.931	10.711	12.585

Table 81

FIG. 105

	Label							
	24	25	26	27	28	29	30	31
Design #								
121	13.944	15.604	18.051	20.406	23.419	26.971	32.120	39.542
122	13.930	15.620	18.050	20.410	23.420	26.980	32.120	39.530
123	13.870	15.679	18.028	20.445	23.423	27.007	32.136	39.467
124	13.807	15.740	18.014	20.479	23.431	27.044	32.151	39.390
125	13.774	15.782	18.011	20.509	23.442	27.079	32.165	39.312
126	13.761	15.810	18.015	20.535	23.457	27.115	32.177	39.233
127	13.764	15.831	18.026	20.558	23.475	27.151	32.188	39.152
128	13.776	15.846	18.041	20.580	23.496	27.186	32.198	39.070
129	13.792	15.859	18.059	20.600	23.519	27.221	32.206	38.986
130	13.810	15.871	18.079	20.620	23.544	27.256	32.212	38.901
131	13.828	15.884	18.100	20.641	23.572	27.292	32.217	38.814
132	13.847	15.897	18.122	20.662	23.602	27.326	32.219	38.725
133	13.865	15.912	18.145	20.684	23.633	27.361	32.220	38.635
134	13.883	15.928	18.168	20.706	23.666	27.396	32.219	38.544
135	13.900	15.945	18.192	20.731	23.700	27.430	32.216	38.451
136	13.918	15.964	18.216	20.756	23.736	27.464	32.212	38.357
137	13.936	15.984	18.241	20.783	23.773	27.497	32.205	38.261
138	13.955	16.005	18.266	20.812	23.812	27.529	32.196	38.164
139	13.960	16.010	18.270	20.820	23.820	27.530	32.190	38.150
140	13.974	16.028	18.292	20.843	23.851	27.561	32.185	38.065
141	13.993	16.051	18.319	20.876	23.892	27.592	32.172	37.965
142	14.014	16.076	18.348	20.910	23.933	27.621	32.157	37.863
143	14.036	16.102	18.377	20.946	23.975	27.650	32.140	37.760
144	14.058	16.129	18.408	20.984	24.017	27.677	32.120	37.656
145	14.082	16.157	18.441	21.024	24.060	27.702	32.099	37.550
146	14.107	16.186	18.475	21.066	24.104	27.727	32.075	37.443
147	14.133	16.217	18.511	21.110	24.147	27.749	32.049	37.334
148	14.161	16.249	18.549	21.155	24.190	27.770	32.021	37.224
149	14.190	16.283	18.589	21.201	24.233	27.789	31.990	37.112
150	14.220	16.318	18.631	21.249	24.276	27.806	31.957	37.000
151	14.252	16.356	18.676	21.298	24.318	27.822	31.923	36.886
152	14.286	16.395	18.722	21.348	24.359	27.835	31.885	36.770
153	14.321	16.437	18.771	21.399	24.399	27.846	31.846	36.654
154	14.359	16.481	18.821	21.450	24.438	27.855	31.805	36.536
155	14.398	16.527	18.873	21.502	24.476	27.862	31.761	36.416
156	14.420	16.550	18.900	21.530	24.490	27.860	31.740	36.360
157	14.440	16.576	18.927	21.554	24.513	27.867	31.715	36.296
158	14.484	16.627	18.983	21.605	24.548	27.869	31.667	36.175
159	14.531	16.681	19.039	21.657	24.581	27.869	31.617	36.052
160	14.581	16.737	19.097	21.708	24.613	27.867	31.565	35.928

Table 82

FIG. 106

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-35.803	-31.511	-27.863	-24.643	-21.758	-19.156	-16.795	-14.633
162	-35.678	-31.455	-27.856	-24.671	-21.808	-19.215	-16.854	-14.688
163	-35.551	-31.397	-27.847	-24.697	-21.856	-19.275	-16.916	-14.745
164	-35.424	-31.337	-27.836	-24.721	-21.903	-19.335	-16.979	-14.805
165	-35.296	-31.275	-27.823	-24.743	-21.949	-19.394	-17.043	-14.867
166	-35.167	-31.212	-27.808	-24.763	-21.994	-19.453	-17.108	-14.932
167	-35.038	-31.147	-27.790	-24.781	-22.037	-19.512	-17.174	-14.998
168	-34.909	-31.081	-27.771	-24.797	-22.078	-19.569	-17.240	-15.065
169	-34.779	-31.014	-27.750	-24.811	-22.117	-19.626	-17.306	-15.134
170	-34.650	-30.946	-27.728	-24.823	-22.155	-19.681	-17.372	-15.203
171	-34.522	-30.877	-27.704	-24.833	-22.191	-19.735	-17.436	-15.273
172	-34.394	-30.808	-27.679	-24.842	-22.226	-19.788	-17.501	-15.342
173	-34.268	-30.739	-27.653	-24.850	-22.259	-19.839	-17.563	-15.411
174	-34.150	-30.670	-27.630	-24.860	-22.290	-19.890	-17.620	-15.480
175	-34.144	-30.671	-27.627	-24.856	-22.290	-19.888	-17.625	-15.479
176	-34.022	-30.603	-27.601	-24.862	-22.320	-19.936	-17.685	-15.546
177	-33.902	-30.537	-27.574	-24.866	-22.348	-19.982	-17.743	-15.611
178	-33.785	-30.472	-27.548	-24.870	-22.375	-20.026	-17.799	-15.673
179	-33.672	-30.409	-27.522	-24.874	-22.401	-20.069	-17.853	-15.734
180	-33.562	-30.347	-27.497	-24.877	-22.426	-20.110	-17.905	-15.793
181	-33.455	-30.288	-27.474	-24.880	-22.450	-20.149	-17.954	-15.849
182	-33.353	-30.231	-27.450	-24.883	-22.473	-20.187	-18.002	-15.903
183	-33.254	-30.176	-27.428	-24.886	-22.495	-20.223	-18.048	-15.954
184	-33.160	-30.124	-27.407	-24.889	-22.516	-20.257	-18.091	-16.003
185	-33.068	-30.073	-27.387	-24.892	-22.536	-20.290	-18.133	-16.050
186	-32.981	-30.026	-27.368	-24.895	-22.556	-20.321	-18.172	-16.094
187	-32.898	-29.980	-27.350	-24.898	-22.574	-20.351	-18.210	-16.136
188	-32.817	-29.936	-27.333	-24.901	-22.592	-20.380	-18.246	-16.176
189	-32.740	-29.894	-27.317	-24.904	-22.609	-20.407	-18.280	-16.214
190	-32.666	-29.854	-27.301	-24.906	-22.626	-20.434	-18.313	-16.250
191	-32.595	-29.816	-27.287	-24.910	-22.642	-20.459	-18.344	-16.285
192	-32.527	-29.780	-27.273	-24.913	-22.657	-20.483	-18.374	-16.318
193	-32.464	-29.746	-27.260	-24.915	-22.671	-20.505	-18.401	-16.348
194	-32.400	-29.712	-27.248	-24.918	-22.686	-20.528	-18.429	-16.379
195	-32.342	-29.681	-27.236	-24.921	-22.699	-20.548	-18.454	-16.406
196	-32.285	-29.652	-27.225	-24.924	-22.712	-20.568	-18.478	-16.433
197	-32.230	-29.623	-27.215	-24.927	-22.724	-20.587	-18.502	-16.459
198	-32.178	-29.595	-27.204	-24.929	-22.736	-20.605	-18.524	-16.483
199	-32.127	-29.568	-27.194	-24.932	-22.747	-20.623	-18.546	-16.507
200	-32.075	-29.542	-27.185	-24.935	-22.759	-20.640	-18.567	-16.530

Table 83

FIG. 107

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	-12.630	-10.749	-8.963	-7.247	-5.586	-3.963	-2.368	-0.788
162	-12.678	-10.790	-8.996	-7.274	-5.606	-3.978	-2.376	-0.790
163	-12.728	-10.833	-9.032	-7.303	-5.628	-3.993	-2.386	-0.794
164	-12.782	-10.879	-9.070	-7.334	-5.652	-4.010	-2.396	-0.797
165	-12.838	-10.927	-9.111	-7.367	-5.677	-4.028	-2.406	-0.800
166	-12.897	-10.979	-9.154	-7.402	-5.704	-4.047	-2.418	-0.804
167	-12.959	-11.034	-9.201	-7.440	-5.734	-4.068	-2.430	-0.808
168	-13.023	-11.091	-9.250	-7.480	-5.765	-4.090	-2.443	-0.813
169	-13.089	-11.151	-9.302	-7.523	-5.798	-4.114	-2.458	-0.817
170	-13.157	-11.214	-9.356	-7.568	-5.834	-4.140	-2.473	-0.823
171	-13.226	-11.278	-9.413	-7.616	-5.871	-4.167	-2.489	-0.828
172	-13.295	-11.344	-9.472	-7.666	-5.911	-4.195	-2.507	-0.834
173	-13.365	-11.410	-9.532	-7.717	-5.952	-4.225	-2.525	-0.840
174	-13.430	-11.470	-9.590	-7.770	-5.990	-4.250	-2.540	-0.850
175	-13.435	-11.477	-9.593	-7.769	-5.994	-4.256	-2.543	-0.846
176	-13.503	-11.544	-9.654	-7.822	-6.037	-4.287	-2.562	-0.852
177	-13.570	-11.609	-9.714	-7.875	-6.080	-4.319	-2.582	-0.859
178	-13.636	-11.673	-9.774	-7.927	-6.122	-4.350	-2.601	-0.865
179	-13.699	-11.735	-9.832	-7.978	-6.164	-4.381	-2.620	-0.872
180	-13.760	-11.796	-9.888	-8.027	-6.204	-4.411	-2.638	-0.878
181	-13.819	-11.853	-9.942	-8.075	-6.243	-4.440	-2.656	-0.884
182	-13.875	-11.909	-9.993	-8.120	-6.281	-4.468	-2.673	-0.890
183	-13.929	-11.961	-10.043	-8.164	-6.317	-4.495	-2.690	-0.895
184	-13.980	-12.011	-10.089	-8.205	-6.351	-4.520	-2.705	-0.901
185	-14.028	-12.059	-10.134	-8.244	-6.383	-4.544	-2.720	-0.906
186	-14.074	-12.104	-10.176	-8.281	-6.413	-4.566	-2.734	-0.910
187	-14.118	-12.147	-10.216	-8.316	-6.442	-4.588	-2.747	-0.915
188	-14.160	-12.188	-10.253	-8.349	-6.469	-4.608	-2.760	-0.919
189	-14.199	-12.226	-10.289	-8.381	-6.495	-4.627	-2.771	-0.923
190	-14.237	-12.263	-10.323	-8.411	-6.520	-4.645	-2.783	-0.927
191	-14.272	-12.298	-10.355	-8.439	-6.543	-4.662	-2.793	-0.930
192	-14.306	-12.331	-10.386	-8.465	-6.565	-4.679	-2.803	-0.934
193	-14.337	-12.361	-10.414	-8.490	-6.585	-4.694	-2.812	-0.937
194	-14.369	-12.392	-10.442	-8.515	-6.605	-4.709	-2.821	-0.940
195	-14.397	-12.419	-10.467	-8.537	-6.623	-4.722	-2.830	-0.943
196	-14.424	-12.446	-10.492	-8.558	-6.640	-4.735	-2.837	-0.945
197	-14.451	-12.472	-10.516	-8.579	-6.658	-4.747	-2.845	-0.948
198	-14.476	-12.496	-10.538	-8.598	-6.673	-4.759	-2.852	-0.950
199	-14.500	-12.519	-10.560	-8.618	-6.689	-4.771	-2.860	-0.953
200	-14.524	-12.542	-10.581	-8.636	-6.704	-4.782	-2.866	-0.955

Table 84

FIG. 108

	Label							
	16	17	18	19	20	21	22	23
Design #								
161	0.788	2.368	3.963	5.586	7.247	8.963	10.749	12.630
162	0.790	2.376	3.978	5.606	7.274	8.996	10.790	12.678
163	0.794	2.386	3.993	5.628	7.303	9.032	10.833	12.728
164	0.797	2.396	4.010	5.652	7.334	9.070	10.879	12.782
165	0.800	2.406	4.028	5.677	7.367	9.111	10.927	12.838
166	0.804	2.418	4.047	5.704	7.402	9.154	10.979	12.897
167	0.808	2.430	4.068	5.734	7.440	9.201	11.034	12.959
168	0.813	2.443	4.090	5.765	7.480	9.250	11.091	13.023
169	0.817	2.458	4.114	5.798	7.523	9.302	11.151	13.089
170	0.823	2.473	4.140	5.834	7.568	9.356	11.214	13.157
171	0.828	2.489	4.167	5.871	7.616	9.413	11.278	13.226
172	0.834	2.506	4.195	5.911	7.666	9.472	11.344	13.295
173	0.840	2.525	4.225	5.952	7.717	9.532	11.410	13.365
174	0.850	2.540	4.250	5.990	7.770	9.590	11.470	13.430
175	0.846	2.543	4.256	5.994	7.769	9.593	11.477	13.435
176	0.853	2.562	4.287	6.037	7.822	9.654	11.544	13.503
177	0.859	2.582	4.319	6.080	7.875	9.714	11.609	13.570
178	0.865	2.601	4.350	6.122	7.927	9.774	11.673	13.636
179	0.872	2.620	4.381	6.164	7.978	9.832	11.735	13.699
180	0.878	2.638	4.411	6.204	8.027	9.888	11.796	13.760
181	0.884	2.656	4.440	6.244	8.075	9.942	11.853	13.819
182	0.890	2.673	4.468	6.281	8.120	9.993	11.909	13.875
183	0.896	2.690	4.495	6.317	8.164	10.043	11.961	13.929
184	0.901	2.705	4.520	6.351	8.205	10.089	12.011	13.980
185	0.906	2.720	4.544	6.383	8.244	10.134	12.059	14.028
186	0.910	2.734	4.566	6.413	8.281	10.176	12.105	14.074
187	0.915	2.747	4.588	6.442	8.316	10.216	12.147	14.118
188	0.919	2.760	4.608	6.469	8.349	10.253	12.188	14.160
189	0.923	2.771	4.627	6.495	8.381	10.289	12.226	14.199
190	0.927	2.783	4.645	6.520	8.411	10.323	12.263	14.237
191	0.930	2.793	4.662	6.543	8.439	10.355	12.298	14.272
192	0.934	2.803	4.679	6.565	8.466	10.386	12.331	14.306
193	0.937	2.812	4.694	6.585	8.490	10.414	12.361	14.337
194	0.940	2.821	4.709	6.605	8.515	10.442	12.392	14.369
195	0.943	2.830	4.722	6.623	8.537	10.467	12.419	14.397
196	0.945	2.837	4.735	6.640	8.558	10.492	12.446	14.424
197	0.948	2.845	4.747	6.658	8.579	10.516	12.472	14.451
198	0.950	2.852	4.759	6.673	8.598	10.538	12.496	14.476
199	0.953	2.860	4.771	6.689	8.618	10.560	12.519	14.500
200	0.955	2.866	4.782	6.704	8.636	10.580	12.542	14.524

Table 85

FIG. 109

	Label							
	24	25	26	27	28	29	30	31
Design #								
161	14.633	16.795	19.156	21.758	24.643	27.863	31.511	35.803
162	14.688	16.854	19.215	21.808	24.671	27.856	31.455	35.678
163	14.745	16.916	19.275	21.856	24.697	27.847	31.397	35.551
164	14.805	16.979	19.335	21.903	24.721	27.836	31.337	35.424
165	14.867	17.043	19.394	21.949	24.743	27.823	31.275	35.296
166	14.932	17.108	19.453	21.994	24.763	27.808	31.212	35.167
167	14.998	17.174	19.512	22.037	24.781	27.790	31.147	35.038
168	15.065	17.240	19.569	22.078	24.797	27.771	31.081	34.909
169	15.134	17.306	19.626	22.117	24.811	27.750	31.014	34.779
170	15.203	17.372	19.681	22.155	24.823	27.728	30.946	34.650
171	15.273	17.436	19.735	22.191	24.833	27.704	30.877	34.522
172	15.342	17.501	19.788	22.226	24.842	27.679	30.808	34.394
173	15.411	17.563	19.839	22.259	24.850	27.653	30.739	34.268
174	15.480	17.620	19.890	22.290	24.860	27.630	30.670	34.150
175	15.479	17.625	19.888	22.290	24.856	27.627	30.671	34.144
176	15.546	17.685	19.936	22.320	24.862	27.601	30.603	34.022
177	15.611	17.743	19.982	22.348	24.866	27.574	30.537	33.902
178	15.673	17.799	20.026	22.375	24.870	27.548	30.472	33.785
179	15.734	17.853	20.069	22.401	24.874	27.522	30.409	33.672
180	15.793	17.905	20.110	22.426	24.877	27.497	30.347	33.562
181	15.849	17.954	20.149	22.450	24.880	27.473	30.288	33.455
182	15.903	18.002	20.187	22.473	24.883	27.450	30.231	33.353
183	15.954	18.048	20.223	22.495	24.886	27.428	30.176	33.254
184	16.003	18.091	20.257	22.516	24.889	27.407	30.124	33.160
185	16.050	18.133	20.290	22.536	24.892	27.387	30.073	33.068
186	16.094	18.172	20.321	22.556	24.895	27.368	30.025	32.981
187	16.136	18.210	20.351	22.574	24.898	27.350	29.980	32.898
188	16.176	18.246	20.380	22.592	24.901	27.333	29.936	32.817
189	16.214	18.280	20.407	22.609	24.904	27.317	29.894	32.740
190	16.250	18.313	20.434	22.626	24.906	27.301	29.854	32.666
191	16.285	18.344	20.459	22.642	24.910	27.287	29.816	32.595
192	16.318	18.374	20.483	22.657	24.913	27.273	29.780	32.527
193	16.348	18.401	20.505	22.671	24.915	27.260	29.746	32.464
194	16.379	18.429	20.528	22.686	24.918	27.248	29.712	32.400
195	16.406	18.454	20.548	22.699	24.921	27.236	29.681	32.342
196	16.433	18.478	20.568	22.712	24.924	27.225	29.652	32.285
197	16.459	18.502	20.587	22.724	24.927	27.215	29.623	32.230
198	16.483	18.524	20.605	22.736	24.929	27.204	29.595	32.178
199	16.507	18.546	20.623	22.747	24.932	27.194	29.568	32.127
200	16.530	18.567	20.640	22.759	24.935	27.185	29.542	32.075

Table 86

FIG. 110

	Label							
	0	1	2	3	4	5	6	7
Design #								
201	-32.030	-29.518	-27.177	-24.937	-22.770	-20.656	-18.586	-16.551
202	-31.991	-29.497	-27.169	-24.939	-22.778	-20.670	-18.603	-16.570
203	-31.943	-29.472	-27.160	-24.941	-22.788	-20.686	-18.623	-16.592
204	-31.897	-29.450	-27.153	-24.944	-22.799	-20.701	-18.641	-16.611
205	-31.863	-29.431	-27.146	-24.946	-22.808	-20.714	-18.656	-16.627
206	-31.831	-29.416	-27.141	-24.948	-22.814	-20.724	-18.668	-16.641
207	-31.803	-29.401	-27.135	-24.950	-22.821	-20.734	-18.681	-16.654
208	-31.800	-29.402	-27.142	-24.960	-22.834	-20.749	-18.697	-16.670
209	-31.732	-29.365	-27.122	-24.953	-22.836	-20.757	-18.709	-16.686
210	-31.670	-29.332	-27.107	-24.951	-22.843	-20.771	-18.727	-16.706
211	-31.644	-29.307	-27.086	-24.937	-22.838	-20.774	-18.735	-16.718
212	-31.640	-29.319	-27.107	-24.959	-22.856	-20.788	-18.746	-16.727

Table 87

FIG. 111

	Label							
	8	9	10	11	12	13	14	15
Design #								
201	-14.545	-12.563	-10.599	-8.652	-6.717	-4.791	-2.872	-0.957
202	-14.564	-12.581	-10.617	-8.667	-6.730	-4.801	-2.878	-0.959
203	-14.587	-12.603	-10.637	-8.684	-6.743	-4.811	-2.884	-0.961
204	-14.606	-12.622	-10.655	-8.701	-6.757	-4.821	-2.891	-0.963
205	-14.622	-12.637	-10.668	-8.712	-6.766	-4.827	-2.894	-0.964
206	-14.637	-12.652	-10.681	-8.723	-6.775	-4.834	-2.899	-0.966
207	-14.650	-12.664	-10.692	-8.733	-6.784	-4.841	-2.902	-0.967
208	-14.664	-12.676	-10.701	-8.737	-6.783	-4.835	-2.893	-0.954
209	-14.683	-12.696	-10.722	-8.759	-6.804	-4.856	-2.912	-0.970
210	-14.704	-12.716	-10.741	-8.776	-6.819	-4.868	-2.922	-0.978
211	-14.717	-12.731	-10.757	-8.793	-6.836	-4.884	-2.936	-0.990
212	-14.724	-12.735	-10.758	-8.790	-6.830	-4.875	-2.923	-0.974

Table 88

FIG. 112

	Label							
	16	17	18	19	20	21	22	23
Design #								
201	0.957	2.872	4.791	6.717	8.652	10.599	12.563	14.545
202	0.959	2.878	4.801	6.730	8.667	10.617	12.581	14.564
203	0.961	2.884	4.811	6.743	8.684	10.637	12.603	14.587
204	0.963	2.891	4.821	6.757	8.701	10.655	12.622	14.606
205	0.964	2.894	4.827	6.766	8.712	10.668	12.637	14.622
206	0.966	2.899	4.834	6.775	8.723	10.681	12.652	14.637
207	0.967	2.902	4.841	6.784	8.733	10.692	12.664	14.650
208	0.984	2.922	4.862	6.807	8.758	10.718	12.689	14.674
209	0.970	2.912	4.856	6.804	8.759	10.722	12.696	14.683
210	0.966	2.910	4.857	6.808	8.765	10.729	12.704	14.692
211	0.956	2.904	4.855	6.809	8.769	10.735	12.712	14.701
212	0.974	2.923	4.875	6.830	8.790	10.758	12.735	14.724

Table 89

FIG. 113

	Label							
	24	25	26	27	28	29	30	31
Design #								
201	16.551	18.586	20.656	22.770	24.937	27.177	29.518	32.030
202	16.570	18.603	20.670	22.778	24.939	27.169	29.497	31.991
203	16.592	18.623	20.686	22.788	24.941	27.160	29.472	31.943
204	16.611	18.641	20.701	22.799	24.945	27.153	29.450	31.897
205	16.627	18.656	20.714	22.808	24.946	27.146	29.431	31.863
206	16.641	18.668	20.724	22.814	24.948	27.141	29.416	31.831
207	16.654	18.681	20.734	22.821	24.950	27.135	29.401	31.803
208	16.676	18.699	20.746	22.825	24.943	27.113	29.357	31.723
209	16.686	18.709	20.757	22.836	24.953	27.122	29.365	31.732
210	16.697	18.720	20.768	22.845	24.959	27.124	29.362	31.726
211	16.706	18.729	20.775	22.850	24.962	27.127	29.366	31.729
212	16.727	18.746	20.788	22.856	24.959	27.107	29.318	31.639

Table 90

FIG. 114

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.87	0.74	0.63	0.53	0.41	0
2	-4.8	1.02	0.78	0.67	0.57	0.43	0
3	-4.6	0.86	0.73	0.62	0.53	0.45	0
4	-4.4	0.76	0.65	0.55	0.47	0.4	0
5	-4.2	1.03	0.88	0.75	0.63	0.54	0
6	-4	1	0.85	0.72	0.61	0.52	0
7	-3.8	1.18	0.9	0.77	0.65	0.55	0
8	-3.6	1.14	0.97	0.83	0.7	0.6	0
9	-3.4	1.19	1.01	0.86	0.73	0.62	0
10	-3.2	1.41	1.2	1.02	0.87	0.66	0
11	-3	1.33	1.13	0.96	0.81	0.62	0
12	-2.8	1.49	1.14	0.97	0.82	0.63	0
13	-2.6	1.43	1.22	1.03	0.88	0.75	0
14	-2.4	1.74	1.48	1.26	1.07	0.91	0
15	-2.2	1.44	1.23	1.04	0.89	0.75	0
16	-2	1.45	1.23	1.05	0.89	0.76	0
17	-1.8	1.81	1.54	1.31	1.11	0.85	0
18	-1.6	1.87	1.59	1.35	1.15	0.88	0
19	-1.4	1.82	1.55	1.32	1.12	0.95	0
20	-1.2	2.01	1.7	1.45	1.23	1.05	0
21	-1	2.01	1.71	1.45	1.23	0.94	0
22	-0.8	2.03	1.73	1.47	1.25	1.06	0
23	-0.6	2.04	1.73	1.47	1.25	1.06	0
24	-0.4	2.27	1.93	1.64	1.39	1.18	0
25	-0.2	2.26	1.93	1.64	1.39	1.06	0
26	0	2.27	1.93	1.64	1.39	1.18	0
27	0	2.27	1.93	1.64	1.39	1.18	0
28	0.2	2.29	1.95	1.66	1.41	1.2	0
29	0.4	2.83	2.4	2.04	1.74	1.48	0
30	0.6	2.55	2.16	1.84	1.56	1.33	0
31	0.8	2.54	2.16	1.84	1.56	1.19	0
32	1	3.26	2.77	2.35	2	1.53	0
33	1.2	3.29	2.79	2.37	2.02	1.72	0
34	1.4	3.3	2.8	2.38	2.02	1.55	0
35	1.6	2.97	2.53	2.15	1.83	1.55	0
36	1.8	3.31	2.82	2.39	2.03	1.73	0
37	2	3.69	3.13	2.66	2.26	1.73	0
38	2.2	3.69	2.83	2.4	2.04	1.74	0
39	2.4	3.84	3.26	2.77	2.36	1.8	0
40	2.6	3.89	3.31	2.81	2.39	1.83	0

Table 91

FIG. 115

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
41	2.8	3.51	2.98	2.53	2.15	1.65	0
42	3	3.49	2.97	2.52	2.15	1.64	0
43	3.2	3.86	3.28	2.79	2.37	1.63	0
44	3.4	3.48	2.96	2.51	2.14	1.63	0
45	3.6	3.48	2.96	2.51	2.14	1.63	0
46	3.8	3.48	2.96	2.51	2.14	1.64	0
47	4	3.63	3.08	2.62	2.23	1.7	0
48	4.2	3.67	3.12	2.65	2.25	1.72	0
49	4.4	3.5	2.98	2.53	2.15	1.65	0
50	4.6	3.45	2.94	2.5	2.12	1.62	0
51	4.78	3.72	3.16	2.68	2.28	1.75	0
52	4.8	3.64	3.1	2.63	2.24	1.71	0
53	5	3.42	2.9	2.47	2.1	1.6	0
54	5.2	2.82	2.4	2.04	1.73	1.47	0
55	5.4	2.27	1.93	1.64	1.4	1.19	0
56	5.6	1.77	1.5	1.28	1.08	0.92	0
57	5.8	2.71	2.31	1.96	1.67	1.42	0
58	6	2.43	2.07	1.76	1.49	1.27	0
59	6.2	3.53	3	2.55	2.17	1.66	0
60	6.4	3.58	3.04	2.58	2.2	1.68	0
61	6.6	3.8	3.23	2.74	2.33	1.78	0
62	6.8	3.87	3.29	2.8	2.38	1.64	0
63	7	3.49	2.97	2.52	2.14	1.82	0
64	7.2	3.5	2.97	2.53	2.15	1.83	0
65	7.4	3.15	2.68	2.28	1.94	1.65	0
66	7.6	3.69	3.14	2.67	2.27	1.73	0
67	7.8	3.33	2.83	2.41	2.04	1.74	0
68	8	3.75	3.19	2.71	2.3	1.76	0
69	8.2	3.77	3.2	2.72	2.32	1.59	0
70	8.4	3.55	3.02	2.57	2.18	1.85	0
71	8.48	3.56	3.02	2.57	2.18	1.86	0
72	8.6	3.54	3.01	2.56	2.17	1.66	0
73	8.8	3.54	3.01	2.56	2.17	1.66	0
74	9	3.72	3.16	2.68	2.28	1.75	0
75	9.2	3.35	2.85	2.42	2.06	1.75	0
76	9.4	3.36	2.85	2.43	2.06	1.75	0
77	9.6	3.36	2.85	2.43	2.06	1.75	0
78	9.8	3.55	3.01	2.56	2.18	1.67	0
79	10	3.56	3.03	2.57	2.19	1.67	0
80	10.2	3.57	3.04	2.58	2.19	1.68	0

Table 92

FIG. 116

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	2.77	2.35	2	1.7	1.44	0
82	10.6	2.83	2.41	2.05	1.74	1.48	0
83	10.8	2.87	2.44	2.07	1.76	1.5	0
84	11	2.76	2.35	1.99	1.7	1.44	0
85	11.2	2.17	1.84	1.57	1.33	1.13	0
86	11.4	2.35	2	1.7	1.44	1.23	0
87	11.6	2.22	1.89	1.6	1.36	1.16	0
88	11.8	2.34	1.99	1.69	1.43	1.22	0
89	11.83	2.33	1.98	1.69	1.43	1.22	0
90	12	2.34	1.99	1.69	1.44	1.22	0
91	12.2	2.34	1.99	1.69	1.44	1.22	0
92	12.4	2.43	2.06	1.75	1.49	1.27	0
93	12.6	2.25	1.92	1.63	1.38	1.18	0
94	12.8	2.26	1.92	1.63	1.39	1.18	0
95	13	2.26	1.92	1.63	1.39	1.18	0
96	13.2	1.96	1.67	1.42	1.21	1.03	0
97	13.4	1.81	1.54	1.31	1.11	0.94	0
98	13.6	1.83	1.55	1.32	1.12	0.95	0
99	13.8	1.64	1.39	1.18	1	0.85	0
100	14	1.39	1.18	1.01	0.86	0.73	0
101	14.2	1.42	1.21	1.03	0.87	0.74	0
102	14.4	1.44	1.23	1.04	0.89	0.75	0
103	14.6	1.44	1.22	1.04	0.88	0.75	0
104	14.8	1.45	1.23	1.04	0.89	0.75	0
105	15	1.31	1.12	0.95	0.81	0.69	0
106	15.05	1.29	1.09	0.93	0.79	0.67	0
107	15.2	0.93	0.79	0.67	0.57	0.48	0
108	15.4	0.98	0.83	0.71	0.6	0.51	0
109	15.6	0.79	0.67	0.57	0.48	0.41	0
110	15.8	0.84	0.72	0.61	0.52	0.44	0
111	16	0.84	0.71	0.61	0.52	0.44	0
112	16.2	0.85	0.72	0.61	0.52	0.44	0
113	16.4	0.85	0.72	0.61	0.52	0.44	0
114	16.6	0.85	0.72	0.61	0.52	0.44	0
115	16.8	0.85	0.72	0.61	0.52	0.44	0
116	17	0.85	0.72	0.62	0.52	0.45	0
117	17.2	0.85	0.72	0.62	0.52	0.44	0
118	17.4	0.74	0.63	0.54	0.45	0.39	0
119	17.6	0.75	0.64	0.54	0.46	0.39	0
120	17.8	0.79	0.67	0.57	0.49	0.41	0

Table 93

FIG. 117

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.7	0.6	0.51	0.43	0.37	0
122	18.2	0.74	0.63	0.54	0.46	0.39	0
123	18.23	0.67	0.57	0.49	0.41	0.35	0
124	18.4	0.69	0.59	0.5	0.43	0.36	0
125	18.6	0.72	0.61	0.52	0.44	0.37	0
126	18.8	0.75	0.64	0.54	0.46	0.39	0
127	19	0.79	0.67	0.57	0.48	0.41	0
128	19.2	0.82	0.7	0.6	0.51	0.43	0
129	19.4	0.86	0.73	0.62	0.53	0.45	0
130	19.6	0.89	0.76	0.65	0.55	0.47	0
131	19.8	0.93	0.79	0.67	0.57	0.48	0
132	20	0.96	0.82	0.7	0.59	0.5	0
133	20.2	1	0.85	0.72	0.61	0.52	0
134	20.4	1.03	0.88	0.75	0.63	0.54	0
135	20.6	1.07	0.91	0.77	0.65	0.56	0
136	20.8	0.99	0.84	0.71	0.61	0.52	0
137	21	1.14	0.97	0.82	0.7	0.52	0
138	21.2	1.07	0.91	0.77	0.65	0.56	0
139	21.4	1	0.85	0.72	0.61	0.52	0
140	21.42	1	0.85	0.73	0.62	0.52	0
141	21.6	1.04	0.88	0.75	0.64	0.49	0
142	21.8	0.97	0.83	0.7	0.6	0.51	0
143	22	1.01	0.86	0.73	0.62	0.47	0
144	22.2	0.94	0.8	0.68	0.58	0.44	0
145	22.4	0.87	0.74	0.63	0.54	0.41	0
146	22.6	0.9	0.77	0.65	0.55	0.42	0
147	22.8	0.84	0.71	0.61	0.52	0.44	0
148	23	0.78	0.66	0.56	0.48	0.41	0
149	23.2	0.8	0.68	0.58	0.49	0.38	0
150	23.4	0.74	0.63	0.54	0.46	0.35	0
151	23.6	0.76	0.65	0.55	0.47	0.36	0
152	23.8	0.7	0.6	0.51	0.43	0.37	0
153	24	0.65	0.55	0.47	0.4	0.34	0
154	24.2	0.66	0.56	0.48	0.41	0.31	0
155	24.4	0.61	0.52	0.44	0.38	0.29	0
156	24.6	0.56	0.48	0.41	0.35	0.26	0
157	24.69	0.57	0.48	0.41	0.35	0.3	0
158	24.8	0.58	0.49	0.42	0.36	0.27	0
159	25	0.54	0.45	0.39	0.33	0.28	0
160	25.2	0.49	0.42	0.36	0.3	0.26	0

Table 94

FIG. 118

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
161	25.4	0.51	0.43	0.37	0.31	0.26	0
162	25.6	0.47	0.4	0.34	0.29	0.24	0
163	25.8	0.48	0.41	0.35	0.29	0.22	0
164	26	0.44	0.37	0.32	0.27	0.23	0
165	26.2	0.41	0.34	0.29	0.25	0.21	0
166	26.4	0.41	0.35	0.3	0.25	0.21	0
167	26.6	0.4	0.34	0.29	0.24	0.19	0
168	26.8	0.35	0.3	0.25	0.21	0.18	0
169	27	0.34	0.29	0.24	0.21	0.18	0
170	27.2	0.32	0.28	0.23	0.2	0.17	0
171	27.4	0.31	0.27	0.23	0.19	0.16	0
172	27.6	0.27	0.23	0.2	0.17	0.14	0
173	27.8	0.26	0.22	0.19	0.16	0.14	0
174	28	0.25	0.21	0.18	0.15	0.13	0
175	28.19	0.24	0.2	0.17	0.15	0.13	0
176	28.2	0.24	0.2	0.17	0.15	0.13	0
177	28.4	0.23	0.2	0.17	0.14	0.12	0
178	28.6	0.22	0.19	0.16	0.14	0.12	0
179	28.8	0.21	0.18	0.15	0.13	0.11	0
180	29	0.2	0.17	0.15	0.12	0.1	0
181	29.2	0.17	0.15	0.13	0.11	0.09	0
182	29.4	0.17	0.14	0.12	0.1	0.09	0
183	29.6	0.16	0.14	0.12	0.1	0.08	0
184	29.8	0.15	0.13	0.11	0.09	0.08	0
185	30	0.15	0.12	0.11	0.09	0.08	0
186	30.2	0.14	0.12	0.1	0.09	0.07	0
187	30.4	0.13	0.11	0.1	0.08	0.07	0
188	30.6	0.13	0.11	0.09	0.08	0.07	0
189	30.8	0.12	0.1	0.09	0.07	0.06	0
190	31	0.12	0.1	0.08	0.07	0.05	0
191	31.2	0.1	0.08	0.07	0.06	0.05	0
192	31.4	0.1	0.08	0.07	0.06	0.05	0
193	31.6	0.09	0.08	0.07	0.06	0.05	0
194	31.8	0.09	0.07	0.06	0.05	0.05	0
195	32	0.08	0.07	0.06	0.05	0.04	0
196	32.2	0.08	0.07	0.06	0.05	0.04	0
197	32.4	0.08	0.07	0.06	0.05	0.04	0
198	32.6	0.07	0.06	0.05	0.04	0.04	0
199	32.8	0.07	0.06	0.05	0.04	0.04	0
200	33	0.07	0.06	0.05	0.04	0.03	0

Table 95

FIG. 119

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
201	33.2	0.06	0.05	0.05	0.04	0.03	0
202	33.4	0.06	0.05	0.04	0.04	0.03	0
203	33.6	0.06	0.05	0.04	0.04	0.03	0
204	33.8	0.06	0.05	0.04	0.03	0.03	0
205	34	0.05	0.04	0.04	0.03	0.03	0
206	34.2	0.05	0.04	0.04	0.03	0.03	0
207	34.4	0.05	0.04	0.04	0.03	0.03	0
208	34.6	0.05	0.04	0.03	0.03	0.02	0
209	34.8	0.05	0.04	0.03	0.03	0.02	0
210	35	0.04	0.04	0.03	0.03	0.02	0
211	35.2	0.04	0.04	0.03	0.03	0.02	0
212	35.4	0.04	0.03	0.03	0.02	0.02	0
213	35.6	0.04	0.03	0.03	0.02	0.02	0

Table 96

FIG. 120

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
1	-5.0000	0.1977	0.1571	0.0407	25.9021
2	-4.8000	0.2058	0.1637	0.0420	25.6676
3	-4.6000	0.2141	0.1707	0.0434	25.4273
4	-4.4000	0.2227	0.1779	0.0448	25.1813
5	-4.2000	0.2316	0.1854	0.0462	24.9291
6	-4.0000	0.2407	0.1931	0.0476	24.6715
7	-3.8000	0.2502	0.2011	0.0491	24.4072
8	-3.6000	0.2600	0.2094	0.0505	24.1364
9	-3.4000	0.2700	0.2180	0.0520	23.8588
10	-3.2000	0.2804	0.2269	0.0535	23.5744
11	-3.0000	0.2910	0.2361	0.0550	23.2828
12	-2.8000	0.3020	0.2456	0.0564	22.9836
13	-2.6000	0.3133	0.2554	0.0579	22.6764
14	-2.4000	0.3248	0.2655	0.0594	22.3611
15	-2.2000	0.3367	0.2759	0.0608	22.0374
16	-2.0000	0.3489	0.2867	0.0622	21.7045
17	-1.8000	0.3613	0.2977	0.0636	21.3620
18	-1.6000	0.3741	0.3092	0.0650	21.0097
19	-1.4000	0.3872	0.3209	0.0663	20.6467
20	-1.2000	0.4005	0.3330	0.0675	20.2727
21	-1.0000	0.4141	0.3454	0.0687	19.8868
22	-0.8000	0.4280	0.3582	0.0698	19.4885
23	-0.6000	0.4421	0.3713	0.0708	19.0766
24	-0.4000	0.4565	0.3847	0.0718	18.6515
25	-0.2000	0.4711	0.3985	0.0726	18.2112
26	0.0000	0.4859	0.4127	0.0733	17.7556
27	0.1871	0.5	0.4262	0.0738	17.3143
28	0.2000	0.5010	0.4272	0.0738	17.2830
29	0.4000	0.5162	0.4420	0.0742	16.7935
30	0.6000	0.5316	0.4571	0.0744	16.2853
31	0.8000	0.5471	0.4726	0.0745	15.7577
32	1.0000	0.5628	0.4885	0.0743	15.2096
33	1.2000	0.5786	0.5047	0.0739	14.6402
34	1.4000	0.5944	0.5212	0.0732	14.0480
35	1.6000	0.6104	0.5380	0.0724	13.4584
36	1.8000	0.6270	0.5552	0.0718	12.9362
37	2.0000	0.6442	0.5727	0.0715	12.4812
38	2.2000	0.6619	0.5905	0.0714	12.0896
39	2.4000	0.6802	0.6087	0.0716	11.7577
40	2.6000	0.6991	0.6271	0.0720	11.4812

Table 97

FIG. 121

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7186	0.6459	0.0727	11.2561
42	3.0000	0.7386	0.6650	0.0737	11.0780
43	3.2000	0.7593	0.6844	0.0749	10.9425
44	3.4000	0.7804	0.7041	0.0764	10.8448
45	3.6000	0.8021	0.7241	0.0781	10.7804
46	3.8000	0.8243	0.7443	0.0800	10.7445
47	4.0000	0.8470	0.7649	0.0821	10.7322
48	4.2000	0.8702	0.7858	0.0844	10.7388
49	4.4000	0.8938	0.8070	0.0868	10.7590
50	4.6000	0.9178	0.8284	0.0894	10.7882
51	4.8000	0.9421	0.8501	0.0920	10.8216
52	5.0000	0.9668	0.8721	0.0947	10.8545
53	5.2000	0.9917	0.8944	0.0973	10.8828
54	5.2657	1.0000	0.9018	0.0982	10.8905
55	5.4000	1.0169	0.9170	0.1000	10.9024
56	5.6000	1.0423	0.9398	0.1025	10.9096
57	5.8000	1.0678	0.9628	0.1050	10.9012
58	6.0000	1.0934	0.9862	0.1072	10.8741
59	6.2000	1.1191	1.0098	0.1093	10.8262
60	6.4000	1.1448	1.0336	0.1112	10.7558
61	6.6000	1.1704	1.0577	0.1128	10.6614
62	6.8000	1.1960	1.0820	0.1141	10.5429
63	7.0000	1.2210	1.1065	0.1145	10.3491
64	7.2000	1.2477	1.1312	0.1164	10.2925
65	7.4000	1.2747	1.1562	0.1185	10.2474
66	7.6000	1.3026	1.1814	0.1212	10.2619
67	7.8000	1.3306	1.2067	0.1239	10.2665
68	8.0000	1.3587	1.2323	0.1264	10.2601
69	8.2000	1.3869	1.2580	0.1288	10.2420
70	8.4000	1.4150	1.2839	0.1311	10.2121
71	8.6000	1.4432	1.3100	0.1332	10.1702
72	8.8000	1.4714	1.3362	0.1352	10.1178
73	8.9903	1.4984	1.3613	0.1371	10.0694
74	9.0000	1.4999	1.3626	0.1373	10.0734
75	9.2000	1.5300	1.3892	0.1409	10.1412
76	9.4000	1.5600	1.4159	0.1442	10.1840
77	9.6000	1.5899	1.4427	0.1472	10.2025
78	9.8000	1.6195	1.4697	0.1499	10.1975
79	10.0000	1.6491	1.4968	0.1523	10.1750
80	10.2000	1.6796	1.5240	0.1555	10.2052

Table 98

FIG. 122

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
81	10.4000	1.7104	1.5514	0.1590	10.2461
82	10.6000	1.7413	1.5790	0.1623	10.2771
83	10.8000	1.7721	1.6067	0.1654	10.2967
84	11.0000	1.8037	1.6345	0.1692	10.3523
85	11.2000	1.8362	1.6625	0.1737	10.4487
86	11.4000	1.8686	1.6906	0.1779	10.5249
87	11.6000	1.9009	1.7189	0.1819	10.5824
88	11.8000	1.9330	1.7474	0.1856	10.6235
89	12.0000	1.9652	1.7760	0.1892	10.6515
90	12.2000	1.9974	1.8048	0.1926	10.6705
91	12.2081	1.9987	1.8060	0.1927	10.6711
92	12.4000	2.0297	1.8338	0.1959	10.6843
93	12.6000	2.0621	1.8629	0.1993	10.6962
94	12.8000	2.0947	1.8921	0.2026	10.7072
95	13.0000	2.1275	1.9215	0.2059	10.7170
96	13.2000	2.1604	1.9511	0.2093	10.7247
97	13.4000	2.1934	1.9808	0.2125	10.7283
98	13.6000	2.2264	2.0107	0.2157	10.7258
99	13.8000	2.2594	2.0407	0.2187	10.7156
100	14.0000	2.2924	2.0709	0.2215	10.6966
101	14.2000	2.3253	2.1011	0.2242	10.6682
102	14.4000	2.3581	2.1315	0.2266	10.6302
103	14.6000	2.3908	2.1620	0.2288	10.5831
104	14.8000	2.4234	2.1926	0.2308	10.5273
105	15.0000	2.4559	2.2233	0.2326	10.4639
106	15.2000	2.4884	2.2541	0.2343	10.3941
107	15.2711	2.4999	2.2651	0.2348	10.3676
108	15.4000	2.5207	2.2850	0.2357	10.3170
109	15.6000	2.5530	2.3160	0.2370	10.2322
110	15.8000	2.5851	2.3471	0.2380	10.1392
111	16.0000	2.6171	2.3783	0.2387	10.0379
112	16.2000	2.6489	2.4097	0.2392	9.9288
113	16.4000	2.6806	2.4411	0.2395	9.8122
114	16.6000	2.7122	2.4727	0.2396	9.6892
115	16.8000	2.7438	2.5043	0.2394	9.5605
116	17.0000	2.7756	2.5362	0.2394	9.4386
117	17.2000	2.8075	2.5682	0.2393	9.3185
118	17.4000	2.8392	2.6003	0.2389	9.1891
119	17.6000	2.8709	2.6326	0.2383	9.0518
120	17.8000	2.9025	2.6651	0.2374	8.9078

Table 99

FIG. 123

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.9341	2.6977	0.2364	8.7621
122	18.2000	2.9657	2.7306	0.2351	8.6105
123	18.4000	2.9973	2.7637	0.2336	8.4532
124	18.4172	3.0000	2.7665	0.2335	8.4394
125	18.6000	3.0288	2.7969	0.2319	8.2922
126	18.8000	3.0605	2.8304	0.2301	8.1301
127	19.0000	3.0923	2.8640	0.2282	7.9695
128	19.2000	3.1243	2.8979	0.2264	7.8128
129	19.4000	3.1566	2.9320	0.2246	7.6606
130	19.6000	3.1891	2.9662	0.2228	7.5114
131	19.8000	3.2216	3.0007	0.2209	7.3626
132	20.0000	3.2543	3.0353	0.2189	7.2125
133	20.2000	3.2870	3.0701	0.2169	7.0633
134	20.4000	3.3197	3.1051	0.2146	6.9116
135	20.6000	3.3523	3.1402	0.2121	6.7543
136	20.8000	3.3847	3.1754	0.2093	6.5913
137	21.0000	3.4170	3.2108	0.2062	6.4231
138	21.2000	3.4491	3.2462	0.2029	6.2502
139	21.4000	3.4811	3.2817	0.1993	6.0734
140	21.5191	3.5000	3.3029	0.1971	5.9667
141	21.6000	3.5128	3.3173	0.1955	5.8936
142	21.8000	3.5445	3.3530	0.1915	5.7115
143	22.0000	3.5760	3.3887	0.1873	5.5275
144	22.2000	3.6073	3.4244	0.1829	5.3421
145	22.4000	3.6385	3.4601	0.1784	5.1557
146	22.6000	3.6695	3.4958	0.1737	4.9688
147	22.8000	3.7003	3.5314	0.1689	4.7819
148	23.0000	3.7310	3.5670	0.1639	4.5957
149	23.2000	3.7615	3.6026	0.1589	4.4107
150	23.4000	3.7919	3.6381	0.1538	4.2274
151	23.6000	3.8221	3.6734	0.1486	4.0462
152	23.8000	3.8522	3.7087	0.1434	3.8675
153	24.0000	3.8821	3.7439	0.1382	3.6919
154	24.2000	3.9119	3.7789	0.1330	3.5202
155	24.4000	3.9417	3.8138	0.1279	3.3529
156	24.6000	3.9714	3.8485	0.1228	3.1909
157	24.7934	4.0000	3.8820	0.1180	3.0401
158	24.8000	4.0010	3.8831	0.1179	3.0351
159	25.0000	4.0306	3.9175	0.1131	2.8869
160	25.2000	4.0603	3.9517	0.1086	2.7487

Table 100

FIG. 124

	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
Design #					
161	25.4000	4.0904	3.9858	0.1046	2.6251
162	25.6000	4.1208	4.0196	0.1012	2.5185
163	25.8000	4.1514	4.0532	0.0982	2.4225
164	26.0000	4.1820	4.0867	0.0953	2.3319
165	26.2000	4.2124	4.1199	0.0925	2.2443
166	26.4000	4.2426	4.1529	0.0896	2.1580
167	26.6000	4.2725	4.1857	0.0867	2.0723
168	26.8000	4.3021	4.2183	0.0838	1.9866
169	27.0000	4.3315	4.2507	0.0808	1.9006
170	27.2000	4.3605	4.2828	0.0777	1.8141
171	27.4000	4.3892	4.3146	0.0745	1.7273
172	27.6000	4.4175	4.3462	0.0713	1.6404
173	27.8000	4.4454	4.3774	0.0680	1.5535
174	28.0000	4.4730	4.4083	0.0647	1.4670
175	28.1991	4.5000	4.4387	0.0613	1.3817
176	28.2000	4.5001	4.4388	0.0613	1.3813
177	28.4000	4.5268	4.4689	0.0580	1.2968
178	28.6000	4.5531	4.4985	0.0546	1.2139
179	28.8000	4.5789	4.5276	0.0513	1.1330
180	29.0000	4.6042	4.5561	0.0480	1.0544
181	29.2000	4.6289	4.5840	0.0449	0.9784
182	29.4000	4.6530	4.6112	0.0417	0.9053
183	29.6000	4.6765	4.6377	0.0387	0.8352
184	29.8000	4.6993	4.6635	0.0358	0.7682
185	30.0000	4.7214	4.6883	0.0330	0.7045

Table 101

FIG. 125

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-18.468	-18.468	-18.468	-18.468	-18.466	-18.466	-18.468	-18.468
2	-18.487	-18.487	-18.464	-18.464	-18.463	-18.463	-18.464	-18.463
3	-18.502	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
4	-18.502	-18.480	-18.480	-18.480	-18.460	-18.460	-18.463	-18.460
5	-18.513	-18.513	-18.513	-18.513	-18.443	-18.464	-18.513	-18.468
6	-18.472	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
7	-18.510	-18.483	-18.483	-18.483	-18.466	-18.466	-18.483	-18.474
8	-18.478	-18.478	-18.472	-18.472	-18.472	-18.472	-18.472	-18.472
9	-18.494	-18.466	-18.464	-18.465	-18.464	-18.464	-18.464	-18.464
10	-18.477	-18.477	-18.475	-18.475	-18.474	-18.474	-18.475	-18.475
11	-18.489	-18.478	-18.478	-18.478	-18.475	-18.475	-18.478	-18.478
12	-18.509	-18.479	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465
13	-18.540	-18.529	-18.451	-18.527	-18.451	-18.451	-18.451	-18.451
14	-18.529	-18.514	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
15	-18.506	-18.505	-18.494	-18.505	-18.444	-18.480	-18.494	-18.480
16	-18.529	-18.504	-18.461	-18.461	-18.459	-18.461	-18.461	-18.461
17	-18.471	-18.470	-18.470	-18.470	-18.470	-18.470	-18.470	-18.470
18	-18.506	-18.505	-18.468	-18.468	-18.465	-18.468	-18.468	-18.468
19	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476
20	-18.482	-18.482	-18.482	-18.482	-18.454	-18.482	-18.482	-18.482
21	-18.474	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
22	-18.478	-18.474	-18.474	-18.474	-18.464	-18.474	-18.474	-18.474
23	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524
24	-18.483	-18.479	-18.479	-18.479	-18.461	-18.466	-18.479	-18.479
25	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496
26	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467
27	-18.474	-18.471	-18.471	-18.471	-18.471	-18.471	-18.471	-18.471
28	-18.538	-18.537	-18.537	-18.537	-18.446	-18.446	-18.446	-18.446
29	-18.499	-18.499	-18.467	-18.467	-18.466	-18.466	-18.466	-18.466
30	-18.495	-18.495	-18.495	-18.495	-18.453	-18.459	-18.486	-18.460
31	-18.494	-18.494	-18.485	-18.485	-18.485	-18.485	-18.485	-18.485
32	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528
33	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488
34	-18.536	-18.535	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464
35	-20.584	-20.584	-20.584	-20.584	-20.584	-20.584	-20.584	-20.584
36	-21.574	-21.574	-21.574	-21.574	-21.573	-21.573	-21.574	-21.573
37	-22.222	-22.222	-22.222	-22.222	-22.220	-22.222	-22.222	-22.222
38	-22.705	-22.705	-22.704	-22.705	-22.703	-22.703	-22.703	-22.703
39	-23.088	-23.087	-23.087	-23.087	-23.087	-23.087	-23.087	-23.087
40	-23.404	-23.404	-23.404	-23.404	-23.402	-23.404	-23.404	-23.404

Table 102

FIG. 126

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	-18.462	-18.463	-18.466	-18.463	-18.466	-18.466	-18.466	-18.466
2	-18.462	-18.462	-18.463	-18.463	-18.463	-18.463	-18.463	-18.463
3	-18.445	-18.455	-18.466	-18.462	-18.466	-18.466	-18.466	-18.466
4	-18.459	-18.459	-18.459	-18.459	-18.460	-18.459	-18.459	-18.459
5	-18.436	-18.438	-18.438	-18.438	-18.442	-18.442	-18.442	-18.442
6	-18.460	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
7	-18.449	-18.450	-18.450	-18.450	-18.466	-18.450	-18.450	-18.450
8	-18.391	-18.447	-18.472	-18.472	-18.472	-18.472	-18.472	-18.472
9	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464
10	-18.444	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
11	-18.424	-18.431	-18.456	-18.450	-18.472	-18.469	-18.458	-18.468
12	-18.428	-18.463	-18.463	-18.463	-18.465	-18.465	-18.465	-18.465
13	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451
14	-18.453	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
15	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444
16	-18.458	-18.458	-18.458	-18.458	-18.459	-18.459	-18.458	-18.458
17	-18.415	-18.463	-18.470	-18.469	-18.470	-18.470	-18.470	-18.470
18	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456
19	-18.455	-18.455	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457
20	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454
21	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465
22	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
23	-18.391	-18.391	-18.391	-18.391	-18.456	-18.456	-18.394	-18.394
24	-18.452	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457
25	-18.399	-18.418	-18.420	-18.420	-18.495	-18.495	-18.422	-18.426
26	-18.465	-18.465	-18.465	-18.465	-18.467	-18.465	-18.465	-18.465
27	-18.459	-18.459	-18.459	-18.459	-18.465	-18.463	-18.462	-18.462
28	-18.408	-18.445	-18.445	-18.445	-18.445	-18.445	-18.445	-18.445
29	-18.456	-18.456	-18.457	-18.456	-18.461	-18.461	-18.458	-18.459
30	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453
31	-18.440	-18.440	-18.440	-18.440	-18.484	-18.440	-18.440	-18.440
32	-18.389	-18.389	-18.390	-18.390	-18.419	-18.419	-18.419	-18.419
33	-18.428	-18.429	-18.430	-18.429	-18.486	-18.455	-18.449	-18.449
34	-18.450	-18.450	-18.450	-18.450	-18.452	-18.450	-18.450	-18.450
35	-16.071	-16.073	-16.073	-16.073	-16.073	-16.073	-16.073	-16.073
36	-14.716	-14.716	-14.716	-14.716	-14.722	-14.722	-14.717	-14.717
37	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720
38	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905
39	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206
40	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589

Table 103

FIG. 127

	Label							
	16	17	18	19	20	21	22	23
Design #								
1	18.471	18.471	18.466	18.466	18.466	18.466	18.466	18.466
2	18.479	18.469	18.469	18.469	18.468	18.468	18.469	18.468
3	18.502	18.502	18.497	18.502	18.481	18.497	18.497	18.497
4	18.502	18.502	18.483	18.483	18.483	18.483	18.483	18.483
5	18.481	18.481	18.481	18.481	18.481	18.481	18.481	18.481
6	18.471	18.471	18.471	18.471	18.471	18.471	18.471	18.471
7	18.509	18.468	18.468	18.468	18.468	18.468	18.468	18.468
8	18.478	18.467	18.467	18.467	18.466	18.466	18.467	18.466
9	18.496	18.496	18.496	18.496	18.496	18.496	18.496	18.496
10	18.541	18.542	18.478	18.479	18.452	18.452	18.452	18.452
11	18.489	18.468	18.467	18.467	18.465	18.465	18.467	18.465
12	18.509	18.484	18.484	18.484	18.466	18.466	18.466	18.466
13	18.540	18.499	18.469	18.469	18.469	18.469	18.469	18.469
14	18.528	18.528	18.528	18.528	18.476	18.476	18.528	18.476
15	18.505	18.464	18.464	18.464	18.464	18.464	18.464	18.464
16	18.528	18.471	18.466	18.469	18.460	18.462	18.465	18.462
17	18.543	18.544	18.456	18.456	18.455	18.455	18.455	18.455
18	18.505	18.505	18.505	18.505	18.440	18.505	18.505	18.505
19	18.541	18.542	18.471	18.471	18.453	18.453	18.453	18.453
20	18.474	18.474	18.474	18.474	18.468	18.468	18.474	18.468
21	18.477	18.477	18.467	18.474	18.466	18.466	18.467	18.466
22	18.478	18.478	18.478	18.478	18.478	18.478	18.478	18.478
23	18.524	18.491	18.491	18.491	18.491	18.491	18.491	18.491
24	18.484	18.484	18.484	18.484	18.484	18.484	18.484	18.484
25	18.483	18.483	18.474	18.474	18.474	18.474	18.474	18.474
26	18.468	18.468	18.468	18.468	18.468	18.468	18.468	18.468
27	18.470	18.468	18.468	18.468	18.468	18.468	18.468	18.468
28	18.519	18.481	18.481	18.481	18.481	18.481	18.481	18.481
29	18.500	18.474	18.474	18.474	18.474	18.474	18.474	18.474
30	18.495	18.495	18.495	18.495	18.495	18.495	18.495	18.495
31	18.483	18.481	18.481	18.481	18.481	18.481	18.481	18.481
32	18.528	18.520	18.499	18.499	18.473	18.477	18.499	18.499
33	18.488	18.485	18.485	18.485	18.485	18.485	18.485	18.485
34	18.534	18.488	18.488	18.488	18.487	18.487	18.487	18.487
35	20.584	20.583	20.583	20.583	20.583	20.583	20.583	20.583
36	21.574	21.572	21.571	21.571	21.571	21.571	21.571	21.571
37	22.218	22.218	22.218	22.218	22.208	22.209	22.218	22.217
38	22.705	22.705	22.705	22.705	22.702	22.702	22.705	22.704
39	23.088	23.088	23.088	23.088	23.088	23.088	23.088	23.088
40	23.404	23.404	23.404	23.404	23.401	23.401	23.401	23.401

Table 104

FIG. 128

	Label							
	24	25	26	27	28	29	30	31
Design #								
1	18.465	18.465	18.465	18.465	18.466	18.466	18.466	18.466
2	18.452	18.463	18.463	18.463	18.468	18.464	18.463	18.463
3	18.434	18.434	18.436	18.435	18.437	18.437	18.437	18.437
4	18.410	18.410	18.445	18.410	18.483	18.483	18.445	18.472
5	18.309	18.456	18.460	18.460	18.481	18.481	18.481	18.481
6	18.438	18.445	18.466	18.453	18.471	18.471	18.471	18.471
7	18.433	18.451	18.462	18.462	18.468	18.468	18.463	18.466
8	18.464	18.464	18.464	18.464	18.465	18.465	18.464	18.464
9	18.408	18.408	18.420	18.420	18.496	18.497	18.420	18.420
10	18.452	18.452	18.452	18.452	18.452	18.452	18.452	18.452
11	18.461	18.464	18.464	18.464	18.465	18.464	18.464	18.464
12	18.453	18.453	18.455	18.453	18.455	18.455	18.455	18.455
13	18.448	18.448	18.448	18.448	18.469	18.448	18.448	18.448
14	18.404	18.417	18.418	18.418	18.476	18.419	18.418	18.418
15	18.457	18.462	18.464	18.462	18.464	18.464	18.464	18.464
16	18.458	18.460	18.460	18.460	18.460	18.460	18.460	18.460
17	18.455	18.455	18.455	18.455	18.455	18.455	18.455	18.455
18	18.421	18.432	18.437	18.437	18.439	18.439	18.439	18.439
19	18.452	18.452	18.452	18.452	18.453	18.453	18.452	18.452
20	18.455	18.455	18.455	18.455	18.467	18.467	18.466	18.467
21	18.459	18.460	18.461	18.460	18.466	18.466	18.462	18.465
22	18.426	18.432	18.432	18.432	18.478	18.478	18.478	18.478
23	18.428	18.428	18.428	18.428	18.490	18.440	18.428	18.428
24	18.415	18.418	18.429	18.419	18.484	18.483	18.472	18.473
25	18.456	18.456	18.456	18.456	18.456	18.456	18.456	18.456
26	18.465	18.465	18.465	18.465	18.465	18.465	18.465	18.465
27	18.464	18.464	18.464	18.464	18.464	18.464	18.464	18.464
28	18.437	18.437	18.453	18.437	18.453	18.453	18.453	18.453
29	18.453	18.453	18.453	18.453	18.473	18.453	18.453	18.453
30	18.427	18.438	18.438	18.438	18.445	18.438	18.438	18.438
31	18.451	18.451	18.451	18.451	18.455	18.451	18.451	18.451
32	18.428	18.428	18.429	18.428	18.451	18.437	18.429	18.434
33	18.447	18.447	18.447	18.447	18.448	18.448	18.447	18.447
34	18.439	18.439	18.439	18.439	18.440	18.439	18.439	18.439
35	16.071	16.071	16.072	16.071	16.085	16.072	16.072	16.072
36	14.720	14.720	14.720	14.720	14.720	14.720	14.720	14.720
37	13.722	13.722	13.728	13.728	13.731	13.728	13.728	13.728
38	12.905	12.905	12.905	12.905	12.905	12.905	12.905	12.905
39	12.205	12.205	12.205	12.205	12.205	12.205	12.205	12.205
40	11.590	11.590	11.590	11.590	11.590	11.590	11.590	11.590

Table 105

FIG. 129

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-23.669	-23.669	-23.669	-23.669	-23.662	-23.662	-23.665	-23.665
42	-23.892	-23.892	-23.892	-23.892	-23.891	-23.891	-23.892	-23.891
43	-24.086	-24.086	-24.086	-24.086	-24.085	-24.085	-24.086	-24.085
44	-24.254	-24.254	-24.253	-24.254	-24.245	-24.250	-24.253	-24.253
45	-24.397	-24.397	-24.397	-24.397	-24.395	-24.395	-24.397	-24.397
46	-24.525	-24.524	-24.524	-24.524	-24.524	-24.524	-24.524	-24.524
47	-24.635	-24.635	-24.635	-24.635	-24.633	-24.634	-24.635	-24.635
48	-24.738	-24.738	-24.730	-24.738	-24.727	-24.729	-24.730	-24.730
49	-24.821	-24.821	-24.816	-24.816	-24.816	-24.816	-24.816	-24.816
50	-24.897	-24.897	-24.892	-24.897	-24.890	-24.891	-24.892	-24.892
51	-24.963	-24.960	-24.960	-24.960	-24.955	-24.960	-24.960	-24.960
52	-25.024	-25.023	-25.015	-25.015	-25.015	-25.015	-25.015	-25.015
53	-25.074	-25.074	-25.074	-25.074	-25.038	-25.041	-25.061	-25.061
54	-25.083	-25.082	-25.082	-25.082	-25.078	-25.078	-25.078	-25.078
55	-25.109	-25.109	-25.108	-25.108	-25.108	-25.108	-25.108	-25.108
56	-25.148	-25.147	-25.143	-25.144	-25.143	-25.143	-25.143	-25.143
57	-25.176	-25.174	-25.174	-25.174	-25.173	-25.173	-25.173	-25.173
58	-25.202	-25.202	-25.202	-25.202	-25.189	-25.192	-25.197	-25.195
59	-25.220	-25.220	-25.215	-25.215	-25.214	-25.214	-25.215	-25.214
60	-25.238	-25.236	-25.236	-25.236	-25.226	-25.226	-25.235	-25.228
61	-25.243	-25.243	-25.243	-25.243	-25.222	-25.222	-25.233	-25.230
62	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243
63	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792
64	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737
65	-38.698	-26.007	-23.569	-23.569	-21.371	-21.371	-21.371	-21.371
66	-39.014	-25.904	-23.497	-23.497	-21.322	-21.322	-21.322	-21.322
67	-39.276	-25.788	-23.433	-23.433	-21.281	-21.281	-21.281	-21.281
68	-39.422	-25.843	-23.419	-23.419	-21.222	-21.222	-21.222	-21.222
69	-39.538	-25.889	-23.430	-23.430	-21.156	-21.156	-21.156	-21.156
70	-39.608	-26.034	-23.434	-23.434	-21.077	-21.077	-21.077	-21.077
71	-39.643	-26.177	-23.477	-23.477	-20.995	-20.995	-20.995	-20.995
72	-39.643	-26.413	-23.533	-23.533	-20.878	-20.878	-20.878	-20.878
73	-31.746	-31.746	-31.746	-31.746	-14.945	-14.945	-14.945	-14.945
74	-31.751	-31.751	-31.734	-31.746	-14.948	-14.948	-14.952	-14.949
75	-31.689	-31.689	-31.688	-31.688	-15.023	-15.023	-15.024	-15.024
76	-31.638	-31.638	-31.622	-31.622	-15.108	-15.108	-15.110	-15.110
77	-31.574	-31.574	-31.564	-31.570	-15.198	-15.198	-15.201	-15.200
78	-31.554	-31.547	-31.456	-31.456	-15.296	-15.297	-15.310	-15.310
79	-38.428	-31.170	-28.865	-28.865	-14.205	-14.668	-15.742	-15.742
80	-38.361	-30.759	-28.491	-28.491	-14.105	-14.576	-15.661	-15.661

Table 106

FIG. 130

	Label							
	8	9	10	11	12	13	14	15
Design #								
41	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041
42	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545
43	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096
44	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688
45	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317
46	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977
47	-8.668	-8.668	-8.668	-8.668	-8.669	-8.668	-8.668	-8.668
48	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385
49	-8.128	-8.128	-8.128	-8.128	-8.129	-8.129	-8.128	-8.129
50	-7.892	-7.892	-7.892	-7.892	-7.897	-7.896	-7.896	-7.896
51	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685
52	-7.494	-7.496	-7.496	-7.496	-7.497	-7.497	-7.496	-7.496
53	-7.337	-7.337	-7.337	-7.337	-7.346	-7.337	-7.337	-7.337
54	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280
55	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183
56	-7.054	-7.054	-7.057	-7.056	-7.058	-7.058	-7.057	-7.057
57	-6.949	-6.949	-6.950	-6.950	-6.951	-6.951	-6.951	-6.951
58	-6.860	-6.860	-6.860	-6.860	-6.869	-6.866	-6.860	-6.866
59	-6.789	-6.794	-6.794	-6.794	-6.797	-6.797	-6.797	-6.797
60	-6.738	-6.739	-6.739	-6.739	-6.742	-6.742	-6.741	-6.741
61	-6.715	-6.715	-6.715	-6.715	-6.718	-6.717	-6.715	-6.715
62	-6.695	-6.696	-6.696	-6.696	-6.698	-6.698	-6.698	-6.698
63	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815
64	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812
65	-5.930	-6.162	-6.424	-6.424	-6.878	-6.878	-6.878	-6.878
66	-5.857	-6.089	-6.361	-6.361	-6.860	-6.860	-6.860	-6.860
67	-5.810	-6.038	-6.309	-6.309	-6.851	-6.851	-6.851	-6.851
68	-5.751	-5.969	-6.255	-6.255	-6.862	-6.862	-6.862	-6.862
69	-5.707	-5.913	-6.203	-6.203	-6.880	-6.880	-6.880	-6.880
70	-5.674	-5.865	-6.164	-6.164	-6.913	-6.913	-6.913	-6.913
71	-5.641	-5.816	-6.119	-6.119	-6.957	-6.957	-6.957	-6.957
72	-5.601	-5.760	-6.071	-6.071	-7.015	-7.015	-7.015	-7.015
73	31.747	31.747	31.746	31.746	-11.518	-11.518	-11.518	-11.518
74	31.742	31.742	31.742	31.742	-11.521	-11.521	-11.521	-11.521
75	31.700	31.700	31.674	31.677	-11.571	-11.571	-11.571	-11.571
76	31.637	31.638	31.625	31.631	-11.616	-11.616	-11.616	-11.616
77	31.574	31.574	31.564	31.570	-11.658	-11.658	-11.658	-11.658
78	31.553	31.553	31.441	31.484	-11.695	-11.695	-11.695	-11.695
79	31.181	31.181	31.181	31.181	-11.375	-11.375	-11.375	-11.375
80	38.361	30.758	28.491	28.491	-11.302	-11.302	-11.302	-11.302

Table 107

FIG. 131

	Label							
	16	17	18	19	20	21	22	23
Design #								
41	23.671	23.671	23.668	23.671	23.657	23.663	23.667	23.666
42	23.892	23.892	23.892	23.892	23.891	23.891	23.891	23.891
43	24.086	24.086	24.084	24.086	24.080	24.080	24.080	24.080
44	24.254	24.254	24.254	24.254	24.249	24.249	24.249	24.249
45	24.397	24.397	24.397	24.397	24.397	24.397	24.397	24.397
46	24.526	24.526	24.526	24.526	24.516	24.517	24.526	24.523
47	24.635	24.635	24.635	24.635	24.635	24.635	24.635	24.635
48	24.735	24.735	24.731	24.735	24.731	24.731	24.731	24.731
49	24.821	24.821	24.821	24.821	24.818	24.819	24.820	24.820
50	24.896	24.895	24.895	24.895	24.891	24.891	24.894	24.891
51	24.963	24.961	24.961	24.961	24.948	24.951	24.960	24.957
52	25.021	25.020	25.013	25.013	25.012	25.012	25.012	25.012
53	25.074	25.074	25.062	25.062	25.061	25.061	25.061	25.061
54	25.081	25.081	25.081	25.081	25.079	25.079	25.079	25.079
55	25.109	25.109	25.106	25.106	25.106	25.106	25.106	25.106
56	25.148	25.148	25.143	25.148	25.138	25.138	25.142	25.140
57	25.176	25.176	25.176	25.176	25.167	25.167	25.173	25.170
58	25.203	25.203	25.203	25.203	25.185	25.185	25.203	25.194
59	25.219	25.219	25.219	25.219	25.204	25.209	25.214	25.214
60	25.230	25.230	25.226	25.230	25.225	25.225	25.225	25.225
61	25.240	25.240	25.240	25.240	25.240	25.240	25.240	25.240
62	25.243	25.243	25.243	25.243	25.228	25.237	25.243	25.243
63	38.522	26.837	24.259	24.259	21.835	21.835	21.835	21.835
64	39.430	26.613	24.103	24.103	21.747	21.747	21.747	21.747
65	38.697	25.964	23.561	23.561	21.380	21.380	21.380	21.380
66	39.013	25.893	23.500	23.500	21.319	21.319	21.319	21.319
67	39.272	25.861	23.457	23.457	21.263	21.263	21.263	21.263
68	39.422	25.861	23.430	23.430	21.215	21.215	21.215	21.215
69	39.550	25.899	23.429	23.429	21.154	21.154	21.154	21.154
70	39.625	26.010	23.440	23.440	21.079	21.079	21.079	21.079
71	39.618	26.207	23.493	23.493	20.983	20.983	20.983	20.983
72	39.638	26.474	23.540	23.540	20.865	20.865	20.865	20.865
73	11.518	11.518	11.518	11.518	0.398	0.398	0.398	0.398
74	11.522	11.522	11.522	11.522	0.402	0.402	0.403	0.403
75	11.574	11.574	11.574	11.574	0.462	0.462	0.462	0.462
76	11.619	11.619	11.619	11.619	0.528	0.528	0.528	0.528
77	11.658	11.658	11.658	11.658	0.595	0.595	0.595	0.595
78	11.692	11.692	11.692	11.692	0.665	0.665	0.665	0.665
79	11.652	11.652	11.652	11.652	0.746	0.760	0.845	0.845
80	11.303	11.303	11.303	11.303	0.716	0.717	0.717	0.717

Table 108

FIG. 132

	Label							
	24	25	26	27	28	29	30	31
Design #								
41	11.041	11.041	11.041	11.041	11.041	11.041	11.041	11.041
42	10.545	10.545	10.545	10.545	10.545	10.545	10.545	10.545
43	10.098	10.099	10.099	10.099	10.099	10.099	10.099	10.099
44	9.688	9.688	9.688	9.688	9.688	9.688	9.688	9.688
45	9.316	9.316	9.316	9.316	9.316	9.316	9.316	9.316
46	8.978	8.978	8.978	8.978	8.978	8.978	8.978	8.978
47	8.666	8.667	8.667	8.667	8.667	8.667	8.667	8.667
48	8.385	8.385	8.385	8.385	8.385	8.385	8.385	8.385
49	8.125	8.125	8.125	8.125	8.126	8.126	8.125	8.125
50	7.893	7.894	7.894	7.894	7.894	7.894	7.894	7.894
51	7.685	7.686	7.686	7.686	7.688	7.687	7.686	7.686
52	7.498	7.498	7.498	7.498	7.501	7.499	7.499	7.499
53	7.332	7.335	7.337	7.337	7.337	7.337	7.337	7.337
54	7.280	7.280	7.280	7.280	7.280	7.280	7.280	7.280
55	7.183	7.184	7.184	7.184	7.186	7.186	7.185	7.185
56	7.054	7.058	7.058	7.058	7.059	7.059	7.059	7.059
57	6.950	6.950	6.952	6.952	6.952	6.952	6.952	6.952
58	6.863	6.863	6.863	6.863	6.864	6.863	6.863	6.863
59	6.791	6.791	6.791	6.791	6.810	6.799	6.798	6.798
60	6.745	6.745	6.745	6.745	6.746	6.746	6.745	6.745
61	6.709	6.709	6.710	6.710	6.711	6.711	6.711	6.711
62	6.693	6.700	6.700	6.700	6.702	6.702	6.700	6.700
63	5.660	5.923	6.256	6.256	6.886	6.886	6.886	6.886
64	5.576	5.836	6.181	6.181	6.844	6.844	6.844	6.844
65	5.939	6.170	6.431	6.431	6.876	6.876	6.876	6.876
66	5.863	6.093	6.361	6.361	6.862	6.862	6.862	6.862
67	5.794	6.017	6.297	6.297	6.856	6.856	6.856	6.856
68	5.748	5.965	6.249	6.249	6.865	6.865	6.865	6.865
69	5.706	5.911	6.201	6.201	6.880	6.880	6.880	6.880
70	5.673	5.865	6.161	6.161	6.912	6.912	6.912	6.912
71	5.638	5.814	6.118	6.118	6.962	6.962	6.962	6.962
72	5.598	5.755	6.066	6.066	7.014	7.014	7.014	7.014
73	14.945	14.945	14.945	14.945	-0.398	-0.398	-0.398	-0.398
74	14.947	14.947	14.947	14.947	-0.397	-0.397	-0.397	-0.397
75	15.017	15.019	15.031	15.029	-0.465	-0.465	-0.466	-0.466
76	15.103	15.103	15.107	15.104	-0.528	-0.528	-0.529	-0.528
77	15.199	15.199	15.203	15.201	-0.597	-0.597	-0.597	-0.597
78	15.293	15.293	15.322	15.314	-0.667	-0.667	-0.668	-0.667
79	15.258	15.258	15.258	15.258	-0.593	-0.593	-0.593	-0.593
80	14.104	14.575	15.660	15.660	-0.716	-0.717	-0.717	-0.717

Table 109

FIG. 133

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-38.770	-30.565	-28.305	-28.305	-14.112	-14.591	-15.728	-15.728
82	-39.045	-30.428	-28.150	-28.150	-14.151	-14.629	-15.813	-15.813
83	-39.237	-30.332	-28.008	-28.008	-14.213	-14.681	-15.922	-15.922
84	-42.842	-27.881	42.842	-25.702	29.062	-24.594	31.203	-24.594
85	-42.655	-27.882	42.853	-25.687	29.031	-24.553	31.121	-24.553
86	-42.464	-27.897	42.842	-25.674	29.001	-24.507	31.047	-24.507
87	-42.259	-27.918	42.803	-25.658	28.972	-24.450	30.979	-24.450
88	-42.044	-27.950	42.732	-25.640	28.944	-24.384	30.918	-24.384
89	-41.810	-27.996	42.639	-25.619	28.920	-24.309	30.866	-24.309
90	-41.564	-28.049	42.521	-25.594	28.899	-24.226	30.820	-24.226
91	-41.554	-28.060	42.517	-25.592	28.899	-24.218	30.819	-24.218
92	-41.313	-28.130	42.382	-25.568	28.883	-24.131	30.781	-24.131
93	-41.061	-28.204	42.235	-25.546	28.871	-24.041	30.748	-24.041
94	-40.815	-28.281	42.094	-25.529	28.867	-23.954	30.725	-23.954
95	-40.588	-28.384	41.955	-25.523	28.866	-23.867	30.704	-23.867
96	-40.382	-28.507	41.826	-25.531	28.868	-23.781	30.688	-23.781
97	-40.205	-28.658	41.701	-25.556	28.869	-23.693	30.674	-23.693
98	-40.054	-28.809	41.580	-25.607	28.865	-23.609	30.656	-23.609
99	-39.931	-28.990	41.461	-25.677	28.857	-23.523	30.638	-23.523
100	-39.837	-29.167	41.338	-25.777	28.842	-23.437	30.617	-23.437
101	-39.769	-29.347	41.210	-25.909	28.820	-23.348	30.594	-23.348
102	-39.729	-29.519	41.072	-26.092	28.790	-23.251	30.569	-23.251
103	-39.717	-29.676	40.928	-26.350	28.749	-23.144	30.538	-23.144
104	-39.740	-29.838	40.770	-26.653	28.695	-23.030	30.504	-23.030
105	-39.801	-30.016	40.599	-27.009	28.627	-22.860	30.464	-22.947
106	-39.861	-30.186	40.421	-27.323	28.552	-22.705	30.427	-22.896
107	-39.873	-30.238	40.366	-27.420	28.529	-22.658	30.421	-22.880
108	-39.892	-30.332	40.263	-27.572	28.484	-22.588	30.408	-22.856
109	-39.907	-30.471	40.107	-27.763	28.409	-22.501	30.409	-22.830
110	-39.901	-30.592	39.964	-27.911	28.334	-22.431	30.432	-22.810
111	-39.885	-30.712	39.825	-28.026	28.255	-22.368	30.473	-22.799
112	-39.845	-30.817	39.722	-28.095	28.173	-22.304	30.566	-22.787
113	-39.795	-30.919	39.633	-28.136	28.094	-22.240	30.679	-22.778
114	-39.727	-31.016	39.585	-28.134	28.013	-22.164	30.850	-22.763
115	-39.632	-31.098	39.588	-28.106	27.930	-22.083	31.071	-22.749
116	-40.170	-32.458	-24.267	-26.000	-15.469	-15.469	-18.490	-17.922
117	-40.113	-32.438	-24.253	-26.017	-15.428	-15.428	-18.613	-18.023
118	-40.042	-32.414	-24.246	-26.045	-15.378	-15.378	-18.749	-18.141
119	-39.959	-32.389	-24.245	-26.083	-15.323	-15.323	-18.892	-18.271
120	-39.866	-32.361	-24.251	-26.129	-15.266	-15.266	-19.035	-18.407

Table 110

FIG. 134

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	38.770	30.564	28.304	28.304	-11.260	-11.260	-11.260	-11.260
82	39.045	30.428	28.150	28.150	-11.220	-11.220	-11.220	-11.220
83	39.236	30.328	28.011	28.011	-11.174	-11.174	-11.174	-11.174
84	-8.991	-12.436	-8.991	-12.638	-8.991	-13.018	-8.991	-13.018
85	-8.995	-12.533	-8.995	-12.741	-8.995	-13.146	-8.995	-13.146
86	-8.991	-12.638	-8.991	-12.852	-8.991	-13.286	-8.991	-13.286
87	-8.970	-12.766	-8.970	-12.985	-8.970	-13.450	-8.970	-13.450
88	-8.931	-12.915	-8.931	-13.140	-8.931	-13.637	-8.931	-13.637
89	-8.879	-13.083	-8.879	-13.313	-8.879	-13.842	-8.879	-13.842
90	-8.810	-13.271	-8.810	-13.505	-8.810	-14.064	-8.810	-14.064
91	-8.807	-13.276	-8.807	-13.512	-8.807	-14.073	-8.807	-14.073
92	-8.729	-13.463	-8.729	-13.703	-8.729	-14.290	-8.729	-14.290
93	-8.644	-13.658	-8.644	-13.899	-8.644	-14.508	-8.644	-14.508
94	-8.563	-13.841	-8.563	-14.081	-8.563	-14.707	-8.563	-14.707
95	-8.488	-13.996	-8.488	-14.234	-8.488	-14.882	-8.488	-14.882
96	-8.419	-14.121	-8.419	-14.357	-8.419	-15.032	-8.419	-15.032
97	-8.355	-14.215	-8.355	-14.447	-8.355	-15.161	-8.355	-15.161
98	-8.299	-14.289	-8.299	-14.513	-8.299	-15.273	-8.299	-15.273
99	-8.244	-14.335	-8.244	-14.548	-8.244	-15.366	-8.244	-15.366
100	-8.199	-14.363	-8.199	-14.561	-8.185	-15.447	-8.190	-15.447
101	-8.165	-14.372	-8.165	-14.551	-8.123	-15.519	-8.124	-15.519
102	-8.133	-14.360	-8.132	-14.514	-8.059	-15.585	-8.059	-15.585
103	-8.103	-14.325	-8.101	-14.448	-7.991	-15.648	-7.991	-15.648
104	-8.070	-14.265	-8.067	-14.358	-7.916	-15.702	-7.916	-15.702
105	-8.033	-14.180	-8.031	-14.237	-7.833	-15.746	-7.833	-15.746
106	-8.001	-14.102	-7.999	-14.128	-7.755	-15.782	-7.755	-15.782
107	-7.994	-14.077	-7.990	-14.094	-7.730	-15.794	-7.730	-15.794
108	-7.980	-14.038	-7.978	-14.041	-7.687	-15.814	-7.687	-15.814
109	-7.965	-13.981	-7.963	-13.981	-7.628	-15.842	-7.628	-15.842
110	-7.959	-13.935	-7.957	-13.935	-7.578	-15.871	-7.578	-15.871
111	-7.958	-13.895	-7.957	-13.895	-7.531	-15.907	-7.531	-15.892
112	-7.967	-13.861	-7.967	-13.861	-7.491	-15.947	-7.495	-15.905
113	-7.981	-13.829	-7.981	-13.829	-7.455	-15.990	-7.459	-15.920
114	-8.002	-13.795	-8.002	-13.795	-7.423	-16.030	-7.431	-15.932
115	-8.027	-13.760	-8.027	-13.760	-7.397	-16.076	-7.403	-15.949
116	40.170	32.459	24.267	26.000	-9.724	-9.724	-8.849	-8.881
117	40.113	32.438	24.253	26.016	-9.774	-9.774	-8.798	-8.827
118	40.041	32.414	24.246	26.044	-9.829	-9.829	-8.735	-8.760
119	39.958	32.388	24.245	26.081	-9.889	-9.889	-8.662	-8.682
120	39.865	32.360	24.250	26.128	-9.954	-9.954	-8.580	-8.596

Table 111

FIG. 135

	Label							
	16	17	18	19	20	21	22	23
Design #								
81	11.260	11.260	11.260	11.260	0.782	0.782	0.782	0.782
82	11.219	11.219	11.219	11.219	0.855	0.855	0.855	0.855
83	11.176	11.176	11.176	11.176	0.939	0.939	0.939	0.939
84	14.917	12.616	14.917	12.616	18.770	12.616	18.440	12.616
85	15.029	12.559	15.029	12.559	18.943	12.559	18.598	12.559
86	15.158	12.498	15.158	12.498	19.110	12.498	18.752	12.498
87	15.323	12.421	15.323	12.421	19.290	12.421	18.924	12.421
88	15.528	12.328	15.528	12.328	19.470	12.328	19.102	12.328
89	15.765	12.227	15.765	12.227	19.641	12.227	19.272	12.227
90	16.040	12.111	16.040	12.111	19.803	12.111	19.434	12.111
91	16.050	12.107	16.050	12.107	19.810	12.107	19.440	12.107
92	16.334	11.988	16.334	11.988	19.950	11.988	19.582	11.988
93	16.630	11.865	16.630	11.865	20.081	11.865	19.714	11.865
94	16.908	11.750	16.908	11.750	20.189	11.750	19.819	11.750
95	17.152	11.664	17.152	11.657	20.285	11.630	19.911	11.630
96	17.361	11.607	17.361	11.606	20.372	11.494	19.990	11.494
97	17.532	11.562	17.532	11.562	20.447	11.378	20.055	11.385
98	17.667	11.531	17.667	11.531	20.512	11.289	20.106	11.300
99	17.779	11.505	17.779	11.505	20.569	11.210	20.149	11.229
100	17.870	11.489	17.870	11.489	20.621	11.149	20.185	11.172
101	17.942	11.482	17.942	11.482	20.671	11.099	20.215	11.126
102	18.000	11.481	18.000	11.481	20.721	11.059	20.244	11.089
103	18.041	11.488	18.041	11.488	20.769	11.027	20.268	11.061
104	18.070	11.499	18.070	11.499	20.816	11.000	20.288	11.037
105	18.084	11.513	18.084	11.513	20.863	10.978	20.303	11.017
106	18.089	11.536	18.089	11.536	20.917	10.958	20.324	11.000
107	18.091	11.544	18.091	11.544	20.940	10.950	20.331	10.993
108	18.090	11.562	18.090	11.562	20.986	10.937	20.352	10.981
109	18.081	11.593	18.081	11.593	21.073	10.915	20.385	10.961
110	18.062	11.627	18.062	11.627	21.179	10.887	20.428	10.936
111	18.031	11.666	18.031	11.666	21.300	10.854	20.480	10.904
112	17.983	11.700	17.983	11.700	21.446	10.807	20.532	10.857
113	17.921	11.735	17.921	11.735	21.603	10.747	20.598	10.796
114	17.840	11.763	17.840	11.763	21.778	10.668	20.661	10.715
115	17.747	11.779	17.747	11.779	21.949	10.577	20.706	10.623
116	9.724	9.724	8.849	8.880	2.894	2.894	3.101	3.098
117	9.774	9.774	8.798	8.827	2.866	2.866	3.128	3.126
118	9.829	9.829	8.736	8.760	2.831	2.831	3.159	3.157
119	9.889	9.889	8.662	8.683	2.787	2.787	3.194	3.192
120	9.954	9.954	8.581	8.596	2.735	2.735	3.234	3.232

Table 112

FIG. 136

	Label							
	24	25	26	27	28	29	30	31
Design #								
81	14.114	14.594	15.729	15.729	-0.783	-0.783	-0.783	-0.783
82	14.152	14.629	15.814	15.814	-0.856	-0.856	-0.856	-0.856
83	14.215	14.684	15.916	15.916	-0.939	-0.939	-0.939	-0.939
84	0.734	2.397	0.734	2.397	0.504	2.397	0.515	2.397
85	0.628	2.487	0.628	2.487	0.412	2.487	0.424	2.487
86	0.514	2.585	0.514	2.585	0.316	2.585	0.329	2.585
87	0.375	2.700	0.375	2.700	0.203	2.700	0.215	2.700
88	0.211	2.836	0.211	2.836	0.069	2.836	0.082	2.836
89	0.029	2.988	0.029	2.988	-0.079	2.988	-0.067	2.988
90	-0.178	3.161	-0.178	3.161	-0.250	3.161	-0.239	3.161
91	-0.186	3.167	-0.186	3.167	-0.257	3.167	-0.246	3.167
92	-0.395	3.346	-0.395	3.346	-0.435	3.346	-0.424	3.346
93	-0.612	3.534	-0.612	3.534	-0.625	3.534	-0.614	3.534
94	-0.808	3.712	-0.808	3.712	-0.808	3.712	-0.808	3.712
95	-0.978	3.870	-0.978	3.870	-0.978	3.870	-0.978	3.870
96	-1.124	4.007	-1.124	4.007	-1.124	4.007	-1.124	4.007
97	-1.244	4.122	-1.244	4.122	-1.244	4.122	-1.244	4.122
98	-1.336	4.218	-1.336	4.218	-1.336	4.218	-1.336	4.218
99	-1.409	4.297	-1.409	4.297	-1.409	4.297	-1.409	4.297
100	-1.464	4.365	-1.464	4.365	-1.464	4.365	-1.464	4.365
101	-1.501	4.420	-1.501	4.420	-1.501	4.420	-1.501	4.420
102	-1.524	4.465	-1.524	4.465	-1.524	4.465	-1.524	4.465
103	-1.533	4.505	-1.533	4.505	-1.533	4.505	-1.533	4.505
104	-1.530	4.540	-1.530	4.540	-1.530	4.540	-1.530	4.540
105	-1.514	4.570	-1.514	4.571	-1.514	4.571	-1.514	4.571
106	-1.495	4.593	-1.495	4.593	-1.495	4.609	-1.495	4.609
107	-1.490	4.596	-1.490	4.596	-1.490	4.619	-1.490	4.618
108	-1.480	4.604	-1.480	4.604	-1.480	4.639	-1.480	4.638
109	-1.466	4.612	-1.466	4.612	-1.466	4.669	-1.466	4.666
110	-1.456	4.614	-1.456	4.614	-1.456	4.693	-1.456	4.691
111	-1.435	4.610	-1.436	4.610	-1.456	4.719	-1.456	4.716
112	-1.421	4.592	-1.422	4.592	-1.468	4.737	-1.468	4.733
113	-1.407	4.566	-1.408	4.566	-1.484	4.751	-1.484	4.748
114	-1.401	4.525	-1.401	4.525	-1.511	4.757	-1.510	4.754
115	-1.397	4.473	-1.397	4.473	-1.545	4.759	-1.545	4.753
116	15.469	15.469	18.491	17.923	-2.894	-2.894	-3.101	-3.099
117	15.428	15.428	18.613	18.023	-2.866	-2.866	-3.128	-3.126
118	15.378	15.378	18.749	18.141	-2.831	-2.831	-3.159	-3.157
119	15.324	15.324	18.891	18.272	-2.787	-2.787	-3.194	-3.192
120	15.266	15.266	19.034	18.406	-2.735	-2.735	-3.233	-3.232

Table 113

FIG. 137

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-39.738	-32.309	-24.163	-26.205	39.739	32.310	-19.642	-19.232
122	-39.649	-32.294	-24.211	-26.262	39.652	32.295	-19.652	-19.205
123	-39.536	-32.264	-24.242	-26.313	39.558	32.276	-19.673	-19.189
124	-39.535	-32.267	-24.247	-26.323	39.535	32.267	-19.676	-19.188
125	-39.425	-32.234	-24.266	-26.372	39.425	32.235	-19.705	-19.182
126	-39.280	-32.184	-24.279	-26.420	39.280	32.185	-19.748	-19.190
127	-39.115	-32.122	-24.285	-26.466	39.114	32.120	-19.802	-19.206
128	-38.948	-32.053	-24.274	-26.504	38.949	32.054	-19.853	-19.220
129	-38.784	-31.991	-24.260	-26.553	38.784	31.991	-19.906	-19.230
130	-38.640	-31.943	-24.236	-26.616	38.640	31.943	-19.954	-19.226
131	-38.523	-31.917	-24.206	-26.700	38.522	31.915	-19.999	-19.205
132	-38.424	-31.907	-24.169	-26.796	38.425	31.908	-20.038	-19.169
133	-38.432	-32.010	38.436	-27.388	-17.451	-21.330	-17.003	-22.982
134	-38.350	-32.013	38.356	-27.381	-17.487	-21.326	-16.995	-23.037
135	-38.274	-32.018	38.274	-27.382	-17.524	-21.318	-16.982	-23.104
136	-38.195	-32.021	38.202	-27.387	-17.562	-21.304	-16.961	-23.182
137	-38.121	-32.028	38.125	-27.401	-17.603	-21.285	-16.930	-23.274
138	-38.052	-32.040	38.050	-27.427	-17.643	-21.261	-16.888	-23.380
139	-37.977	-32.047	37.977	-27.455	-17.685	-21.234	-16.841	-23.485
140	-37.932	-32.052	37.936	-27.475	-17.710	-21.218	-16.811	-23.549
141	-37.904	-32.056	37.909	-27.490	-17.726	-21.207	-16.788	-23.592
142	-37.835	-32.069	37.833	-27.533	-17.771	-21.185	-16.735	-23.696
143	-37.760	-32.075	37.765	-27.574	-17.819	-21.165	-16.683	-23.789
144	-37.689	-32.084	37.687	-27.619	-17.873	-21.153	-16.633	-23.875
145	-37.610	-32.087	37.614	-27.661	-17.935	-21.147	-16.586	-23.953
146	-37.532	-32.090	37.534	-27.705	-18.009	-21.151	-16.540	-24.027
147	-37.452	-32.088	37.452	-27.746	-18.093	-21.162	-16.497	-24.094
148	-37.366	-32.082	37.368	-27.784	-18.187	-21.184	-16.457	-24.159
149	-37.280	-32.074	37.279	-27.821	-18.284	-21.215	-16.423	-24.220
150	-37.189	-32.062	37.189	-27.855	-18.380	-21.253	-16.398	-24.278
151	-37.094	-32.045	37.094	-27.885	-18.474	-21.299	-16.383	-24.334
152	-36.994	-32.023	36.996	-27.912	-18.564	-21.350	-16.383	-24.388
153	-36.891	-31.997	36.891	-27.935	-18.649	-21.405	-16.394	-24.439
154	-36.781	-31.964	36.781	-27.953	-18.731	-21.462	-16.421	-24.487
155	-36.668	-31.927	36.666	-27.965	-18.808	-21.519	-16.460	-24.531
156	-36.538	-31.876	36.541	-27.969	-18.887	-21.579	-16.517	-24.571
157	-36.412	-31.824	36.413	-27.969	-18.956	-21.631	-16.575	-24.605
158	-36.408	-31.822	36.408	-27.968	-18.959	-21.633	-16.577	-24.606
159	-36.258	-31.750	36.259	-27.953	-19.032	-21.685	-16.653	-24.631
160	-36.085	-31.658	36.082	-27.921	-19.106	-21.732	-16.740	-24.645

Table 114

FIG. 138

	Label							
	8	9	10	11	12	13	14	15
Design #								
121	-6.470	-6.470	-11.613	-11.613	-5.566	-5.562	-13.954	-14.014
122	-6.536	-6.536	-11.556	-11.556	-5.489	-5.486	-14.052	-14.129
123	-6.623	-6.623	-11.505	-11.505	-5.407	-5.404	-14.171	-14.262
124	-6.629	-6.629	-11.499	-11.499	-5.397	-5.393	-14.181	-14.273
125	-6.729	-6.729	-11.456	-11.456	-5.311	-5.308	-14.303	-14.405
126	-6.870	-6.870	-11.425	-11.425	-5.212	-5.210	-14.452	-14.563
127	-7.045	-7.045	-11.413	-11.413	-5.115	-5.114	-14.605	-14.723
128	-7.239	-7.239	-11.420	-11.420	-5.034	-5.033	-14.742	-14.866
129	-7.422	-7.422	-11.438	-11.438	-4.968	-4.967	-14.857	-14.991
130	-7.572	-7.572	-11.454	-11.454	-4.920	-4.919	-14.940	-15.088
131	-7.681	-7.681	-11.460	-11.460	-4.881	-4.880	-14.989	-15.157
132	-7.759	-7.759	-11.460	-11.460	-4.850	-4.850	-15.014	-15.205
133	-3.256	-6.122	-3.256	-6.122	-13.075	-9.682	-13.146	-9.682
134	-3.230	-6.176	-3.230	-6.176	-13.104	-9.682	-13.191	-9.682
135	-3.208	-6.216	-3.208	-6.216	-13.123	-9.681	-13.228	-9.681
136	-3.192	-6.246	-3.192	-6.246	-13.132	-9.682	-13.259	-9.677
137	-3.178	-6.265	-3.178	-6.265	-13.130	-9.680	-13.283	-9.668
138	-3.163	-6.275	-3.163	-6.275	-13.115	-9.673	-13.302	-9.654
139	-3.154	-6.283	-3.154	-6.283	-13.098	-9.670	-13.318	-9.640
140	-3.149	-6.285	-3.149	-6.286	-13.083	-9.667	-13.326	-9.630
141	-3.145	-6.285	-3.145	-6.286	-13.070	-9.664	-13.331	-9.622
142	-3.135	-6.282	-3.135	-6.286	-13.038	-9.659	-13.344	-9.602
143	-3.129	-6.279	-3.129	-6.287	-13.003	-9.657	-13.359	-9.583
144	-3.122	-6.273	-3.122	-6.287	-12.964	-9.656	-13.376	-9.561
145	-3.118	-6.266	-3.117	-6.287	-12.921	-9.658	-13.397	-9.538
146	-3.113	-6.255	-3.109	-6.285	-12.871	-9.660	-13.423	-9.510
147	-3.109	-6.242	-3.103	-6.284	-12.815	-9.664	-13.452	-9.477
148	-3.106	-6.224	-3.094	-6.282	-12.752	-9.670	-13.489	-9.438
149	-3.102	-6.203	-3.085	-6.280	-12.685	-9.677	-13.531	-9.391
150	-3.099	-6.176	-3.073	-6.278	-12.618	-9.687	-13.582	-9.338
151	-3.096	-6.143	-3.058	-6.278	-12.552	-9.703	-13.645	-9.277
152	-3.095	-6.103	-3.040	-6.280	-12.490	-9.726	-13.724	-9.209
153	-3.094	-6.054	-3.016	-6.284	-12.433	-9.759	-13.815	-9.132
154	-3.096	-5.995	-2.986	-6.293	-12.389	-9.805	-13.919	-9.050
155	-3.103	-5.925	-2.947	-6.310	-12.359	-9.868	-14.026	-8.968
156	-3.119	-5.848	-2.899	-6.346	-12.357	-9.959	-14.140	-8.897
157	-3.146	-5.767	-2.841	-6.400	-12.374	-10.067	-14.245	-8.842
158	-3.148	-5.764	-2.839	-6.402	-12.375	-10.071	-14.248	-8.841
159	-3.206	-5.686	-2.765	-6.502	-12.429	-10.216	-14.364	-8.819
160	-3.322	-5.636	-2.677	-6.667	-12.520	-10.390	-14.488	-8.853

Table 115

FIG. 139

	Label							
	16	17	18	19	20	21	22	23
Design #								
121	19.643	19.233	13.955	14.016	24.164	26.206	11.618	11.618
122	19.652	19.205	14.052	14.129	24.210	26.261	11.560	11.560
123	19.674	19.187	14.170	14.260	24.246	26.321	11.504	11.504
124	19.676	19.189	14.182	14.275	24.247	26.323	11.502	11.502
125	19.706	19.184	14.305	14.408	24.268	26.372	11.458	11.458
126	19.750	19.190	14.454	14.564	24.281	26.422	11.426	11.426
127	19.800	19.206	14.605	14.722	24.283	26.463	11.416	11.416
128	19.854	19.222	14.744	14.868	24.276	26.504	11.421	11.421
129	19.905	19.230	14.857	14.992	24.259	26.553	11.438	11.438
130	19.955	19.227	14.941	15.089	24.237	26.615	11.455	11.455
131	19.998	19.205	14.990	15.157	24.205	26.698	11.461	11.461
132	20.038	19.170	15.015	15.206	24.170	26.797	11.460	11.460
133	27.394	22.988	32.015	21.350	13.255	16.943	13.194	17.422
134	27.387	23.044	32.018	21.341	13.270	16.949	13.193	17.467
135	27.386	23.111	32.020	21.328	13.286	16.947	13.190	17.512
136	27.393	23.191	32.027	21.311	13.300	16.932	13.180	17.556
137	27.406	23.282	32.032	21.289	13.313	16.907	13.166	17.599
138	27.426	23.382	32.038	21.262	13.322	16.870	13.144	17.640
139	27.457	23.490	32.048	21.235	13.332	16.826	13.117	17.684
140	27.479	23.555	32.055	21.219	13.337	16.796	13.098	17.709
141	27.495	23.598	32.061	21.208	13.340	16.775	13.084	17.726
142	27.533	23.697	32.067	21.184	13.350	16.725	13.050	17.770
143	27.577	23.791	32.079	21.165	13.362	16.675	13.011	17.817
144	27.619	23.876	32.084	21.152	13.378	16.627	12.971	17.872
145	27.664	23.955	32.090	21.147	13.398	16.581	12.925	17.934
146	27.704	24.026	32.090	21.149	13.422	16.537	12.876	18.006
147	27.745	24.093	32.088	21.161	13.452	16.495	12.819	18.091
148	27.784	24.158	32.083	21.182	13.488	16.455	12.756	18.184
149	27.821	24.219	32.074	21.214	13.531	16.422	12.688	18.282
150	27.855	24.278	32.062	21.253	13.582	16.397	12.620	18.379
151	27.885	24.334	32.045	21.299	13.646	16.384	12.553	18.474
152	27.912	24.388	32.024	21.350	13.723	16.382	12.491	18.563
153	27.935	24.439	31.997	21.405	13.816	16.395	12.434	18.649
154	27.953	24.488	31.965	21.463	13.920	16.422	12.389	18.732
155	27.966	24.532	31.926	21.521	14.028	16.462	12.360	18.810
156	27.971	24.572	31.879	21.578	14.139	16.515	12.356	18.886
157	27.969	24.605	31.824	21.631	14.245	16.575	12.373	18.956
158	27.968	24.606	31.822	21.633	14.248	16.577	12.375	18.959
159	27.953	24.631	31.751	21.684	14.363	16.651	12.427	19.031
160	27.919	24.643	31.656	21.730	14.488	16.739	12.520	19.105

Table 116

FIG. 140

	Label							
	24	25	26	27	28	29	30	31
Design #								
121	0.241	0.241	5.559	5.557	-0.243	-0.243	6.468	6.468
122	0.299	0.299	5.486	5.482	-0.300	-0.300	6.535	6.535
123	0.369	0.369	5.397	5.395	-0.376	-0.376	6.618	6.618
124	0.379	0.379	5.393	5.389	-0.380	-0.380	6.629	6.629
125	0.462	0.462	5.306	5.303	-0.465	-0.465	6.728	6.728
126	0.579	0.579	5.208	5.207	-0.581	-0.581	6.870	6.870
127	0.724	0.724	5.115	5.113	-0.723	-0.723	7.046	7.046
128	0.881	0.881	5.031	5.030	-0.883	-0.883	7.239	7.239
129	1.033	1.033	4.967	4.966	-1.033	-1.034	7.422	7.422
130	1.159	1.159	4.918	4.917	-1.160	-1.160	7.572	7.572
131	1.255	1.255	4.881	4.880	-1.255	-1.255	7.681	7.681
132	1.326	1.326	4.849	4.849	-1.328	-1.328	7.759	7.759
133	0.172	2.924	0.172	2.924	9.485	6.389	9.485	6.389
134	0.132	2.974	0.132	2.974	9.533	6.378	9.533	6.378
135	0.102	3.014	0.102	3.014	9.568	6.370	9.569	6.370
136	0.077	3.042	0.077	3.042	9.589	6.361	9.596	6.361
137	0.059	3.064	0.059	3.064	9.602	6.353	9.615	6.353
138	0.047	3.080	0.047	3.080	9.605	6.345	9.627	6.345
139	0.035	3.089	0.035	3.089	9.603	6.335	9.634	6.335
140	0.029	3.093	0.029	3.093	9.598	6.329	9.636	6.329
141	0.026	3.094	0.026	3.094	9.593	6.324	9.637	6.323
142	0.021	3.100	0.021	3.100	9.582	6.317	9.641	6.314
143	0.015	3.101	0.015	3.101	9.566	6.309	9.642	6.301
144	0.012	3.103	0.012	3.103	9.550	6.304	9.646	6.291
145	0.008	3.101	0.008	3.102	9.528	6.298	9.650	6.278
146	0.007	3.100	0.007	3.103	9.505	6.295	9.656	6.266
147	0.005	3.095	0.004	3.102	9.474	6.291	9.662	6.250
148	0.005	3.089	0.002	3.100	9.435	6.288	9.668	6.230
149	0.006	3.081	-0.001	3.098	9.390	6.284	9.676	6.207
150	0.007	3.069	-0.004	3.095	9.336	6.281	9.687	6.179
151	0.010	3.055	-0.008	3.094	9.276	6.279	9.703	6.145
152	0.015	3.038	-0.013	3.093	9.207	6.280	9.726	6.104
153	0.022	3.014	-0.021	3.093	9.130	6.284	9.759	6.054
154	0.033	2.984	-0.033	3.094	9.048	6.293	9.805	5.994
155	0.048	2.945	-0.049	3.101	8.966	6.310	9.868	5.924
156	0.072	2.899	-0.071	3.119	8.896	6.345	9.958	5.849
157	0.106	2.841	-0.105	3.146	8.842	6.400	10.066	5.768
158	0.107	2.839	-0.106	3.148	8.840	6.402	10.071	5.765
159	0.163	2.767	-0.161	3.206	8.819	6.501	10.213	5.688
160	0.257	2.680	-0.254	3.324	8.854	6.668	10.390	5.639

Table 117

FIG. 141

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-35.860	-31.521	35.860	-27.852	-19.181	-21.766	-16.846	-24.633
162	-35.666	-31.406	35.663	-27.799	-19.246	-21.799	-16.934	-24.628
163	-35.507	-31.322	35.508	-27.769	-19.301	-21.835	-17.000	-24.637
164	-35.373	-31.256	35.373	-27.752	-19.353	-21.874	-17.056	-24.654
165	-35.247	-31.195	35.249	-27.738	-19.403	-21.913	-17.108	-24.672
166	-35.132	-31.143	35.131	-27.730	-19.453	-21.954	-17.158	-24.695
167	-35.013	-31.087	35.014	-27.719	-19.503	-21.994	-17.210	-24.716
168	-34.901	-31.036	34.902	-27.711	-19.552	-22.034	-17.260	-24.737
169	-34.790	-30.985	34.787	-27.702	-19.603	-22.074	-17.313	-24.758
170	-34.677	-30.930	34.676	-27.689	-19.652	-22.112	-17.366	-24.776
171	-34.564	-30.875	34.560	-27.677	-19.703	-22.150	-17.421	-24.793
172	-34.448	-30.817	34.449	-27.660	-19.751	-22.185	-17.476	-24.807
173	-34.335	-30.759	34.335	-27.643	-19.800	-22.220	-17.531	-24.820
174	-34.219	-30.700	34.221	-27.624	-19.848	-22.254	-17.587	-24.832
175	-34.107	-30.641	-24.843	-27.605	34.107	30.641	-22.286	-19.895
176	-34.106	-30.640	-24.843	-27.604	34.106	30.640	-22.287	-19.896
177	-33.992	-30.579	-24.851	-27.583	33.993	30.580	-22.316	-19.941
178	-33.880	-30.519	-24.858	-27.561	33.881	30.520	-22.346	-19.986
179	-33.770	-30.459	-24.864	-27.538	33.770	30.459	-22.373	-20.029
180	-33.661	-30.399	-24.870	-27.515	33.661	30.399	-22.400	-20.071
181	-33.554	-30.341	-24.875	-27.493	33.554	30.341	-22.426	-20.111
182	-33.452	-30.284	-24.878	-27.470	33.451	30.284	-22.449	-20.149
183	-33.348	-30.227	-24.881	-27.447	33.350	30.228	-22.472	-20.187
184	-33.253	-30.175	-24.886	-27.427	33.252	30.175	-22.495	-20.223
185	-33.159	-30.123	-24.888	-27.406	33.159	30.123	-22.515	-20.257

Table 118

FIG. 142

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	-3.564	-5.671	-2.596	-6.941	-12.671	-10.618	-14.638	-8.985
162	-3.775	-5.733	-2.552	-7.160	-12.793	-10.792	-14.757	-9.108
163	-3.899	-5.774	-2.533	-7.290	-12.873	-10.904	-14.841	-9.183
164	-3.977	-5.800	-2.521	-7.375	-12.931	-10.982	-14.907	-9.233
165	-4.034	-5.822	-2.516	-7.440	-12.982	-11.045	-14.964	-9.274
166	-4.073	-5.837	-2.510	-7.487	-13.025	-11.095	-15.016	-9.306
167	-4.107	-5.856	-2.511	-7.531	-13.070	-11.143	-15.068	-9.339
168	-4.133	-5.872	-2.511	-7.566	-13.112	-11.187	-15.118	-9.369
169	-4.156	-5.888	-2.512	-7.600	-13.157	-11.229	-15.170	-9.401
170	-4.177	-5.908	-2.518	-7.632	-13.203	-11.272	-15.222	-9.433
171	-4.198	-5.929	-2.524	-7.666	-13.253	-11.318	-15.277	-9.470
172	-4.220	-5.953	-2.533	-7.701	-13.305	-11.366	-15.333	-9.509
173	-4.242	-5.979	-2.543	-7.737	-13.359	-11.416	-15.390	-9.551
174	-4.265	-6.008	-2.555	-7.776	-13.416	-11.468	-15.449	-9.596
175	-7.818	-9.641	-13.473	-11.523	-6.035	-4.291	-15.508	-17.643
176	-7.819	-9.642	-13.474	-11.524	-6.035	-4.291	-15.509	-17.644
177	-7.860	-9.691	-13.532	-11.579	-6.070	-4.315	-15.568	-17.698
178	-7.903	-9.742	-13.592	-11.635	-6.104	-4.339	-15.626	-17.752
179	-7.947	-9.794	-13.651	-11.692	-6.140	-4.365	-15.685	-17.805
180	-7.992	-9.846	-13.711	-11.749	-6.177	-4.392	-15.743	-17.858
181	-8.037	-9.897	-13.768	-11.804	-6.213	-4.418	-15.798	-17.908
182	-8.081	-9.947	-13.823	-11.859	-6.249	-4.445	-15.852	-17.956
183	-8.126	-9.999	-13.879	-11.914	-6.286	-4.472	-15.906	-18.004
184	-8.165	-10.044	-13.930	-11.963	-6.318	-4.496	-15.955	-18.049
185	-8.206	-10.091	-13.980	-12.013	-6.352	-4.521	-16.003	-18.091

Table 119

FIG. 143

	Label							
	16	17	18	19	20	21	22	23
Design #								
161	27.855	24.636	31.523	21.769	14.639	16.849	12.670	19.184
162	27.798	24.627	31.405	21.799	14.757	16.934	12.794	19.246
163	27.769	24.637	31.322	21.836	14.841	17.000	12.873	19.302
164	27.751	24.654	31.255	21.874	14.906	17.056	12.931	19.353
165	27.740	24.675	31.197	21.914	14.964	17.108	12.981	19.404
166	27.729	24.694	31.142	21.953	15.016	17.157	13.025	19.452
167	27.721	24.717	31.089	21.995	15.068	17.209	13.069	19.503
168	27.711	24.737	31.037	22.034	15.118	17.260	13.112	19.553
169	27.699	24.755	30.982	22.072	15.170	17.312	13.158	19.601
170	27.689	24.775	30.930	22.111	15.221	17.365	13.203	19.652
171	27.673	24.791	30.872	22.148	15.277	17.421	13.254	19.702
172	27.660	24.808	30.818	22.186	15.333	17.476	13.305	19.752
173	27.644	24.822	30.760	22.221	15.390	17.531	13.359	19.800
174	27.625	24.833	30.701	22.254	15.449	17.587	13.415	19.848
175	22.286	19.895	15.508	17.643	24.843	27.605	13.473	11.523
176	22.286	19.895	15.508	17.643	24.843	27.604	13.473	11.523
177	22.317	19.942	15.568	17.698	24.852	27.584	13.532	11.578
178	22.346	19.986	15.627	17.753	24.859	27.562	13.592	11.635
179	22.374	20.029	15.685	17.806	24.864	27.539	13.652	11.692
180	22.400	20.071	15.742	17.858	24.869	27.515	13.710	11.749
181	22.425	20.111	15.798	17.908	24.874	27.493	13.768	11.805
182	22.450	20.150	15.852	17.957	24.879	27.471	13.824	11.859
183	22.473	20.188	15.906	18.004	24.882	27.448	13.879	11.913
184	22.495	20.223	15.955	18.048	24.885	27.427	13.930	11.964
185	22.516	20.257	16.004	18.092	24.889	27.407	13.981	12.013

Table 120

FIG. 144

	Label							
	24	25	26	27	28	29	30	31
Design #								
161	0.424	2.591	-0.428	3.561	8.983	6.939	10.617	5.667
162	0.573	2.552	-0.572	3.776	9.108	7.160	10.793	5.734
163	0.655	2.532	-0.655	3.899	9.183	7.290	10.904	5.774
164	0.707	2.522	-0.706	3.977	9.233	7.376	10.982	5.801
165	0.740	2.513	-0.742	4.032	9.272	7.438	11.044	5.820
166	0.767	2.511	-0.766	4.073	9.306	7.488	11.095	5.838
167	0.784	2.509	-0.786	4.106	9.338	7.530	11.143	5.855
168	0.799	2.510	-0.799	4.133	9.369	7.566	11.186	5.871
169	0.812	2.516	-0.809	4.159	9.403	7.602	11.231	5.891
170	0.820	2.519	-0.819	4.178	9.434	7.633	11.272	5.908
171	0.829	2.527	-0.826	4.201	9.472	7.668	11.320	5.931
172	0.833	2.533	-0.834	4.220	9.509	7.701	11.366	5.953
173	0.839	2.542	-0.841	4.241	9.550	7.737	11.416	5.979
174	0.845	2.554	-0.846	4.264	9.595	7.776	11.468	6.007
175	0.855	2.563	6.035	4.291	-0.856	-2.563	7.818	9.641
176	0.856	2.564	6.036	4.292	-0.855	-2.562	7.819	9.641
177	0.859	2.577	6.069	4.314	-0.860	-2.578	7.859	9.691
178	0.864	2.593	6.104	4.339	-0.865	-2.594	7.902	9.742
179	0.869	2.610	6.140	4.365	-0.870	-2.610	7.947	9.794
180	0.875	2.627	6.177	4.392	-0.874	-2.626	7.993	9.846
181	0.880	2.643	6.213	4.419	-0.880	-2.642	8.037	9.898
182	0.885	2.658	6.248	4.444	-0.886	-2.660	8.080	9.947
183	0.890	2.675	6.285	4.471	-0.891	-2.676	8.125	9.998
184	0.896	2.691	6.319	4.496	-0.896	-2.691	8.166	10.045
185	0.900	2.705	6.352	4.520	-0.902	-2.707	8.206	10.091

Table 121

FIG. 145

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
63	7	2.79	2.51	2.26	2.03	1.48	0
64	7.2	2.88	2.59	2.33	2.1	1.38	0
65	7.4	2.75	2.47	2.23	2	1.31	0
66	7.6	2.74	2.47	2.22	2	1.31	0
67	7.8	2.74	2.47	2.22	2	1.31	0
68	8	2.72	2.45	2.2	1.98	1.3	0
69	8.2	2.7	2.43	2.18	1.96	1.29	0
70	8.4	2.66	2.39	2.15	1.94	1.27	0
71	8.6	2.62	2.36	2.12	1.91	1.25	0
72	8.8	2.56	2.3	2.07	1.87	1.22	0
73	8.99	2.59	1.28	1.15	1.04	0.93	0
74	9	2.59	1.28	1.15	1.04	0.93	0
75	9.2	2.57	1.25	1.13	1.02	0.91	0
76	9.4	2.54	1.23	1.1	0.99	0.89	0
77	9.6	2.51	1.2	1.08	0.97	0.87	0
78	9.8	2.48	1.18	1.06	0.95	0.86	0
79	10	2.39	2.15	1.93	1.74	1.14	0
80	10.2	2.13	1.92	1.73	1.56	1.13	0
81	10.4	2.31	2.08	1.87	1.69	1.11	0
82	10.6	2.26	2.03	1.83	1.64	1.08	0
83	10.8	2.2	1.98	1.78	1.6	1.17	0
84	11	2.19	1.97	1.77	1.59	1.05	0
85	11.2	2.15	1.93	1.74	1.56	1.03	0
86	11.4	2.1	1.89	1.7	1.53	1.12	0
87	11.6	2.05	1.84	1.66	1.49	1.09	0
88	11.8	2.2	1.98	1.78	1.61	1.05	0
89	12	2.13	1.91	1.72	1.55	1.02	0
90	12.2	2.04	1.84	1.65	1.49	1.09	0
91	12.21	2.04	1.83	1.65	1.49	1.08	0
92	12.4	2.17	1.96	1.76	1.58	1.04	0
93	12.6	2.12	1.91	1.72	1.39	1.02	0
94	12.8	2.07	1.87	1.68	1.51	0.99	0
95	13	2.02	1.82	1.64	1.47	0.97	0
96	13.2	1.96	1.76	1.59	1.43	1.04	0
97	13.4	1.89	1.7	1.53	1.38	1.01	0
98	13.6	1.83	1.65	1.48	1.33	0.97	0
99	13.8	1.76	1.58	1.42	1.28	0.93	0
100	14	1.89	1.7	1.53	1.38	1	0
101	14.2	1.85	1.66	1.5	1.35	0.88	0
102	14.4	1.81	1.63	1.47	1.32	0.96	0

Table 122

FIG. 146

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
103	14.6	1.77	1.59	1.43	1.29	0.94	0
104	14.8	1.72	1.55	1.39	1.25	0.91	0
105	15	1.86	1.67	1.5	1.35	0.89	0
106	15.2	1.81	1.63	1.47	1.32	0.87	0
107	15.27	1.8	1.62	1.46	1.32	0.86	0
108	15.4	1.79	1.61	1.45	1.17	0.86	0
109	15.6	1.58	1.43	1.28	1.15	0.84	0
110	15.8	1.72	1.55	1.4	1.26	0.82	0
111	16	1.54	1.38	1.25	1.12	0.82	0
112	16.2	1.53	1.37	1.24	1.11	0.81	0
113	16.4	1.51	1.36	1.22	1.1	0.8	0
114	16.6	1.65	1.49	1.34	1.08	0.79	0
115	16.8	1.62	1.46	1.31	1.18	0.77	0
116	17	1.57	1.41	1.27	1.14	0.75	0
117	17.2	1.55	1.39	1.25	1.02	0.74	0
118	17.4	1.52	1.37	1.23	1	0.73	0
119	17.6	1.34	1.21	1.09	0.98	0.71	0
120	17.8	1.46	1.31	1.18	1.06	0.7	0
121	18	1.27	1.14	1.03	0.92	0.67	0
122	18.2	1.36	1.22	1.1	0.99	0.65	0
123	18.4	1.31	1.18	1.06	0.95	0.63	0
124	18.42	1.3	1.17	1.05	0.95	0.62	0
125	18.6	1.25	1.12	1.01	0.91	0.66	0
126	18.8	1.3	1.17	1.05	0.95	0.56	0
127	19	1.2	1.08	0.97	0.88	0.57	0
128	19.2	1.18	1.06	0.96	0.86	0.57	0
129	19.4	1.1	0.99	0.89	0.81	0.59	0
130	19.6	1.14	1.02	0.92	0.83	0.54	0
131	19.8	1.15	1.04	0.93	0.84	0.55	0
132	20	1.04	0.94	0.84	0.76	0.55	0
133	20.2	1.03	0.93	0.84	0.75	0.49	0
134	20.4	1.05	0.94	0.85	0.69	0.5	0
135	20.6	1.05	0.95	0.85	0.69	0.5	0
136	20.8	0.95	0.86	0.77	0.69	0.51	0
137	21	0.95	0.86	0.77	0.7	0.46	0
138	21.2	0.95	0.86	0.77	0.63	0.46	0
139	21.4	0.95	0.86	0.77	0.62	0.45	0
140	21.52	0.85	0.77	0.69	0.62	0.45	0
141	21.6	0.85	0.77	0.69	0.62	0.45	0
142	21.8	0.85	0.76	0.68	0.62	0.45	0
143	22	0.78	0.71	0.64	0.57	0.42	0

Table 123

FIG. 147

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
144	22.2	0.81	0.73	0.66	0.53	0.39	0
145	22.4	0.75	0.68	0.61	0.55	0.36	0
146	22.6	0.78	0.7	0.63	0.57	0.37	0
147	22.8	0.72	0.65	0.59	0.53	0.35	0
148	23	0.67	0.6	0.54	0.49	0.36	0
149	23.2	0.69	0.62	0.56	0.5	0.33	0
150	23.4	0.64	0.57	0.52	0.47	0.34	0
151	23.6	0.66	0.59	0.53	0.43	0.31	0
152	23.8	0.61	0.55	0.49	0.44	0.29	0
153	24	0.56	0.5	0.45	0.41	0.3	0
154	24.2	0.57	0.52	0.46	0.42	0.27	0
155	24.4	0.53	0.48	0.43	0.39	0.28	0
156	24.6	0.54	0.49	0.44	0.39	0.26	0
157	24.79	0.5	0.45	0.4	0.36	0.27	0
158	24.8	0.5	0.45	0.41	0.36	0.24	0
159	25	0.46	0.42	0.38	0.34	0.25	0
160	25.2	0.48	0.43	0.39	0.35	0.25	0
161	25.4	0.44	0.4	0.36	0.32	0.23	0
162	25.6	0.45	0.41	0.37	0.33	0.22	0
163	25.8	0.42	0.38	0.34	0.3	0.22	0
164	26	0.38	0.34	0.31	0.28	0.2	0
165	26.2	0.38	0.34	0.31	0.28	0.2	0
166	26.4	0.37	0.33	0.3	0.27	0.2	0
167	26.6	0.36	0.32	0.29	0.26	0.19	0
168	26.8	0.35	0.32	0.28	0.26	0.19	0
169	27	0.34	0.31	0.28	0.25	0.16	0
170	27.2	0.3	0.27	0.24	0.22	0.16	0
171	27.4	0.29	0.26	0.23	0.21	0.15	0
172	27.6	0.28	0.25	0.23	0.2	0.15	0
173	27.8	0.27	0.24	0.22	0.2	0.14	0
174	28	0.26	0.23	0.21	0.19	0.14	0
175	28.2	0.25	0.23	0.2	0.18	0.13	0
176	28.2	0.25	0.23	0.2	0.18	0.13	0
177	28.4	0.24	0.22	0.2	0.18	0.13	0
178	28.6	0.21	0.19	0.17	0.15	0.11	0
179	28.8	0.22	0.2	0.18	0.16	0.12	0
180	29	0.19	0.17	0.16	0.14	0.1	0
181	29.2	0.18	0.17	0.15	0.13	0.1	0
182	29.4	0.2	0.18	0.16	0.13	0.09	0
183	29.6	0.17	0.15	0.14	0.12	0.09	0
184	29.8	0.16	0.15	0.13	0.12	0.09	0
185	30	0.15	0.14	0.12	0.11	0.08	0

Table 124

FIG. 148

METHODS AND APPARATUSES FOR SIGNALING WITH GEOMETRIC CONSTELLATIONS

CROSS-REFERENCE TO RELATED APPLICATIONS

The present invention is a continuation application of U.S. patent application Ser. No. 15/826,579 filed Nov. 29, 2017 entitled "Methods and Apparatuses for Signaling with Geometric Constellations" and issued as U.S. Pat. No. 10,530,629, which application is a continuation application of U.S. patent application Ser. No. 13/608,838 filed Sep. 10, 2012 entitled "Methods and Apparatuses for Signaling with Geometric Constellations" and issued as U.S. Pat. No. 9,887,870 on Feb. 6, 2018, which application is a continuation application of U.S. patent application Ser. No. 12/650,532 filed Dec. 30, 2009 entitled "Methods and Apparatuses for Signaling with Geometric Constellations" and issued as U.S. Pat. No. 8,265,175 on Sep. 11, 2012, which application claims priority as a Continuation-In-Part to U.S. patent application Ser. No. 12/156,989 filed Jun. 5, 2008 entitled "Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellation" and issued as U.S. Pat. No. 7,978,777 on Jul. 12, 2011, which claims priority to U.S. Provisional Application Ser. No. 60/933,319 filed Jun. 5, 2007 entitled "New Constellations for Communications Signaling: Design Methodology and Method and Apparatus for the New Signaling Scheme" to Barsoum et al. U.S. patent application Ser. No. 12/650,532 also claims priority to U.S. Provisional Application Ser. No. 61/141,662 filed Dec. 30, 2008 and U.S. Provisional Application Ser. No. 61/141,935 filed Dec. 31, 2008, both of which are entitled "PAM-8, 16, 32 Constellations Optimized for Joint and PD Capacity" and are to Barsoum et al. The disclosure of U.S. patent application Ser. Nos. 15/826,579, 13/608,838, 12/650,532, 12/156,989 and U.S. Provisional Application Nos. 60/933,319, 61/141,662 and 61/141,935 are expressly incorporated by reference herein in its entirety.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempt to mimic a Gaussian distribution.

Other approaches increase the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a PAM-8 symbol constellation having constellation points within at least one of the ranges specified in FIGS. **25-46**.

In a further embodiment, the code is a Turbo code. In another embodiment, the code is a LDPC code.

In a still further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

In still another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 15% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

In a yet further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 30% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

In yet another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 45% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

A further embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

In another embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

In a further additional embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

In another additional embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 40% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

In a still yet further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 50% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

In still yet another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

In a still further embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 70% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

In still another embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

A still further additional embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a PAM-16 symbol constellation having constellation points within at least one of the ranges specified in FIGS. **47-84**.

Still another additional embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a PAM-32 symbol constellation having constellation points within at least one of the ranges specified in FIGS. **85-148**.

Another further embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a N-Dimensional symbol constellation, where the constellation points in at least one dimension are within at least one of the ranges specified in FIGS. **25-167**.

BRIEF DESCRIPTION OF DRAWINGS

FIG. **1** is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

FIG. **2** is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

FIG. **3** is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

FIG. **4*a*** is a conceptual illustration of the joint capacity of a channel.

FIG. **4*b*** is a conceptual illustration of the parallel decoding capacity of a channel.

FIG. **5** is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

FIG. **6*a*** is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

FIG. **6*b*** is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8, 16,32.

FIG. **7** is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2,4,8,16,32 constellations.

FIG. **8*a*** is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **8***b* is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

FIG. **9** is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

FIGS. **10***a*-**10***d* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **11***a* and **11***b* are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

FIGS. **12***a*-**12***d* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **13***a* and **13***b* are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

FIGS. **14***a*-**14***d* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **15***a* and **15***b* are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

FIGS. **16***a*-**16***d* are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

FIGS. **17***a* and **17***b* are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

FIG. **18** is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

FIG. **19** is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

FIG. **20** is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

FIG. **21** illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

FIGS. **22***a* and **22***b* are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **23***a* and **23***b* are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **24***a* and **24***b* are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

FIGS. **25-28** are tables showing the performance of geometric PAM-8 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **29-32** are tables listing the constellation points corresponding to the geometric PAM-8 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **25-28**.

FIGS. **33-36** are tables showing maximum ranges for the geometric PAM-8 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **25-28**.

FIGS. **37-40** are tables showing the performance of geometric PAM-8 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **41-44** are tables listing the constellation points corresponding to the geometric PAM-8 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. **37-40**.

FIGS. **45-46** are tables showing maximum ranges for the geometric PAM-8 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. **37-40**.

FIGS. **47-51** are tables showing the performance of geometric PAM-16 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **52-61** are tables listing the constellation points corresponding to the geometric PAM-16 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **47-51**.

FIGS. **62-66** are tables showing maximum ranges for the geometric PAM-16 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **47-51**.

FIGS. **67-71** are tables showing the performance of geometric PAM-16 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **72-81** are tables listing the constellation points corresponding to the geometric PAM-16 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. **67-71**.

FIGS. **82-84** are tables showing maximum ranges for the geometric PAM-16 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. **67-71**.

FIGS. **85-90** are tables showing the performance of geometric PAM-32 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **91-114** are tables listing the constellation points corresponding to the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **85-90**.

FIGS. **115-120** are tables showing maximum ranges for the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **85-90**.

FIGS. **121-125** are tables showing the performance of geometric PAM-32 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

FIGS. **126-145** are tables listing the constellation points corresponding to the geometric PAM-32 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. **121-125**.

FIGS. **146-148** are tables showing maximum ranges for the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. **121-125**.

DETAILED DESCRIPTION OF THE INVENTION

Turning now to the detailed description of the invention, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In many embodiments, the communication systems use specific geometric constellations that are capacity optimized at a specific SNR. In addition, ranges within which the constellation points of a capacity optimized constellation can be perturbed and are still likely to achieve a given percentage of the optimal capacity increase compared to a constellation that maximizes d_{min}, are also described. Capacity measures that are used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. Constellations points for geometric PAM-8, PAM-16, and PAM-32 constellations that are optimized for joint capacity or PD capacity at specific SNRs are also provided. Additional geometric PAM-8, PAM-16, and PAM-32 constellations that are probabilistically likely to provide performance gains compared to constellations that maximize d_{min}, which were identified by perturbing the constellation points of geometric PAM-8, PAM-16, and PAM-32 constellations optimized for joint capacity or PD capacity, are also described. The constellations are described as being probabilistically likely to provide performance gains, because all possible constellations within the ranges have not been exhaustively searched. Within each disclosed range, a large number of constellations were selected at random, and it was verified that all the selected constellations provided a gain that exceeds the given percentage of the optimal capacity increase achieved by the optimized constellations relative to a constellation that maximizes d_{min} (i.e. a PAM equally spaced constellation). In this way, ranges that are probabilistically likely to provide a performance gain that is at least a predetermined percentage of the optimal increase in capacity can be identified and a specific geometric constellation can be compared against the ranges as a guide to the increase in capacity that is likely to be achieved. In a number of embodiments, the communication systems can adapt the location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

Communication Systems

A communication system in accordance with an embodiment of the invention is shown in FIG. 1. The communication system 10 includes a source 12 that provides user bits to a transmitter 14. The transmitter transmits symbols over a channel to a receiver 16 using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

A transmitter in accordance with an embodiment of the invention is shown in FIG. 2. The transmitter 14 includes a coder 20 that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper 22 is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator 24, which modulates the symbols for transmission via the channel.

A receiver in accordance with an embodiment of the invention is illustrated in FIG. 3. The receiver 16 includes a demodulator 30 that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper 32 provides the likelihoods to a decoder 34 that decodes the encoded bit stream to provide a sequence of received bits to a sink.

Geometrically Shaped Constellations

Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. In many embodiments, geometrically shaped symbol constellations, which include constellation points within predetermined ranges of the constellation points of a capacity optimized constellation, and that provide improved capacity compared to constellations that maximize d_{min} are used. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

Selection of a Geometrically Shaped Constellations

Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

Upgrading Existing Communication Systems

A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Another approach is to select a constellation from a range that is probabilistically likely to yield a constellation having at least a predetermined percentage of the optimal capacity increase. Such an approach can prove useful in circumstances, for example, where an optimized constellation is unable to be implemented. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

Joint Capacity and Parallel Decoding Capacity

A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. 4*a*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **40**. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{l-1} I(X_i; Y)$$

where X_i is the ith bit of the l-bits transmitted symbol, and Y is the received symbol, and I(A;B) denotes the mutual information between random variables A and B.

Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4*b*. The portions of the communication system that are considered part of the channel are indicated by the ghost line **42**. The joint capacity of the channel is given by:

$$C_{JOINT}=I(X;Y)$$

Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

Selecting a Constellation Having an Optimal Capacity

Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. **5**. The process **50** commences with the selection (**52**) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (**52**) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known a priori. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (**56**) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

The SNR at which the optimized M-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (**57**). A determination (**58**) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (**55**). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellations at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

The process illustrated in FIG. **5** can maximize PD capacity or joint capacity of an M-ary constellation for a given SNR. Although the process illustrated in FIG. **5** shows selecting an M-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an M-ary constellation where the SNR gap to

Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. For example, the effect of perturbations on the constellation points of optimized constellations can be used to identify ranges in which predetermined performance improvements are probabilistically likely to be obtained. The ranges can then be used to select geometrically shaped constellations for use in a communication system. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q), or with multi-stage decoding. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

Optimizing the Capacity of an M-ary Constellation at a Given Snr

Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process **56** shown in FIG. **5**, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged.

The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. **5**.

Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

Gains Achieved by Optimized Geometrically Spaced Constellations

The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G=\tfrac{1}{2}\log_2(1+\mathrm{SNR})$$

Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. **6a** and **6b**. FIG. **6a** includes a chart **60** showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps **62** exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. **6b** includes a chart **64** showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}, Gaps **66** exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

In order to gain a better view of the differences between the curves shown in FIGS. **6a** and **6b** at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. **7**. It is interesting to note from the chart **70** in FIG. **7** that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

FIGS. **8a** and **8b** summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. **8a** shows the SNR gap for each constellation optimized for PD capacity. FIG. **8b** shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

Referring to FIG. **8a**, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of ½. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate ½ LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. **9**. A chart **90** includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. **8***a*.

Capacity Optimized Pam Constellations

Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. **10***a*-**17***b*. The locus plots and design tables show PAM-4, 8, 16, and 32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

In FIG. **10***a*, a locus plot **100** shows the location of the points of PAM-4 constellations optimized for joint capacity plotted against achieved capacity. A similar locus plot **105** showing the location of the points of joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. **10***b*. In FIG. **10***c*, the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. **10***d* the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the joint capacity optimized constellations have **3**. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. **11***a* and FIG. **11***b* where vertical slices of FIGS. **10***ab* and **10***cd* are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities={0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4)=2$ bps, these design points represent systems with information code rates R={¼, ⅜, ½, ⅝, ¾} respectively.

FIGS. **12***ab* and **12***cd* present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. **13***a* and **13***b* provide slices from these plots at SNRs corresponding to achievable capacities η={0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate R=η bps/$\log_2(8)$, resulting in R={⅙, ⅓, ½, ⅔, ⅚}. As an example of the relative performance of the constellations in these tables, consider FIG. **13***b* which shows a PD capacity optimized PAM-8 constellation optimized for SNR=9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. **8***a* and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=½). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

Similar information is presented in FIGS. **14***a*-**14***d*, and **15***a*-**15***b* which provide loci plots and design tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. **16***a*-**16***d*, **17***a* and **17***b* provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

Capacity Optimized PSK Constellations

Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart **180** comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. **18** where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4, 8, 16, and 32 and for PD capacity optimized PSK-4, 8, 16, and 32.

The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. **19**, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. **20**. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR=8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR=14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR=2 and 15 dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

We note now that the locus of points for PD optimized PSK-32 in FIG. **19** in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. **18** implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8, and 16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. **19**. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate ⅘ to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to ⅕ to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. **18**.

Adaptive Rate Design

In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation/code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper/decoder configuration in order to match the new transmit-side configuration.

Conversely, the receiver could implement a quality of performance based optimized constellation/code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation/code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper/decoder modules.

Geometrically Shaped QAM Constellations

Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM in phase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

In FIG. **21** we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. **13***b*). The label-point pairs in this PAM-8 constellation are {(000, −1.72), (001, −0.81), (010, 1.72), (011, −0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, −0.02)}. Examination of FIG. **21** shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8×8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-Dimensional Constellation Optimization

Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

Capacity Optimized Constellations for Fading Channels

Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y=a(t)\cdot X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and a(t) is the fading distribution, which is a function of time.

In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

Many fading channels follow a Rayleigh distribution. FIGS. **22***a***-24***b* are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

Geometric PAM-8, PAM-16, and PAM-32 Constellations

As described above, geometric constellations can be obtained that are optimized for joint or PD capacity at specific SNRs. In addition, ranges can be specified for the constellation points of a geometric constellation that are probabilistically likely to result in geometric constellations that provide at least a predetermined performance improvement relative to a constellation that maximizes d_{min}. Turning now to FIGS. **25-167**, geometric PAM-8, PAM-16, and PAM-32 constellations optimized for joint and PD capacity over the Additive White Gaussian Noise (AWGN) channel at specific SNRs are listed. The performances of the optimal constellations are compared to the performances of traditional constellations that maximize d_{min}. Ranges for the constellation points are also defined at specific SNRs, where constellations having constellation points selected from within the ranges are probabilistically likely (with probability close to one) to result in at least a predetermined performance improvement at the specified SNR relative to a traditional constellation that maximizes d_{min}.

The geometric constellations disclosed in FIGS. **25-167** are defined by points y(i) such that y(i)=k(x(i)+r(i))+c. Values for x(i) and bounds on r(i) are provided in FIGS. **25-167** for PAM-8, PAM-16, and PAM-32 optimized for joint and PD capacity. For PAM-8 0≤i≤7, PAM-16 0≤i≤15, and for PAM-32 0≤i≤31. To achieve optimal power efficiency, c should be set to zero. In addition to optimized constellations, FIGS. **25-167** specify ranges for the points of a geometric constellation, where selecting the points of a constellation from within the ranges is probabilistically likely to provide a geometric constellation having at least a predetermined performance improvement relative to a constellation that maximizes d_{min}. The ranges are expressed as a maximum value for the constellation range parameter, r(i), which specifies the amount by which the point x(i) in the constellation is perturbed relative to the location of the corresponding point in the optimal constellation. A communication system using a constellation formed from constellations points selected from within the ranges specified by the maximum value (i.e. $-r_{max}\leq r(i)\leq r_{max}$) is probabilistically likely to achieve a predetermined performance improvement relative to a constellation that maximizes d_{min}. The predetermined performance improvements associated with the ranges specified in FIGS. **25-167** are expressed in terms of a percentage of the increase in capacity achieved by the optimized constellation relative to a constellation that maximizes d_{min}. Constellations formed from constellation points selected from within the ranges are probabilistically likely to achieve an increase in capacity at least as great as the indicated percentage.

With regard to the specific tables shown in FIGS. **25-167**, each table is one of three different types of table. A first set of tables shows the performance of specific geometric constellations optimized for joint capacity or PD capacity. These tables include 6 columns. The first column enumerates a design number. The second column provides the SNR at which the constellation was optimized for the design defined by the entry in the first column. The third column provides the capacity achieved by the optimized constellation (Opt. Cap) at the SNR given in the second column. The fourth column provides the capacity achieved (Std. Cap) by a traditional uniformly spaced constellation i.e. a PAM constellation that maximizes d_{min} (with the same number of points as the optimized constellation and where binary reflective gray labeling is assumed) at the SNR given in the second column. The fifth column shows the gain in bits per transmission provided by the optimized constellation over a constellation that maximizes d_{min}. The sixth column shows the percentage gain in capacity provided by the optimized constellation over the capacity provided by the traditional uniformly spaced constellation.

A second set of tables lists the constellation points of the designs indicated in the first set of tables. These tables contain 9 columns. The first column enumerates a design number. The remaining 8 columns describe a constellation point x(i) enumerated by label in the second row of the table. Labels are given in decimal number format. With PAM 8 as an example, a label of 011 is given as the decimal number 3.

The third set of tables specifies maximum perturbation ranges for the capacity optimized constellations indicated in the first set of tables, where the maximum ranges correspond to a high probabilistic likelihood of at least a predetermined performance improvement relative to a constellation that maximizes d_{min}. These tables contain 8 columns. The first enumerates a design number (corresponding to a design from one of the aforementioned tables). The second column provides the SNR for the design defined by the entry in the first column. The remaining 5 columns describe parameter r_{max} which is the maximum amount any point in the designed constellation may be perturbed (in either the positive or negative direction) and still retain, with probability close to unity, at least the gain noted by each column header of the joint or PD capacity as a percentage of the gain provided by the corresponding optimized point design over a traditional constellation that maximizes d_{min} (all at the given SNR). Each table has a last column showing that if 100% of the gain afforded by the optimized constellation is desired, then parameter r(i) must be equal to zero (no deviation from designed points described in the point specification tables).

Example of Performance Achieved by Constellation within Predetermined Ranges

By way of example, a constellation can be selected using the ranges specified with respect to the constellation points of a geometric PAM-8 constellation optimized with respect to PD capacity at SNR=9 dB. The optimized constellation points are as follows:

−7.8780	−3.7100	7.8780	−2.8590	2.8590	0.0990	3.7100	−0.0990

The PD capacity of the above constellation at 9 dB=1.4999 bits. FIGS. **26-167** define a range around each constellation r_{max} of 0.47 that is probabilistically likely to result in a constellation that can be used by a communication system to achieve at least 5% of the gain of the optimized constellation (compared to an equally spaced constellation).

An example of a PAM-8 constellation formed using constellation points selected from within the specified ranges is as follows:

−7.8462	−3.9552	7.7361	−3.2614	2.9395	0.5152	3.3867	0.0829

The distance between each of the constellation points and the constellation points of the optimized constellation are as follows:

0.0318	−0.2452	−0.1419	−0.4024	0.0805	0.4162	−0.3233	0.1819

The magnitude of each of the distances is less than r_{max} at 9 dB (i.e. 0.47). The capacity of the selected constellation=1.4884. The capacity of a constellation that maximizes d_{min} at 9 dB=1.435 bits. Therefore, the selected constellation achieves 82% of the gain made possible by the optimal constellation (i.e. at least 5%).

Labelling of Constellations using Cyclically Rotated Binary Reflective Gray Labels

In performing optimization with respect to PD capacity, a conjecture can be made that constraining the optimization process to the subsequently described class of labelings results in no or negligible loss in PD capacity (the maximum observed loss is 0.005 bits, but in many cases there is no loss at all). Use of this labeling constraint speeds the optimization process considerably. We note that joint capacity optimization is invariant to choice of labeling. Specifically, joint capacity depends only on point locations whereas PD capacity depends on point locations and respective labelings.

The class of cyclically rotated binary reflective gray labels can be used. The following example, using constellations with cardinality 8, describes the class of cyclically rotated binary reflective gray labels. Given for example the standard gray labeling scheme for PAM-8:

000, 001, 011, 010, 110, 111, 101, 100

Application of a cyclic rotation, one step left, yields:

001, 011, 010, 110, 111, 101, 100, 000

Application of a cyclic rotation, two steps left, yields:

011, 010, 110, 111, 101, 100, 000, 001

For a constellation with cardinality 8, cyclic rotations of 0 to 7 steps can be applied. It should be noted that within this class of labelings, some labelings perform better than others. It should also be noted that different rotations may yield labelings that are equivalent (through trivial column swapping and negation operations). In general, labelings can be expressed in different but equivalent forms through trivial operations such as column swapping and negation operations. For example the binary reflective gray labels with one step rotation:

001, 011, 010, 110, 111, 101, 100, 000

Can be shown to be equivalent to:

000, 001, 011, 111, 101, 100, 110, 010

The above equivalence can be shown by the following steps of trivial operations:

1) Negate the third column. This gives

000, 010, 011, 111, 110, 100, 101, 001

2) Swap the second and third columns. This gives

000, 001, 011, 111, 101, 100, 110, 010

The two labelings are considered equivalent because they yield the same PD Capacity as long as the constellation points locations are the same.

In the constellation point specifications shown in FIGS. **25-167**, a labeling can be interchanged by any equivalent labeling without affecting the performance parameters. A labeling used in the specifications may not directly appear to be a cyclically rotated binary reflective gray labeling, but it can be shown to be equivalent to one or more cyclically rotated binary reflective gray labelings.

Prior Art Geometric Constellations

Geometric constellations have been specified in the prior art in attempts to achieve performance gains relative to constellations that maximize drain. Examples of such constellations are disclosed in Sommer and Fettweis, "Signal Shaping by Non-Uniform QAM for AWGN Channerls and Applications Using Turbo Coding" *ITG Conference Source and Channel Coding*, p. 81-86, 2000. The specific constellations disclosed by Sommer and Fettweis for PAM-8, PAM-16, and PAM-32 are as follows:

				PAM-8:				
−1.6630	−0.9617	−0.5298	−0.1705	0.1705	0.5298	0.9617	1.6630	
				PAM-16:				
−1.9382	−1.3714	−1.0509	−0.8079	−0.6026	−0.4185	−0.2468	−0.0816	0.0816
0.2468	0.4185	0.6026	0.8079	1.0509	1.3714	1.9382		
				PAM-32:				
−2.1970	−1.7095	−1.4462	−1.2545	−1.0991	−0.9657	−0.8471	−0.7390	−0.6386
−0.5441	−0.4540	−0.3673	−0.2832	−0.2010	−0.1201	−0.0400	0.0400	0.1201
0.2010	0.2832	0.3673	0.4540	0.5441	0.6386	0.7390	0.8471	0.9657
1.0991	1.2545	1.4462	1.7095	2.1970				

Another class of geometric constellations is disclosed in Long et al., "Approaching the AWGN Channel Capacity without Active Shaping" *Proceedings of International Symposium on Information Theory*, p. 374, 1997. The specific PAM-8, PAM-16, and PAM-32 constellations disclosed by Long et al. are as follows:

							PAM-8:								
−3	−1	−1	−1	1	1	1	3								
							PAM-16:								
−4	−2	−2	−2	−2	0	0	0	0	0	0	2	2	2	2	4
							PAM-32:								
−5	−3	−3	−3	−3	−3	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	5

The above prior art constellations are geometric and can provide performance improvements at some SNRs relative to constellations that maximize d_{min}. The performance of the constellations varies with SNR and at certain SNRs the constellations are proximate to capacity optimized constellations. Therefore, the ranges specified in FIGS. **25-167** are defined so that prior art constellations are excluded at the specific SNRs at which these constellations are proximate to a capacity optimized constellation.

Constructing Multidimensional Constellations

The tables shown in FIGS. **25-167** can be used to identify optimal N-dimensional constellations. The optimized multi-dimensional constellation can be determined by finding the Cartesian power X^n and the resulting labeling constructed by finding the corresponding Cartesian power of L^n. Ranges within which the multi-dimensional constellation points can be selected (i.e. perturbed), can then be defined with respect to each constellation point of the constructed multi-dimensional constellation, using an n-dimensional perturbation vector, such that each component of the perturbation vector has a magnitude that is less than r_{max} defined by the range tables.

Example of a QAM Constellation

The optimized constellation points for a PAM-8 constellation optimized for PD capacity at SNR=9 dB are as follows:

−7.8780	−3.7100	7.8780	−2.8590	2.8590	0.0990	3.7100	−0.0990

The labelings corresponding to the above PAM-8 constellation points are:

000 001 010 011 100 101 110 111

Using this PAM-8 constellation, it is possible to construct a QAM-64 constellation. While PAM-8 maps 3 bits to one dimension, QAM-64 maps 6 bits to two dimensions. The first three bits will determine the location in the X-dimension and the second three bits will determine the location in the Y-dimension. The resulting QAM-64 constellation for example will map the bits 000 010 to the two dimensional constellation point (−7.878, 7.878), and 111 110 to the two dimensional constellation point (−0.099, 3.71). The points corresponding to the remaining labels can be derived in a similar manner.

The ranges shown in FIGS. **25-167** can be utilized to select QAM constellations in a similar manner to that outlined above with respect to the selection of a PAM-8 constellation based upon ranges specified with respect to a PAM-8 constellation optimized for PD capacity at 9 dB. A range of 0.47 can be applied to every component of each two dimensional constellation point. For example, the two points two points (−7.787, 8.078) and (0.201, 3.31) are within the ranges as they are spaced distances (−0.1, 0.2) and (0.3, −0.4) respectively from the optimized constellation points. In this way, the ranges can be used to identify constellations that are probabilistically likely to result in a performance improvement relative to a constellation that maximizes d_{min}.

The same procedure can apply to a constellation optimized for joint capacity. However, the choice of labeling does not affect joint capacity. The above procedure can similarly be applied to an N-dimensional constellation constructed from a PAM constellation.

Although the present invention has been described in certain specific embodiments, many additional modifications and variations would be apparent to those skilled in the art. It is therefore to be understood that the present invention may be practiced otherwise than specifically described, without departing from the scope and spirit of the present invention. Thus, embodiments of the present invention should be considered in all respects as illustrative and not restrictive.

What is claimed is:

1. A digital communication system, comprising:

a transmitter configured to transmit signals via a communication channel;

wherein the transmitter comprises:

a coder capable of receiving bits and outputting encoded bits using a Low Density Parity Check (LDPC) code;

a mapper, coupled to the coder, capable of mapping the encoded bits to symbols in a non-uniform quadrature amplitude modulation 1024-point symbol constellation (NU-QAM 1024); and

a modulator, coupled to the mapper, capable of producing a signal for transmission via the communication channel based upon symbols selected by the mapper;

wherein the NU-QAM 1024 constellation comprises an in-phase component and a quadrature component, where each component comprises 32 levels of amplitude such that the amplitudes scaled by a scaling factor are within 0.55 from the following set of amplitudes: −38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

2. The digital communication system of claim 1, where each of the in-phase and quadrature components of the NU-QAM 1024 constellation comprises 32 levels of amplitude such that the amplitudes scaled by the scaling factor are from the following set of amplitudes: −38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

3. The digital communication system of claim 1, wherein the LDPC code rate has a code rate that is equal to or less than 0.65086.

4. The digital communication system of claim 1, wherein the transmitter is configured to select the NU-QAM 1024 constellation from a plurality of symbol constellations.

5. The digital communication system of claim 4, wherein the NU-QAM 1024 constellation is characterized in that the NU-QAM 1024 constellation provides greater parallel decode capacity at a specific signal-to-noise ratio (SNR) compared to a QAM 1024 constellation that maximizes d_{min} at the specific SNR.

6. The digital communication system of claim 4, wherein the NU-QAM 1024 constellation is characterized in that selection of the NU-QAM 1024 constellation from the plurality of symbol constellations in combination with an LDPC code rate that is equal to or less than 0.65086 enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 20.0 dB and 21.2 dB.

7. The digital communication system of claim 4, wherein the NU-QAM 1024 constellation is characterized in that selection of the NU-QAM 1024 constellation from the plurality of symbol constellations enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is an SNR between 19.2 dB and 21.4 dB.

8. The digital communication system of claim 4, wherein the NU-QAM 1024 constellation is characterized in that selection of the NU-QAM 1024 constellation from the plurality of symbol constellations enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 19.2 dB and 20 dB.

9. The digital communication system of claim 4, wherein the NU-QAM 1024 constellation is characterized in that selection of the NU-QAM 1024 constellation from the plurality of symbol constellations enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver of 19.2 dB.

10. The digital communication system of claim 7, wherein the transmitter is configured to select the NU-QAM 1024 constellation in combination with an LDPC code rate that is equal to or less than 0.65086.

11. The communication system of claim 4, wherein the transmitter is capable of selecting an LDPC code rate and the NU-QAM 1024 symbol constellation from the plurality of symbol constellations as a pair from a plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

12. The communication system of claim 11, wherein each of the plurality of non-uniform symbol constellations is only included in one of the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

13. The communication system of claim 12 wherein:

the transmitter is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate greater than 0.65634 and less than or equal to 0.68982; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 21.2 dB and 21.4 dB.

14. The communication system of claim 12, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate greater than or equal to 0.66346 and less than or equal to 0.67046; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is greater than or equal to 20.6 dB and less than 21.6 dB.

15. The communication system of claim 12, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate that is greater than or equal to 0.26724 and less than or equal to 0.27738; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is greater than or equal to 8.2 dB and less than 8.8 dB.

16. The communication system of claim **12**, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.46506 and 0.5106; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 14.2 dB and 15.6 dB.

17. The communication system of claim **12**, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.52978 and 0.57418; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 16.2 dB and 17.6 dB.

18. The communication system of claim **12**, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.59946 and 0.63782; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 18.4 dB and 19.6 dB.

19. The communication system of claim **12**, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.66394 and 0.70256; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 20.4 dB and 21.6 dB.

20. The communication system of claim **12**, wherein the transmitter is configured to select:

an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.75838 and 0.78238; and

the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair is characterized in that selection of the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair enables a receiver to decode the transmitted signals when the communication channel has a signal-to-noise ratio (SNR) at the receiver that is between 23.4 dB and 24.2 dB.

21. The communication system of claim **4**, wherein the plurality of symbol constellations includes multiple different sixty-four-point non-uniform symbol constellations, multiple different two-hundred-fifty-six-point non-uniform symbol constellations, and multiple different one-thousand-twenty-four-point non-uniform symbol constellations.

22. The digital communication system of claim **1**, wherein the symbols in the NU-QAM 1024 are labelled using Gray labels.

23. The digital communication system of claim **1**, wherein the symbols in the NU-QAM 1024 are labelled using binary reflective Gray labels.

24. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a non-uniform quadrature amplitude modulation 1024-point symbol constellation (NU-QAM 1024); and

a decoder, coupled to the demapper, capable of using likelihoods determined by the demapper to provide a sequence of received bits based upon a Low Density Parity Check (LDPC) code;

wherein the NU-QAM 1024 constellation comprises an in-phase component and a quadrature component, where each component comprises 32 levels of amplitude such that the amplitudes scaled by a scaling factor are within 0.55 from the following set of amplitudes: −38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

25. The digital communication system of claim **24**, where each of the in-phase and quadrature components of the NU-QAM 1024 constellation comprises 32 levels of amplitude such that the amplitudes scaled by the scaling factor are from the following set of amplitudes: −38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

26. The digital communication system of claim **24**, wherein the LDPC code rate has a code rate that is equal to or less than 0.65086.

27. The digital communication system of claim **24**, wherein the receiver is configured to select the NU-QAM 1024 constellation from a plurality of symbol constellations.

28. The digital communication system of claim 27, wherein the NU-QAM 1024 constellation is characterized in that the NU-QAM 1024 constellation provides greater parallel decode capacity at a specific signal-to-noise ratio (SNR) compared to a QAM 1024 constellation that maximizes d_{min} at the specific SNR.

29. The digital communication system of claim 27, wherein the receiver is configured to select the NU-QAM 1024 constellation from the plurality of symbol constellations in combination with an LDPC code rate that is equal to or less than 0.65086 and the receiver is capable of decoding the signals received via the communication channel using the LDPC code rate and the NU-QAM 1024 symbol constellation when the communication channel SNR is between 20.0 dB and 21.2 dB.

30. The digital communication system of claim 27, wherein the receiver is capable of decoding the signals received via the communication channel using the NU-QAM 1024 constellation when the communication channel SNR is between 19.2 dB and 21.4 dB.

31. The digital communication system of claim 27, wherein the receiver is capable of decoding the signals received via the communication channel using the NU-QAM 1024 constellation when the communication channel SNR is between 19.2 dB and 20 dB.

32. The digital communication system of claim 27, wherein the receiver is capable of decoding the signals received via the communication channel using the NU-QAM 1024 constellation when the communication channel SNR is 19.2 dB.

33. The digital communication system of claim 27, wherein the receiver is configured to select the NU-QAM 1024 constellation in combination with an LDPC code rate that is equal to or less than 0.65086.

34. The communication system of claim 27, wherein the receiver is capable of selecting an LDPC code rate and the NU-QAM 1024 symbol constellation from the plurality of symbol constellations as a pair from a plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

35. The communication system of claim 34, wherein each of the plurality of non-uniform symbol constellations is only included in one of the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

36. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate greater than 0.65634 and less than or equal to 0.68982; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 21.2 dB and 21.4 dB.

37. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate greater than or equal to 0.66346 and less than or equal to 0.67046; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is greater than or equal to 20.6 dB and less than 21.6 dB.

38. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate that is greater than or equal to 0.26724 and less than or equal to 0.27738; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is greater than or equal to 8.2 dB and less than 8.8 dB.

39. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.46506 and 0.5106; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 14.2 dB and 15.6 dB.

40. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.52978 and 0.57418; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 16.2 dB and 17.6 dB.

41. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.59946 and 0.63782; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 18.4 dB and 19.6 dB.

42. The communication system of claim 35, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.66394 and 0.70256; and the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 20.4 dB and 21.6 dB.

43. The communication system of claim **35**, wherein:

the receiver is configured to select an alternative LDPC code rate and NU-QAM 1024 symbol constellation pair from the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs, where the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair includes an LDPC code rate between 0.75838 and 0.78238; and

the receiver is capable of decoding the signals received via the communication channel using the alternative LDPC code rate and NU-QAM 1024 symbol constellation pair when the communication channel SNR is between 23.4 dB and 24.2 dB.

44. The communication system of claim **27**, wherein the plurality of symbol constellations includes multiple different sixty-four-point non-uniform symbol constellations, multiple different two-hundred-fifty-six-point non-uniform symbol constellations, and multiple different one-thousand-twenty-four-point non-uniform symbol constellations.

45. The digital communication system of claim **24**, wherein the symbols in the NU-QAM 1024 are labelled using Gray labels.

46. The digital communication system of claim **24**, wherein the symbols in the NU-QAM 1024 are labelled using binary reflective Gray labels.

47. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter comprises:

a coder capable of receiving bits and outputting encoded bits using a Low Density Parity Check (LDPC) code;

a mapper, coupled to the coder, capable of mapping the encoded bits to symbols in a non-uniform quadrature amplitude modulation 1024-point symbol constellation (NU-QAM 1024); and

a modulator, coupled to the mapper, capable of producing a signal for transmission via the communication channel based upon symbols selected by the mapper; and

a receiver capable of receiving signals via the communication channel at a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the NU-QAM 1024; and

a decoder, coupled to the demapper, capable of using likelihoods determined by the demapper to provide a sequence of received bits based upon the LDPC;

wherein the NU-QAM 1024 constellation comprises an in-phase component and a quadrature component, where each component comprises 32 different of amplitude such that the amplitudes scaled by a scaling factor are within 0.55 from the following set of amplitudes: −38.424, −31.907, −24.169, −26.796, 38.425, 31.908, −20.038, −19.169, −7.759, −7.759, −11.460, −11.460, −4.850, −4.850, −15.014, −15.205, 20.038, 19.170, 15.206, 15.015, 24.170, 26.797, 11.460, 11.460, 1.326, 1.326, 4.849, 4.849, −1.328, −1.328, 7.759, and 7.759.

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CERTIFICATE OF SERVICE AND FILING

I certify that on August 19, 2024, I electronically filed the foregoing **NON-CONFIDENTIAL OPENING BRIEF** of Defendants-Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc. using the Court's CM/ECF filing system. Counsel for Plaintiff-Appellee Constellation Designs, LLC were electronically served by and through the Court's CM/ECF filing system per Fed. R. App. P. 25 and Fed. Cir. R. 25(e).

/s/ Michael J. McKeon
Michael J. McKeon

CERTIFICATE OF COMPLIANCE

The **NON-CONFIDENTIAL OPENING BRIEF** of Defendants-Appellants LG Electronics Inc., LG Electronics U.S.A., Inc., and LG Electronics Alabama, Inc. is submitted in accordance with the type-volume limitation of Fed. Cir. R. 32(b). The brief contains 13,993 words, excluding the parts of the brief exempted by Fed. R. App. P. 32(f) and Fed. Cir. R. 32(b)(2). This brief has been prepared in a proportionally spaced typeface using Microsoft® Word for Microsoft 365 in Times New Roman, 14 Point.

Dated: August 19, 2024

/s/ Michael J. McKeon
Michael J. McKeon

CERTIFICATE OF COMPLIANCE WITH CONFIDENTIALITY REQUIREMENTS

The **NON-CONFIDENTIAL OPENING BRIEF** of Defendants-Appellants contains one image marked confidential. This number does not exceed the maximum of 15 words permitted by Fed. Cir. R. 25.1(d)(1)(A).

Dated: August 19, 2024

/s/ Michael J. McKeon
Michael J. McKeon